

WEAPONS OF MATHS CONSTRUCTION

NEW CALCULATORS FOR THE 21ST CENTURY

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KEYWORDS

- New user interfaces
- Easy maths
- Calculators
- Gesture recognition

Imagine writing a calculation down on paper and the paper magically working out the answers. This exhibit is about a calculator that works like this, which is ideal for pen-based computers and interactive whiteboards in classrooms.

We have always used instruments to aid our mental arithmetic and to help us with mathematics. The abacus dates from around 3000BC and the slide rule from 1650. The first successful four-function calculator was invented in the 1820s, and calculators grew steadily more sophisticated, reaching a pinnacle in the 1870s with the work of Charles Babbage. With the development of electronics, a whole century later, by the 1970s calculators became popular and affordable consumer products.

Today, over thirty years later, when desktop and handheld computers can do almost anything, today's calculators merely imitate the designs of the early, rather crude electronic forebears.

Everybody takes handheld calculators for granted, yet they are harder to use and more unreliable than we think. We use calculators to work out sums we don't know the answers to, so it is very important that they work reliably.

We did some experiments with people using ordinary calculators, doing sums from $4x-5$ to harder ones like 2^{π} . We got surprising results:

51%	of calculators gave wrong answers (e.g., $4x-5=-1$, not -20)
27%	gave the right answers
22%	gave error messages

A slip or an unnoticed oddity of calculation can cause disaster, like paying the wrong bills, getting the wrong dose of medicine, or throwing an aircraft off course. It is very important for everyone to be able to do mathematics accurately and easily!

We've built a new calculator that is far easier to use, and our experiments with it show people rarely make mistakes with it. A key difference from conventional calculators is that our calculator shows you *everything* you need to know: it's all there too see. This makes it much simpler and far more reliable to use. Using it feels like using magic paper.

Of course you can get very sophisticated calculators, and some can do calculus and draw graphs. These are all based on pressing keys and

using templates. For example, to get $\frac{5}{9}$ you need to select $\frac{\blacksquare}{\blacksquare}$ from a menu of templates (of fractions, square roots, and so on), then type 5 over the top black square, and then type 9 over the bottom. Once you'd done that, you can't easily change it to, say, $5-9$ or even $5/9$. So tedious!

Our approach does a lot better than imitating mechanical calculators or using templates. Instead, you just write what you want. You could write 5 over 9, or write a dash and put 5 on top of it and 9 below, or even write -59 and then move the 5 and 9 around to their final positions. *You can write just as you would on paper — and you can edit it freely!*

If the user wrote $3x=18$, the calculator would immediately show the correct missing number, **6**, in red so that it is easier to see. Now, here's our new calculator letting the user change the sum, to one where it is all divided by 5:

$$\frac{3 \times 6}{5} = 18$$

You can see the user's wobbly blue handwriting of the division bar and their digit 5. Immediately, the calculator recognises the handwritten 5 and presents it formally as a typeset digit; at the same time it also re-solves the new equation:

$$\frac{3 \times 30}{5} = 18$$

It gives the answer **30**. Now the user might want to edit this by moving the $3x$ to the bottom of the equation, alongside the 5 they wrote earlier. They can select and drag the $3x$ easily:

$$\frac{3 \times 30}{5} = 18$$

...and the calculator immediately works out the new answer, **270**:

$$\frac{270}{3 \times 5} = 18$$

In an ordinary calculator, you have to rearrange every problem so that the answer comes last after an equals sign — not too hard in this case, but sometimes very tricky! So on a conventional calculator this last example would need to have been entered as $18 \times 3 \times 5 =$ and you'd

Babbage's Difference Engine

The background image on this page is one of Charles Babbage's working drawings for his difference engine, arguably the mechanical predecessor of modern computers.

www.cs.swansea.ac.uk/calculators

You can download our calculator from our web site, where we've also put lots more stuff for you to read or try out. The program works on Macs (OSX), PCs and Linux — it was written in Java so should run on most things. It needs a tablet, Mimio or SMARTboard or equivalent for best results.

