Future Trends in Hypercomputation

hypercomputation.net

Mike Stannett
Computer Science, Sheffield University

M. Stannett (2006) The Case for Hypercomputation. Applied Mathematics and Computation (to appear).

Introduction

- What is hypercomputation?
  - Non-recursive physical behaviours
- Get involved this September!
  - EPSRC workshop in Sheffield
  - Several student bursaries available

See `http://hypercomputation.net` from June 2006

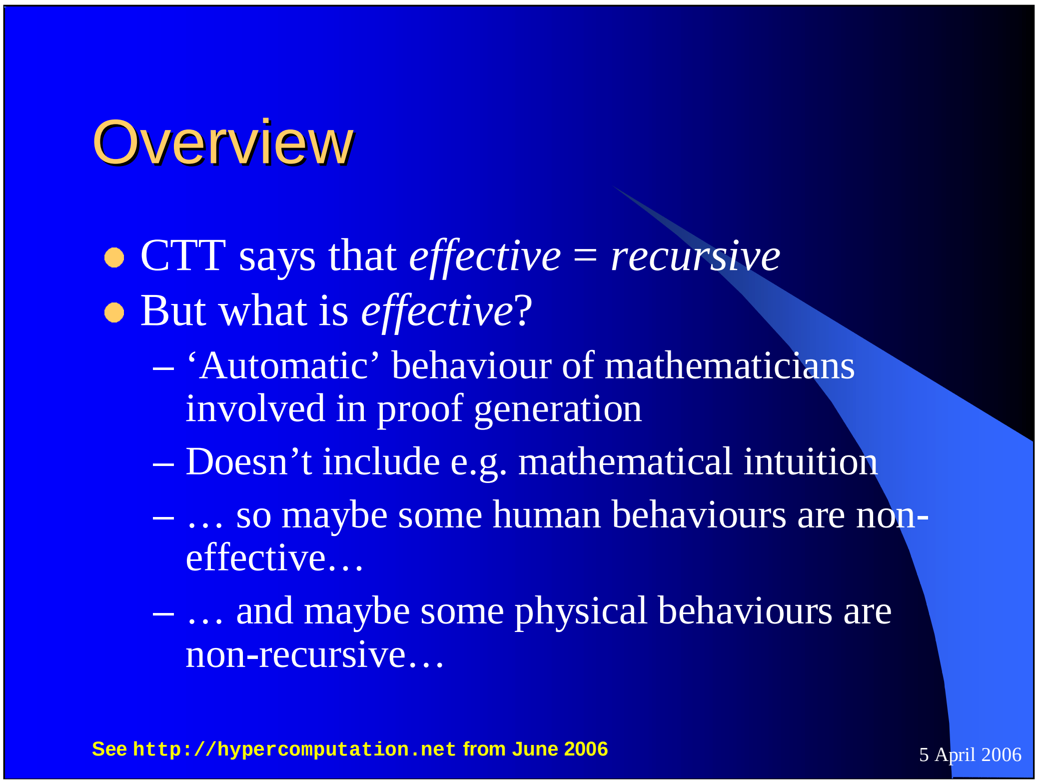
5 April 2006

Agenda

- Church-Turing Thesis
- Computer Science vs Physics
- Examples of hypercomputation
- Unresolved questions
- Workshop details

