LTL with the Freeze Quantifier and Register Automata

Stéphane Demri
CNRS & ENS Cachan & INRIA Futurs

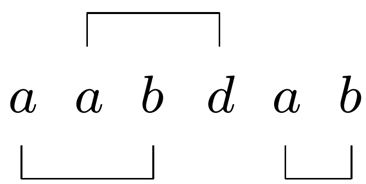
Ranko Lazić
University of Warwick

Data words

Finite alphabet & infinite domain:
XML documents, Timed words, . . .

$$\begin{array}{cccccc} a & a & b & d & a & b \\ URL_1 & URL_2 & URL_1 & URL_2 & URL_3 & URL_3 \end{array}$$

$$\begin{array}{cccccc} a & a & b & d & a & b \\ 3 & 2.5 & 3 & 2.5 & 4 & 4 \end{array}$$



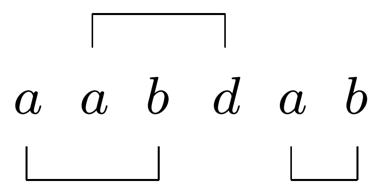
$\mathbf{LTL}_n^{\downarrow}$

$$\begin{aligned}\phi \; ::= \;& \top \mid a \mid \uparrow_r \sim \mid \\ & \neg\phi \mid \phi \wedge \phi \mid \\ & \mathtt{O}(\phi, \ldots, \phi) \mid \\ & \downarrow_r \phi\end{aligned}$$

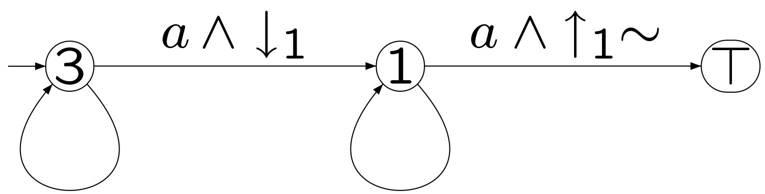
$$\begin{aligned} r &\in \{1, \ldots, n\} \\ \mathtt{O} &\in \{\mathtt{X}, \mathtt{X}^{-1}, \mathtt{F}, \mathtt{F}^{-1}, \mathtt{U}, \mathtt{U}^{-1}, \ldots\}\end{aligned}$$

Example: nonces

$$\mathtt{F}(a \wedge {\downarrow_1} \, \mathtt{XF}(a \wedge {\uparrow_1}{\sim}))$$



Register Automata



1-way or **2**-way, **N**ondeterministic or **A**lternating, n registers

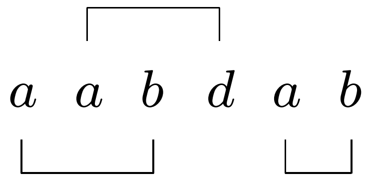
Over infinite data words: *weak parity* acceptance.
Special case of *Büchi* and *co-Büchi* acceptance.

Register automata. [Kaminski & Francez, TCS '94]

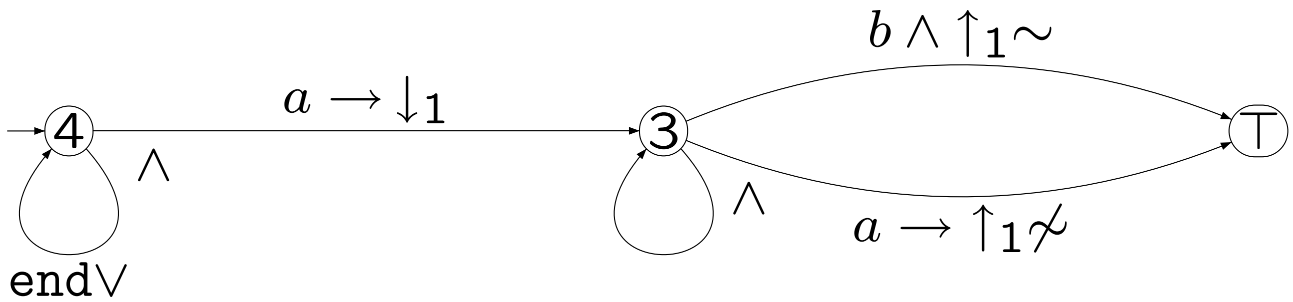
Pebble automata. [Neven, Schwentick & Vianu, ACM ToCL '04]