The large image on this page shows Will Thimbleby using his fingers directly to write onto a large whiteboard. The computer recognises how he moves his finger, and uses a projector to show what he is drawing: a sum about π . The calculator can also do complex sums like $e^{-i\pi}$ too.

have to work out that a / (division) on one side should be an x (multiply) on the other side of the equation. So with our new calculator, rather than devising *instructions* like this to get an answer, you write an *equation* without having to rearrange it and get the calculator to solve it just like that, wherever the answer is supposed to go.

These ideas and others make our new calculator both very easy to use, and great fun.

Suppose you want to convert Fahrenheit to Celsius, the equation is

$$F = 32 + \frac{9 \times C}{5}$$

If you use a normal calculator, this needs rearranging to get the Celsius unknown on one side, on its own. On our new one, you can write it directly, just putting in the numbers you know:

$$98.6 = 32 + \frac{9 \times 37}{5}$$

You simply miss out the Celsius number altogether. The calculator corrects this omission, and inserts the correct answer, 37. So 98.6F is 37C.

A more sophisticated example is to find the power of 2 that is 100. To do this on a conventional calculator involves log keys (such as 100 log/ 2log=, but in different ways on different calculators), but it can be done *directly* on the new one.

A problem like $x! = 5040$ would be next to impossible on a conventional calculator, but is trivial with the new one. You can write $! = 5040$, and again the 'mistake' of the missing answer is filled in by our calculator. The answer is 7.

Because no two-dimensional templates are needed, as in conventional 'advanced' 2D calculators, on ours you can change, say, 2^3 into 3^2 very easily, in just one gesture. People smile when they see the answer change so easily from 8 to 9. It is fun to mess around and try to fool the calculator — you can even write $2^3 = 3^2$ and it will do something sensible.

You can see how the user interacts by writing on it using a pen (or finger) and how the calculator recognises the input and completes the calculation. The beauty of this is watching and playing with it in motion, as the calculator recognises and 'morphs' handwriting and lets you play with any expression instantly.

The benefits are immediately apparent when you see people using it. People enjoy using it and have fun exploring mathematics.

The potential of this sort of calculator for learning mathematics and in enabling users to calculate and solve problems they were not able to before is very exciting. Hopefully this calculator will prompt us to rethink and redesign how we interact with and use calculators. There are also exciting possibilities for its use on tablet PCs, small-screen handhelds, and in school classrooms on interactive whiteboards.

It's a nice irony that using handwriting to make calculators better uses computers to simulate a technology that was invented thousands of years ago: pen and paper.

Why not collaborate?

The ideas are patented, and we are looking for industrial and educational collaborators to help transform the handheld and educational markets.

Gresham College

The Royal Society was formed at Gresham College, and famous scientists, including Robert Hooke and Christopher Wren, are numbered amongst its Professors. Prof Harold Thimbleby was 28th Gresham Professor of Geometry. Gresham College is an independent institution, and was named after Sir Thomas Gresham, who also founded the Royal Exchange. Since 1597 Gresham Professors have given free public lectures. Information about Gresham College can be found at www.gresham.ac.uk.



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Some research questions — for fun, or even for a degree or research project at Swansea University

How would you make the calculator work well in the classroom, setting problems? How would you make the dock sharable over the web? How would you make the calculator handle huge numbers and division by zero? How would you extend the interface to work well on small screens, which require more gestures to zoom and scroll? How would you get it to handle several equations, more like a spreadsheet? How would you make the calculator work with a keyboard and mouse, rather than purely a pen? How would you extend it to do calculus (e.g., integrals) and other problems from symbolic maths? How would you get it to draw graphs? How would you make the calculator do primary level maths, using apples, counters or knobs instead of Arabic numerals?

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