See `http://hypercomputation.net` from June 2006

5 April 2006



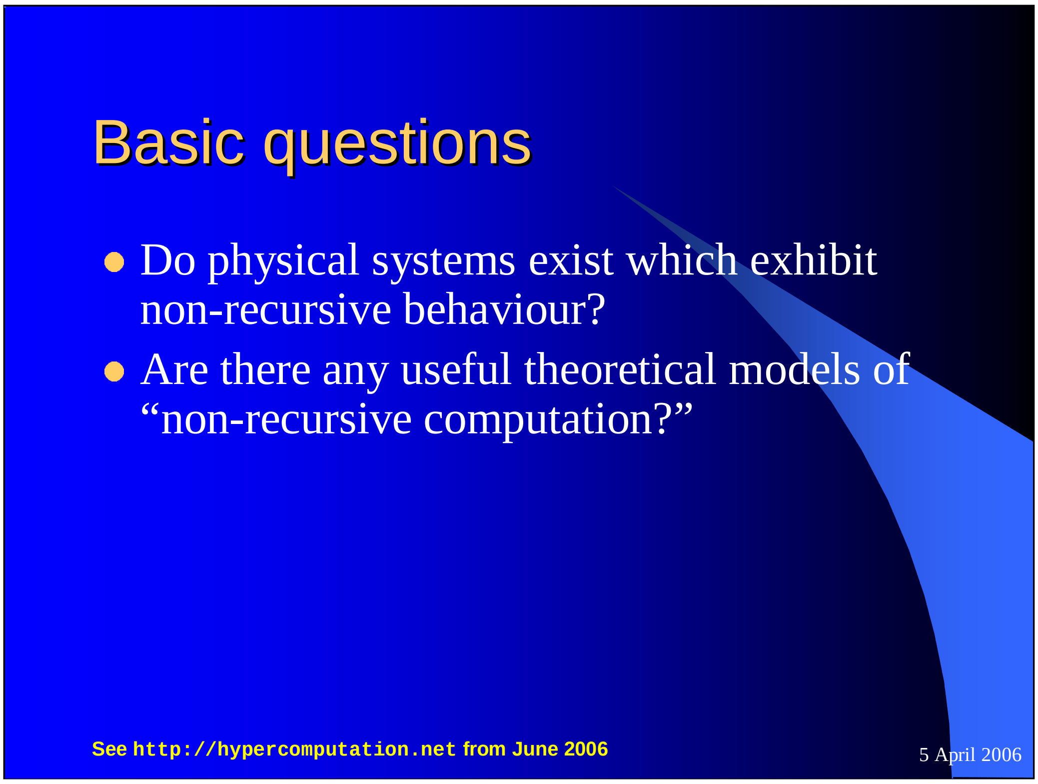
Turing wanted to model the way mathematicians behave when they prove things; his "machine" simulates the way we make notes, do calculations on scraps of paper, and so on. The result is a general representation of 'algorithmic mathematical behaviour' – the sort of stuff a person can be instructed to do when you hire them to work as a clerk. According to Copeland, who wrote Turing's Royal Society obituary, the mode, was not intended to represent intuitive thinking, just the basic `automatic' stuff we all do when we follow tried and trusted standard procedures.

Since Turing machines were never intended to represent *all* human behaviours, it's worth wondering if maybe some human behaviours simply *aren't* rote-like. Since humans are physical systems, it would follow that some physical behaviours are non-effective, and hence (by CTT) non-recursive. However, humans are hard to reason about formally, so it makes sense to think about other, more formal, scientific models.