Timed automata. [Alur & Dill, TCS '94]

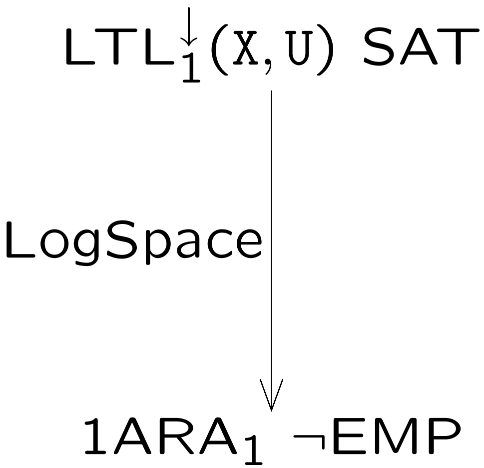
Data automata. [Bouyer, Petit & Thérien, IC '03]



$$\mathtt{G}\,(a \rightarrow {\downarrow_1}\,\mathtt{X}((a \rightarrow \neg{\uparrow_1}{\sim})\,\mathtt{U}\,(b \wedge {\uparrow_1}{\sim})))$$

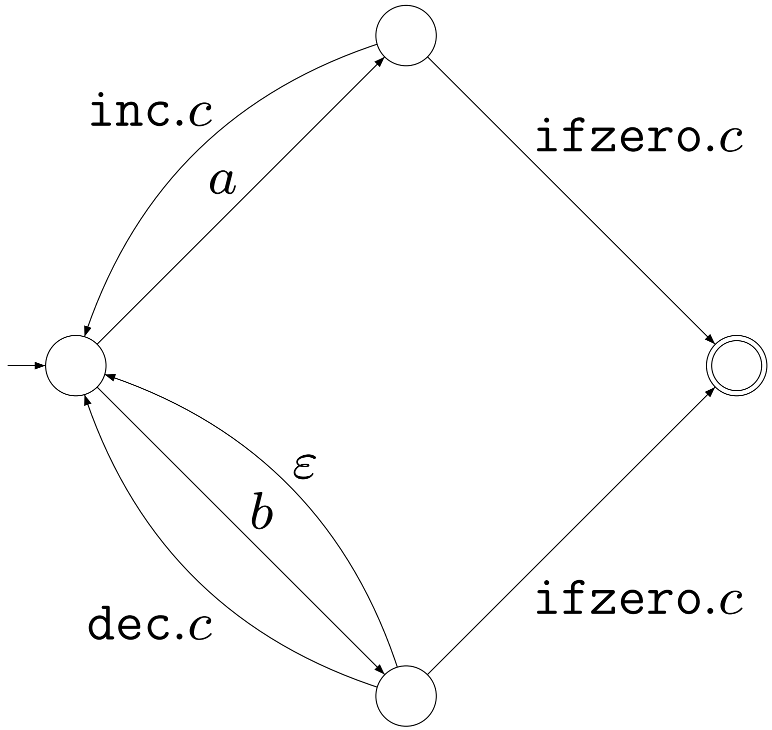


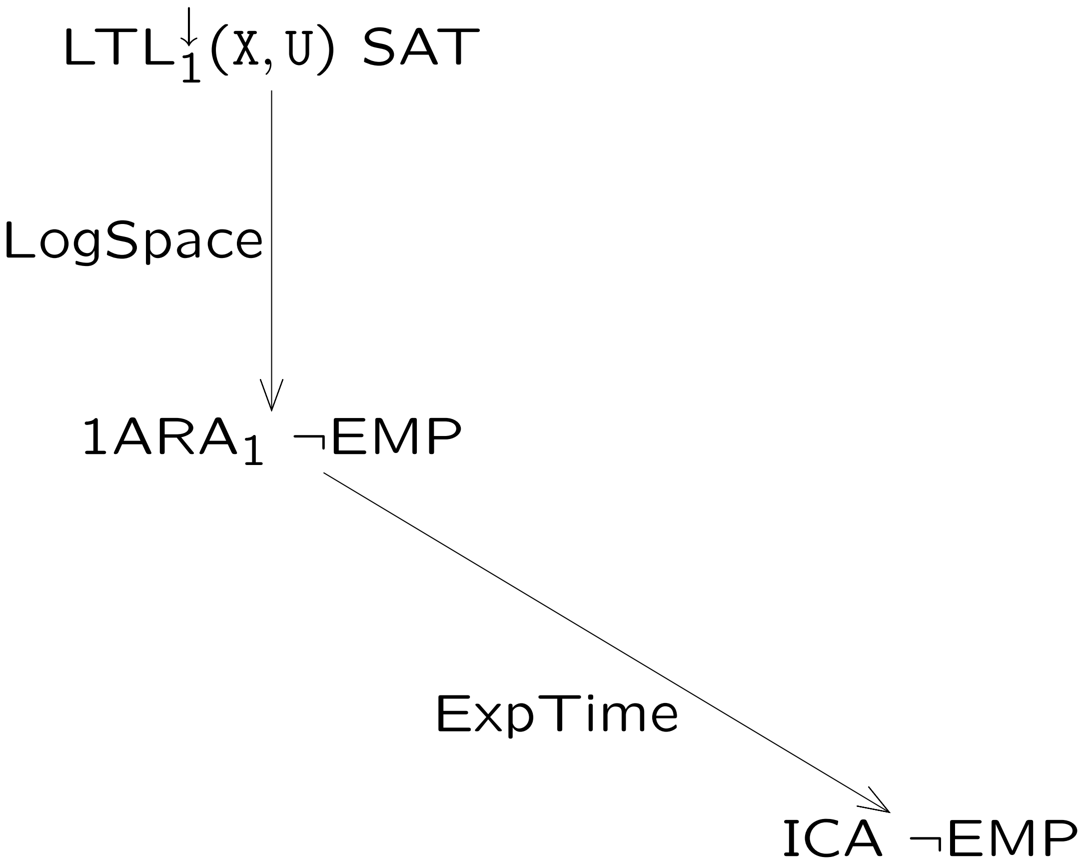
Theorem. $\mathsf{LTL}^{\downarrow}_{n}(\mathtt{X}, \mathtt{X}^{-1}, \mathtt{U}, \mathtt{U}^{-1}) \xrightarrow{\mathsf{LogSpace}} \mathsf{2ARA}_{n}$.



LTL$_1^{\downarrow}$(X, U) SAT
LogSpace
1ARA$_1$ ¬EMP

Incrementation Counter Automata





$\mathsf{LTL}_1^{\downarrow}(\mathtt{X},\mathtt{U})$ SAT
LogSpace
$1\mathsf{ARA}_1$ $\neg$EMP
ExpTime
ICA $\neg$EMP

Proof:

1. Quotient runs by $\sim$.

2. Represent levels and steps using counters.

3. For infinite data words, use [Miyano & Hayashi, TCS '84]:
   weak parity alternating $\mapsto$ Büchi nondeterministic.

