

Linear Temporal Logics and Grammars

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Overview – Word Systems

Regular expressiveness

Linear temporal logic

ν TL, QPTL, ETL, ...

Büchi-automata

over infinite words

Right-linear grammars

over infinite words

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 ν TL, QPTL, ETL, ...

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over infinite words

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Beyond context-free expressiveness

Linear temporal logic
+
chop/concatenation
LFLC

Alternating
context-free grammars
over finite/infinite words

Temporal Logic

Linear-time temporal logic with chop (LF_{LC}):

- propositional constants p, q, \dots
- special “empty” proposition ε
- connectives \vee, \wedge
- concatenation $;$
- fixed-point variables X, Y, \dots
- fixed-point operators μ, ν

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p, q, \dots

ε

\vee, \wedge

;

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Temporal Logic

Linear-time temporal logic with chop (LF_{LC}):

- propositional variables
- special “empty” proposition
- connectives
- concatenation
- fixed-point variables
- fixed-point operators

$$p \equiv \{\neg a, \neg b\}$$

$$q \equiv \{\neg a, b\}$$

$$r \equiv \{a, \neg b\}$$

$$s \equiv \{a, b\}$$

p, q, \dots

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Models:

p
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$M \models p$

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Models:

$$M \models p; q; p; q$$



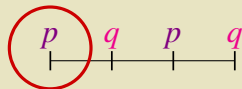
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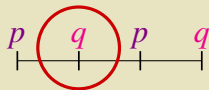
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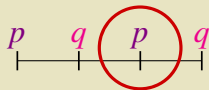
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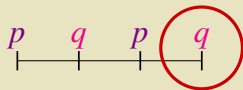
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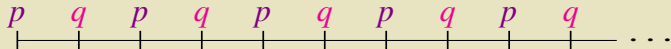
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Models:

$$M \models \nu X.(p; q; X)$$



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Models: $M \models p \vee p; q; p; q \vee \nu X.(p; q; X)$

p

$p \quad q \quad p \quad q$

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Grammars

Alternating Context-Free Grammar (ACFG):

- terminals $p, q, \dots \in \Sigma$
- non-terminals $X, Y, \dots \in N$
- production rules $N \rightarrow (N \cup \Sigma)^*$
- designated initial symbol $S \in N$
- alternation function $\lambda : N \rightarrow \{\forall, \exists\}$
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- terminals
- non-terminals
- production rules
- designated start symbol
- alternating production rules
- parity function

$$X \rightarrow pq$$

$$X \rightarrow pYq$$

$$X \rightarrow \varepsilon$$

$$p, q, \dots \in \Sigma$$

$$X, Y, \dots \in N$$

$$N \rightarrow (N \cup \Sigma)^*$$

$$S \in N$$

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Languages:

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$$\mathcal{L} \left(\begin{array}{l} S \rightarrow p \\ \Omega(S) = 0 \end{array} \right)$$

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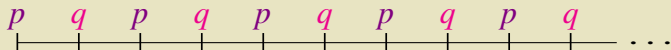
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Relationship of LFLC and ACFGs

$$\mu X.(a; X \vee b)$$
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$$\mathcal{L} \left(\begin{array}{l} S \rightarrow aS \mid b \\ \Omega(S) = 1 \end{array} \right)$$
$$S$$