A. Church, An Unsolvable Problem of Elementary Number Theory, Amer. J. Math. 58 (1936) 345–363.

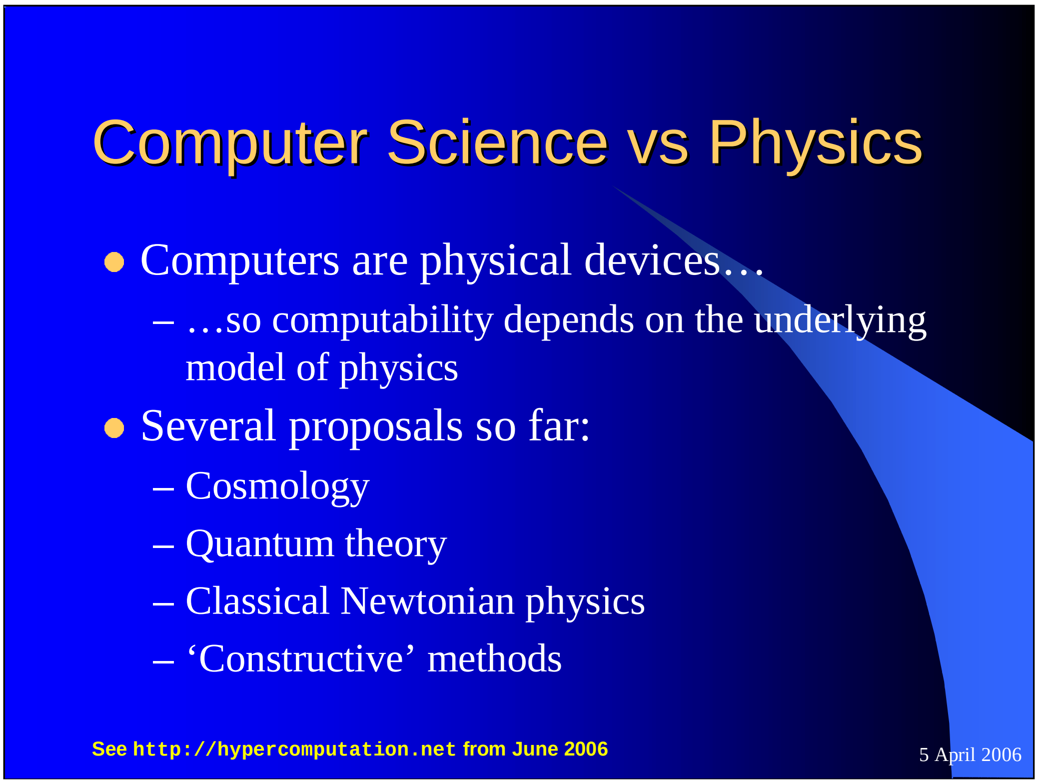
A. Turing, On Computable Numbers, with an Application to the Entscheidungsproblem, Proc. London Math. Soc., Series 2 42 (1936) 230–265, correction: Ibid. 43, 544-546.

M. Newman, Alan Mathison Turing 1912 - 1954, Biographical Memoirs



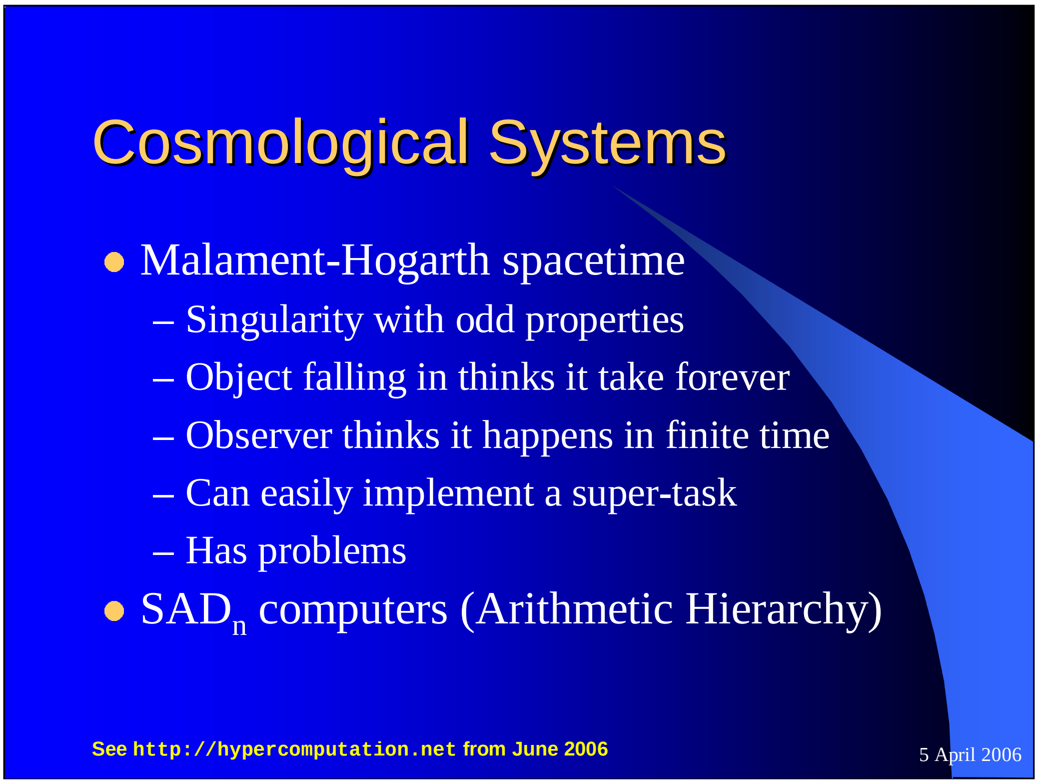
Hypercomputation Theory asks whether there exist (or could exist) physical systems whose behaviour cannot be simulated recursively. Such systems, if they exist, could potentially be used to launch a new IT industry based on super-Turing technologies. It nis not currently believed that "hypercomputers" will be developed any time soon – but maybe you'll be the person to prove everyone wrong!