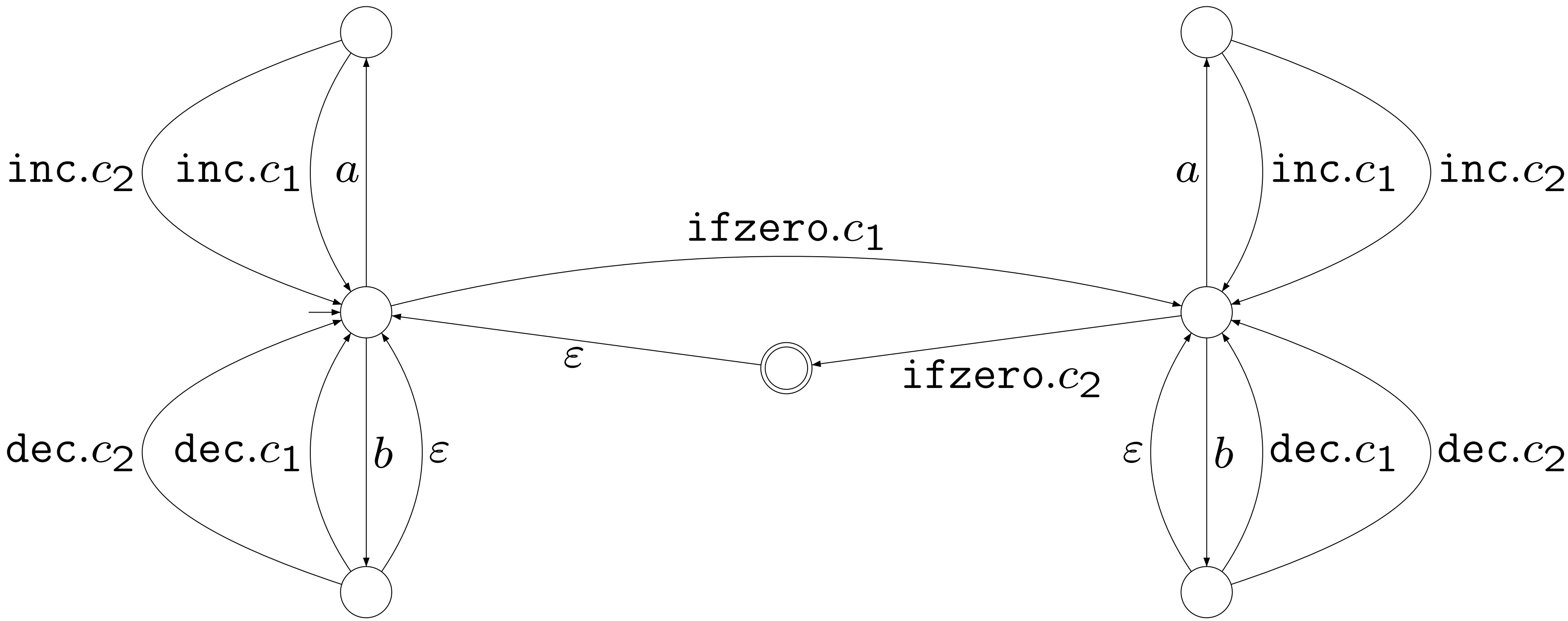
4. Incrementation errors cannot cause a false acceptance.

