Classical propositional logic and decidability of variables in intuitionistic propositional logic

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Let $\vdash_c$ and $\vdash_i$ denote derivability in classical and intuitionistic propositional logic, respectively. Then it is known that if $\vdash_c A$, then $\Pi_{\mathcal{V}(A)} \vdash_i A$, where $\mathcal{V}(A)$ is the set of propositional variables in a formula A and $\Pi_V = \{p \vee \neg p \mid p \in V\}$ for a set V of propositional variables; see, for example, [1], and [4, p. 27] which was originally given in [5].

In this talk, we consider a problem: *what set* V *of propositional variables suffices for* $\Pi_V, \Gamma \vdash_i A$ *whenever* $\Gamma \vdash_c A$, and show, employing a technique in [2, 3], that $V = (\mathcal{V}^-(\Gamma) \cup \mathcal{V}^+(A)) \cap (\mathcal{V}^+_{ns}(\Gamma) \cup \mathcal{V}^-(A))$ suffices, where $\mathcal{V}^+$, $\mathcal{V}^-$ and $\mathcal{V}^+_{ns}$ are the sets of propositional variables occurring positively, negatively and non-strictly positively, respectively.

References

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