Even if physics can't provide examples of hypercomputation (yet?), maybe it's still worthwhile asking the questions. They might prompt new ways of reasoning about computation in general, and hence provide new solutions to existing problems.



Theoretical Computer Science tends to reason about computers as if they were Turing machines, but (of course) they’re not. Real computers are physical devices operating in the real world and subject to physical laws. Consequently, we cannot say what is *actually* computable without first asking what physics itself looks like.

We’ll look very briefly at some models associated with cosmology, quantum theory, classical Newtonian physics, and old-fashioned `constructive’ DIY approaches.



This model is based on the properties of Malament-Hogarth space-time. Although the model is idealistic (and has definite problems) it has generated some interesting theoretical results. Hogarth has shown that "SAD(n)" computers (essentially those using n singularities) compute things at the n'th level of the Arithmetic Hierarchy.

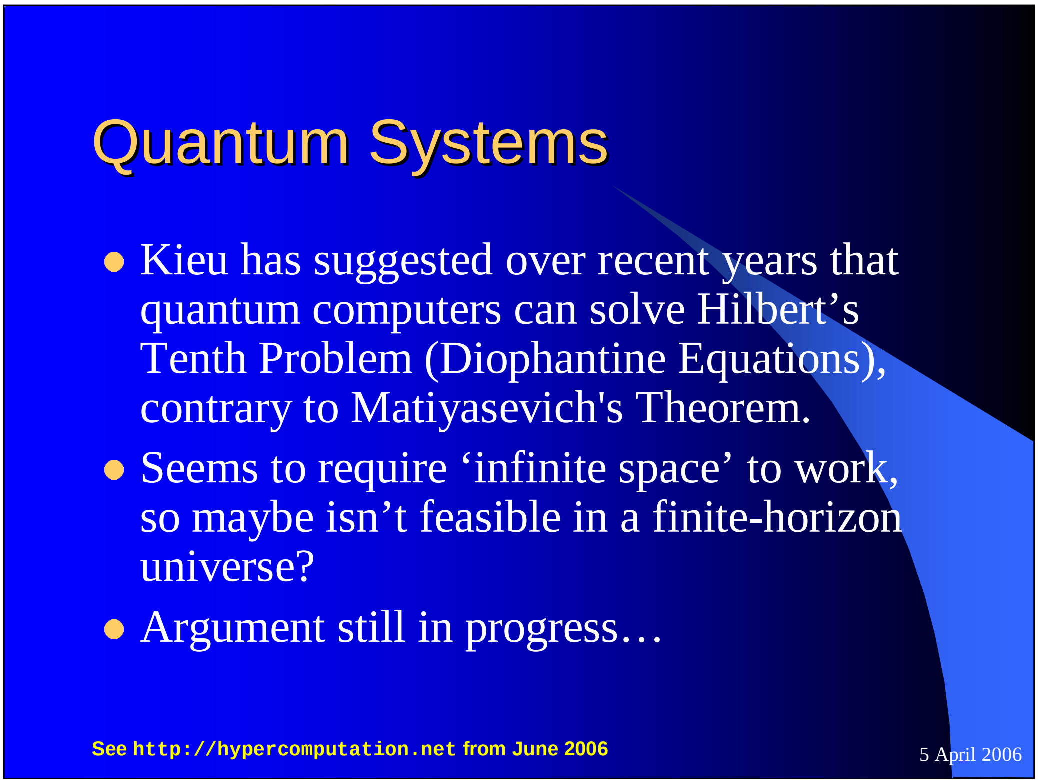
M. Hogarth, Deciding Arithmetic using SAD Computers, Brit. J. Phil. Sci. 55 (2004) 681–691.