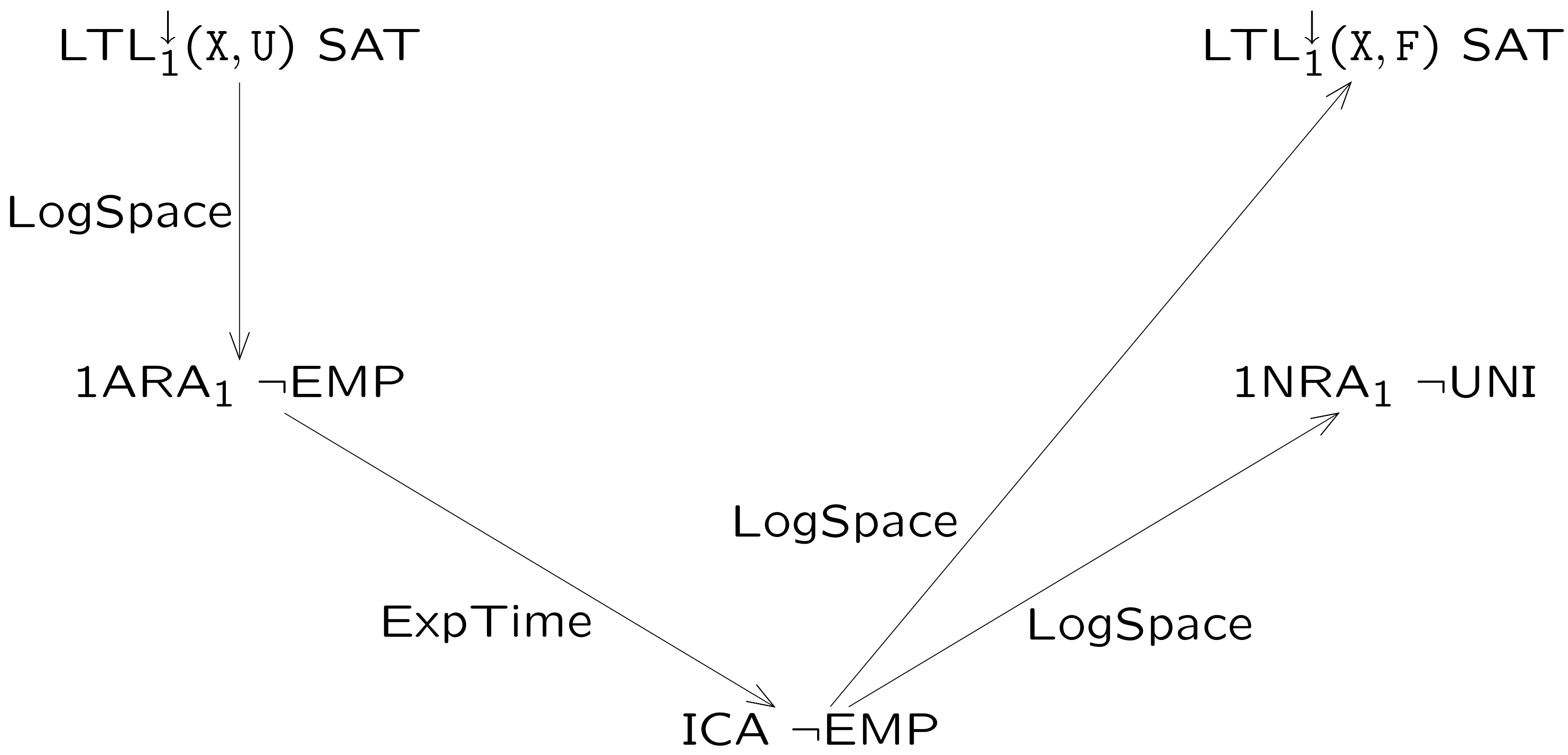
We should accept:

$$a \quad a \quad b \quad a \quad b \quad a \quad b \quad a \quad b \quad \cdots$$

but reject:

$$a \quad a \quad b \quad a \quad b \quad a \quad a \quad a \quad a \quad \cdots$$


inc.c_2
inc.c_1
a
ifzero.c_1
a
inc.c_1
inc.c_2
ε
ifzero.c_2
dec.c_2
dec.c_1
b
ε
ε
b
dec.c_1
dec.c_2