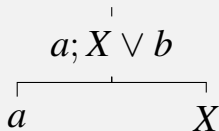
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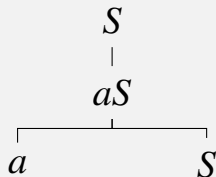
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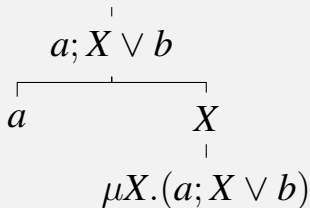
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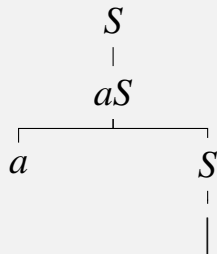
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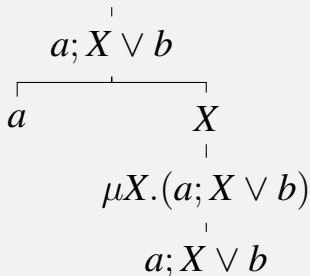
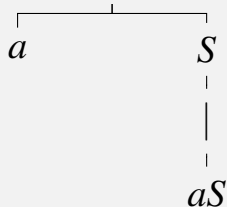
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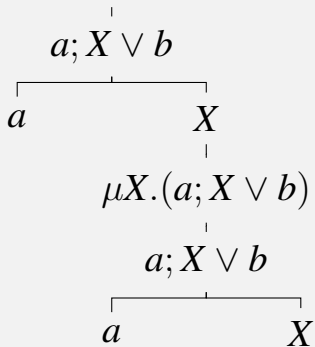
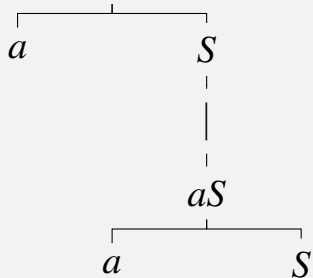
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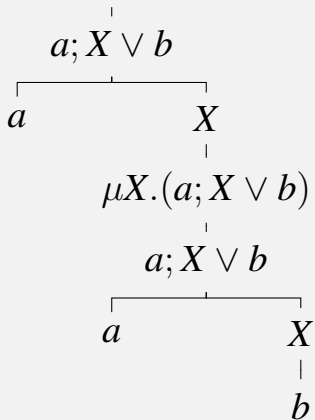
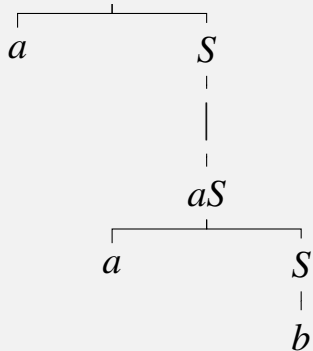
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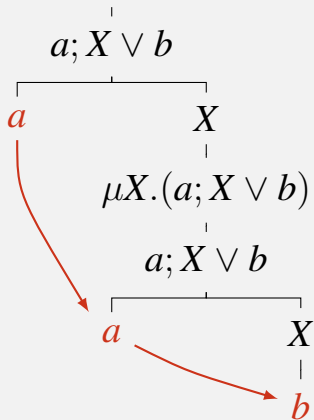
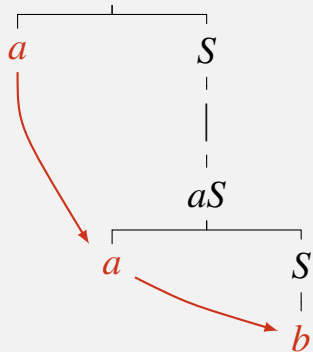
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 S
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Expressiveness of LFLC and ACFGs

- beyond context-free expressiveness

$$\mathcal{L} = \{a^n b^n c^n \mid n \geq 0\}$$

- satisfiability is undecidable

$\mathcal{L} = \mathcal{L}_1 \cap \mathcal{L}_2$, where $\mathcal{L}_1, \mathcal{L}_2$ are context-free
due to

$$\varphi = \varphi_1 \wedge \varphi_2, \varphi_1, \varphi_2 \text{ are context-free}$$

- model-checking of finite words and ultimately periodic infinite words is decidable

$$w \models \varphi \text{ or } w \in \mathcal{L}(G)$$

ε is Obsolete

As long as $\varepsilon \not\models \varphi$:

- φ can be rewritten as an ε -free formula φ'
- since $\varepsilon \stackrel{?}{\models} \varphi$ is decidable:
 - φ becomes either $\varphi' = \varphi$ or $\varphi' = \varphi \vee \varepsilon$
 - new initial symbol S' , such that either $S' \rightarrow S$ or $S' \rightarrow S \mid \varepsilon, \lambda(S) = \exists$

\Rightarrow LFLC's syntax/semantics is not concise

Alternation Hierarchy

- fixed-point alternation:

φ	$\mathcal{L}(\varphi)$	depth(φ)
$\nu X.(\mu Y.(\varepsilon \vee b; Y) \vee b; X)$	$(a^*b)^\omega$	0
$\nu X.\mu Y.(a; Y \vee b; X)$	$(a^*b)^\omega$	1

- private communication with Martin Lange:
 - Logic's fixed-point alternation:
hierarchy collapses at level 0
 - Grammar's acceptance condition:
parity acceptance is equiv. to weak parity acc.
c.f. his submitted "Specifying Non-Regular Properties of Runs"

Future Work

- true expressiveness of LFLC/ACFGs
 - proper inclusion in context-sensitive languages
- model-checking decidability reconsidered
 - non-periodic infinite words
- decidable model-checking fragments
 - extending $w \models \varphi$ to regular- $\mathcal{L} \models \varphi$