M. Hogarth, Does General Relativity Allow an Observer to View an Eternity in a Finite Time, Foundations of Physics Letters 5 (1992) 73–81.

J. Earman, J. Norton, Forever is a Day: Supertasks in Pitowsky and Malament-Hogarth Spacetimes, Philosophy of Science 5 (1993) 22–42.

G. Etesi, I. Nemeti, Non-Turing Computability via Malament-Hogarth Space-Times, International Journal of Theoretical Physics 41 (2) (2002) 341–370.

J. Earman, Bangs, Crunches, Whimpers and Shrieks - Singularities and Acausalities in Relativistic Spacetimes, Oxford University Press, Oxford, 1995.



T. Kieu, Numerical simulations of a quantum algorithm for Hilbert’s tenth problem, in: E. Donkor, A. Pirich, H. Brandt (Eds.), Quantum Information and Computation, Vol. 1505 of Proc. SPIE, SPIE, Bellingham, WA, 2003, pp. 89–95.

T. Kieu, Quantum algorithm for Hilbert’s tenth problem., Internat. J. Theoret. Phys. 42 (2003) 1451–1468.

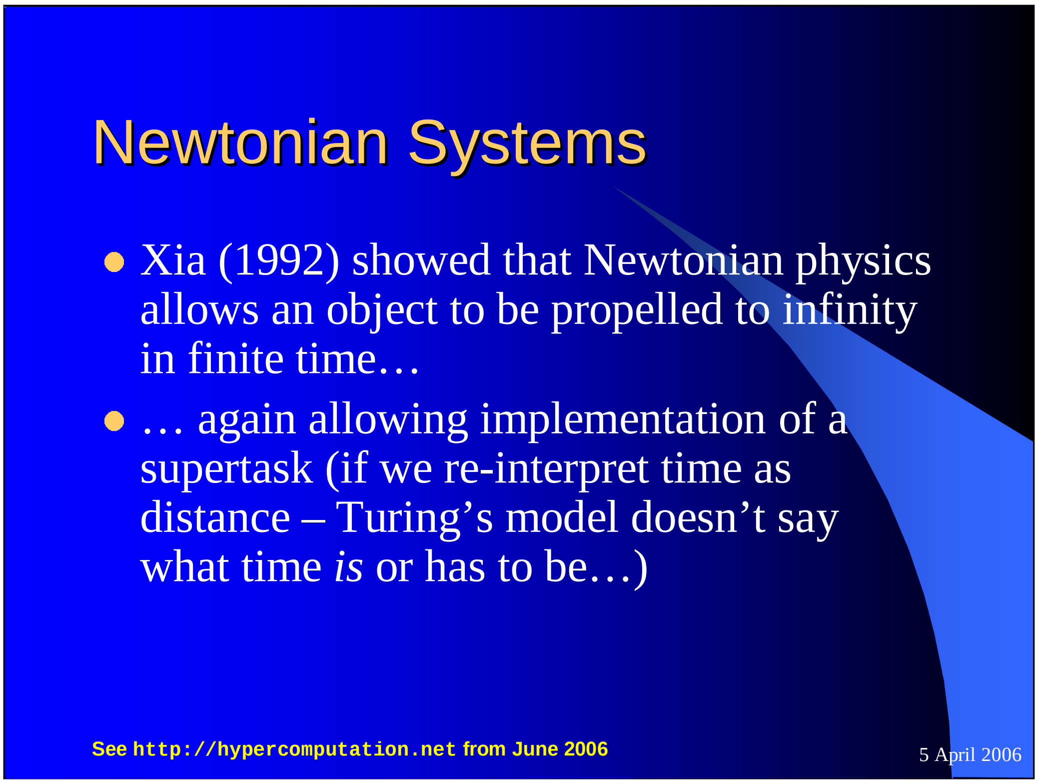
T. Kieu, Quantum adiabatic algorithm for Hilbert’s tenth problem: I. The Algorithm. Online: quant-ph/0310052.