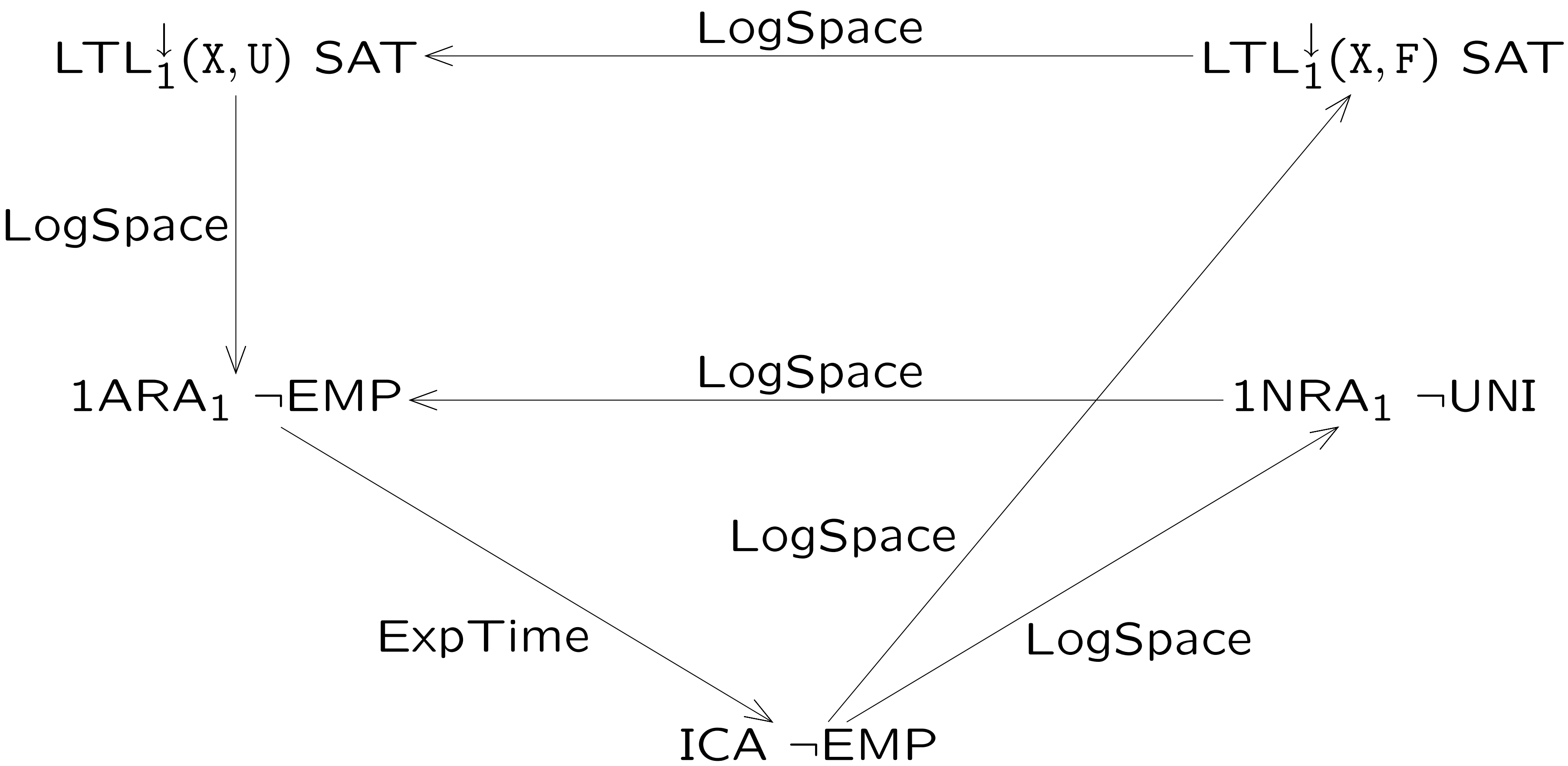


$LTL_1^{\downarrow}(\mathtt{X}, \mathtt{U})$ SAT
$LTL_1^{\downarrow}(\mathtt{X}, \mathtt{F})$ SAT
LogSpace
$1ARA_1$ ¬EMP
$1NRA_1$ ¬UNI
LogSpace
ExpTime
LogSpace
ICA ¬EMP

Proof:

Encode computations of ICA as data words.