T. Kieu, Hypercomputation with quantum adiabatic processes, Theoret. Computer Sci. 317 (2004) 93–104.

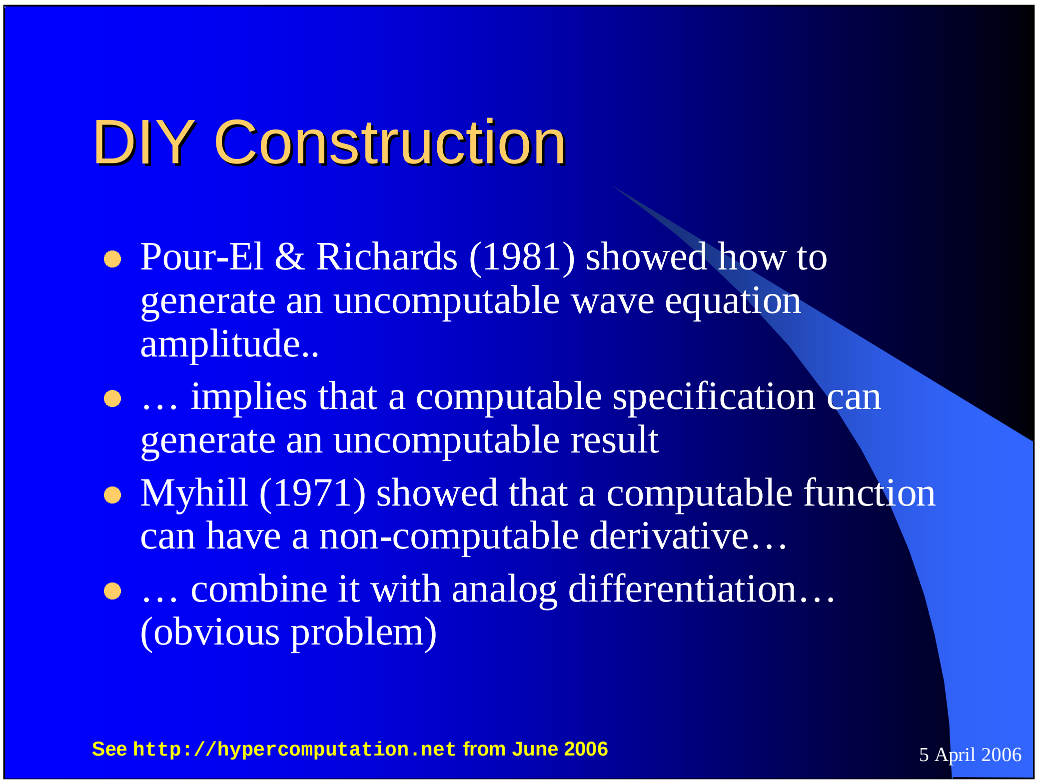
Y. Matiyasevich, Hilbert’s Tenth Problem, MIT Press, Cambridge, MA, 1993.

GENERAL QUANTUM COMPUTATION

D. Deutsch, Quantum theory, the Church-Turing principle, and the universal quantum Turing machine, Proc. Royal Soc. 400 (1985) 97–117.



Z. Xia. The existence of non-collision singularities in the n-body problem. Annals of Maths, 135(3):411–468, 1992.



J. Myhill, A recursive function, defined on a compact interval and having a continuous derivative that is not recursive, Michigan Math. J. 18 (1971) 97–98.

M. Pour-El, J. Richards, A Computable Ordinary Differential Equation Which Possesses No Computable Solution, Annals of Mathematical Logic 17 (1979) 61–90.

M. Pour-El, J. Richards, The Wave Equation with Computable Initial Data such that its Unique Solution is not Computable, Advances in Mathematics 39 (1981) 215–239.

OVERVIEW

M. Stannett, Computation and Hypercomputation, Minds and Machines 13 (1) (2003) 115–153.

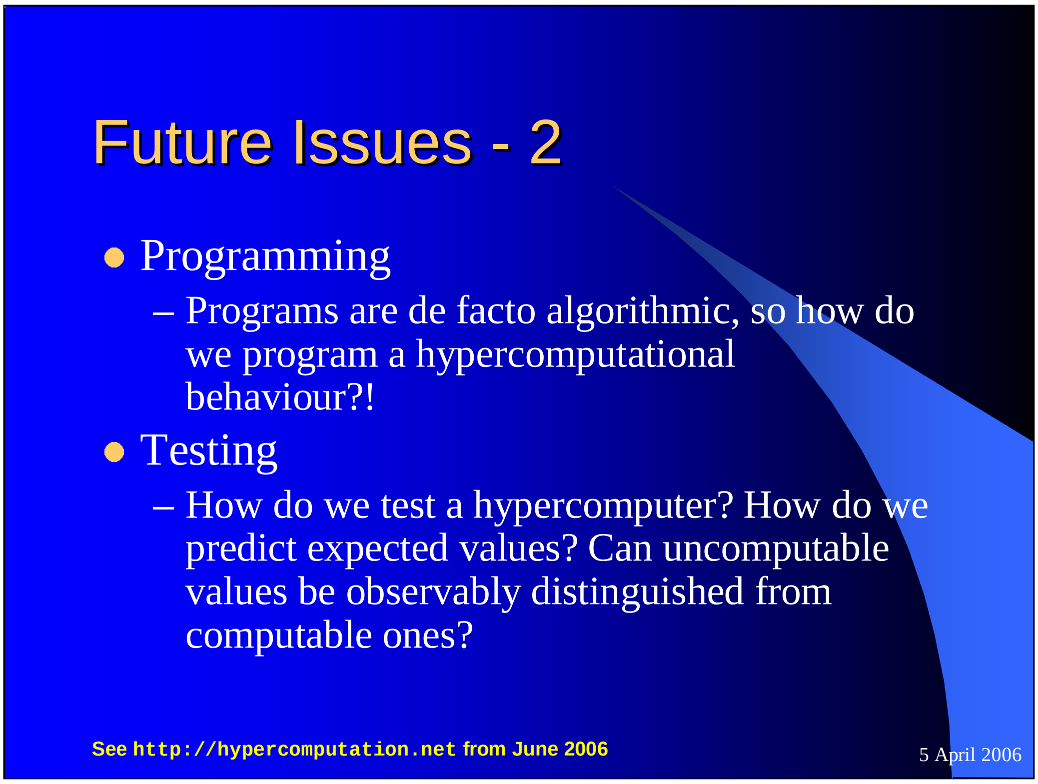
Future Issues - 1

- Realistic physical context
  - Newtonian physics wrong, so can't use it…
  - Quantum and cosmological models seem to require each other's falsity.
  - Is there a combined-model solution?
- Specification
  - Pour-El & Richards work shows we can specify hypercomputers computably. Is there a sensible theory of such specifications? How does refinement work? What should formal representations look like?

See `http://hypercomputation.net` from June 2006

5 April 2006

Kieu's idea seems to require an infinite observable universe; is this allowed by general relativity? Hogarth's cosmological model seems to require absolute determinism, so can't work in quantum universe. The real world appears to include both general relativistic AND quantum theoretical features – can we find a hypercomputational model that works when BOTH are present?



M. Stannett (2006) Programming for Hypercomputation. (submitted to Unconventional Computation 2006, York).

M. Stannett (2006) The Case for Hypercomputation. Applied Mathematics and Computation (to appear).

Summary

- Nature may be more powerful than computation…
  - … but the question is open…
- If hypercomputation is feasible…
  - How do we exploit it?
  - How do we represent it?
  - How do we control it?

See `http://hypercomputation.net` from June 2006

5 April 2006

Where to Get More Information

- Notes attached in this PowerPoint file
- EPSRC workshop in Sheffield this September
- Follows the Unconventional Computation 2006 meeting in York
- Significant student bursaries available (for both)
- Limited number of places (for both)
- See http://www.cs.york.ac.uk/nature/uc06 now
- See http://hypercomputation.net (from June 2006)

See http://hypercomputation.net from June 2006

5 April 2006