`inc.`c `inc.`c `dec.`c `inc.`c `dec.`c `dec.`c `iszero.`c



$LTL^{\downarrow}_1(\mathtt{X}, \mathtt{U})$ SAT
LogSpace
$LTL^{\downarrow}_1(\mathtt{X}, \mathtt{F})$ SAT
LogSpace
$1ARA_1$ $\neg$EMP
LogSpace
$1NRA_1$ $\neg$UNI
LogSpace
ExpTime
LogSpace
ICA $\neg$EMP

Theorem.

	Minsky CA	Incrementation CA
$\neg\mathrm{EMP}^{<\omega}$	Σ^0_1-complete	R **(a)**, not PR **(b)**
$\neg\mathrm{EMP}^{\omega}$	Σ^1_1-complete	Π^0_1-complete **(c)**

Proof:

(a) Reverse computations, and obtain Reset Petri net Coverability [Dufourd, Finkel & Schnoebelen, ICALP '98].

(b) Reverse computations, and adapt [Schnoebelen, IPL '02]: Reachability is not PR for Lossy Channel Systems.

(c) Adapt [Ouaknine & Worrell, FoSSaCS '06]: Recurrent State for Insertion Channel Mach. with Empt. Testing.

[Kaminski & Francez, TCS '94]:

'. . . it is very likely that the results of this paper can be extended to infinite data words.'

[French, TIME '03],
[Demri, Lazić & Nowak, TIME '05],
[Lisitsa & Potapov, TIME '05]:

	$\mathsf{SAT}^{<\omega}$		SAT^{ω}	
Registers	1	2	1	2
$\mathsf{LTL}^{\downarrow}(\mathtt{X}, \mathtt{F})$				
$\mathsf{LTL}^{\downarrow}(\mathtt{X}, \mathtt{U})$		Σ^0_1-comp.		Σ^1_1-comp.
$\mathsf{LTL}^{\downarrow}(\mathtt{X}, \mathtt{F}, \mathtt{F}^{-1})$				Σ^1_1-comp.

$1NRA_1(\sim)$ $\neg UNI^{\omega}$ is Π^0_1-complete!

Registers	$SAT^{<\omega}$ 1	2	SAT^{ω} 1	2
$LTL^{\downarrow}(\mathtt{X},\mathtt{F})$	$R \setminus PR$	Σ^0_1-comp.	Π^0_1-comp.	Σ^1_1-comp.
$LTL^{\downarrow}(\mathtt{X},\mathtt{U})$	$R \setminus PR$	Σ^0_1-comp.	Π^0_1-comp.	Σ^1_1-comp.
$LTL^{\downarrow}(\mathtt{X},\mathtt{F},\mathtt{F}^{-1})$	Σ^0_1-comp.	Σ^0_1-comp.	Σ^1_1-comp.	Σ^1_1-comp.

Theorem.

$$\overset{\text{Memory-1}}{\mathsf{LTL}_1^{\downarrow}(\mathtt{X},\mathtt{X}^{-1},\mathtt{F},\mathtt{F}^{-1})} \quad \underset{\overleftarrow{\text{2ExpTime}}}{\overrightarrow{\text{LogSpace}}} \quad \mathsf{FO}^2(\sim,<,+1)$$

[Etessami, Vardi & Wilke, IC '02]:
$\mathsf{LTL}(\mathtt{X},\mathtt{X}^{-1},\mathtt{F},\mathtt{F}^{-1})$ is equivalent to $\mathsf{FO}^2(<,+1)$.

Memory-1 $\mathrm{LTL}_1^{\downarrow}(\mathtt{X}, \mathtt{X}^{-1}, \mathtt{F}, \mathtt{F}^{-1})$

$$\exists x\,(a(x)\ \wedge\ \exists y\,(x < y\ \wedge\ a(y)\ \wedge\ x \sim y))$$

$$\mathtt{F}(a \wedge \downarrow_1 \mathtt{XF}(a \wedge \uparrow_1 \sim))$$

$$\mathtt{F}(a \wedge \downarrow_1 \mathtt{XF}(b \wedge \mathtt{XF}(a \wedge \uparrow_1 \sim)))$$