Expansions with P = NP

Christine Gaßner
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Expansions with P = NP

1. The uniform model of computation

2. The complexity classes P_Σ, NP_Σ, DEC_Σ

3. Why is it difficult to find an R with $P_{\Sigma_R} = NP_{\Sigma_R}$?

4. How to construct a relation R such that $SAT_{\Sigma_R} \in P_{\Sigma_R}$?

1. The uniform model of computation over structures Σ

A structure:

constants, operations, relations

$$\Sigma = (U\,;\, c_1,\ldots,\, c_u\,;\; f_1,\ldots,\, f_v\,;\; R_1,\ldots,\, R_w\,,\, [=])$$

finite signature

finite universe
or
infinite universe

Examples:

$\Sigma_{\mathrm{bin}} = (\{0, 1\};\, 0, 1;\, ;\, =)$ (⇨ Turing machines),

$\Sigma_R = (\{a, b\}^*;\, \varepsilon;\, add_a,\, add_b,\, sub_a,\, sub_b;\, R, =)$

the empty string (ε)

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The Σ-machines

M

The uniform model of computation

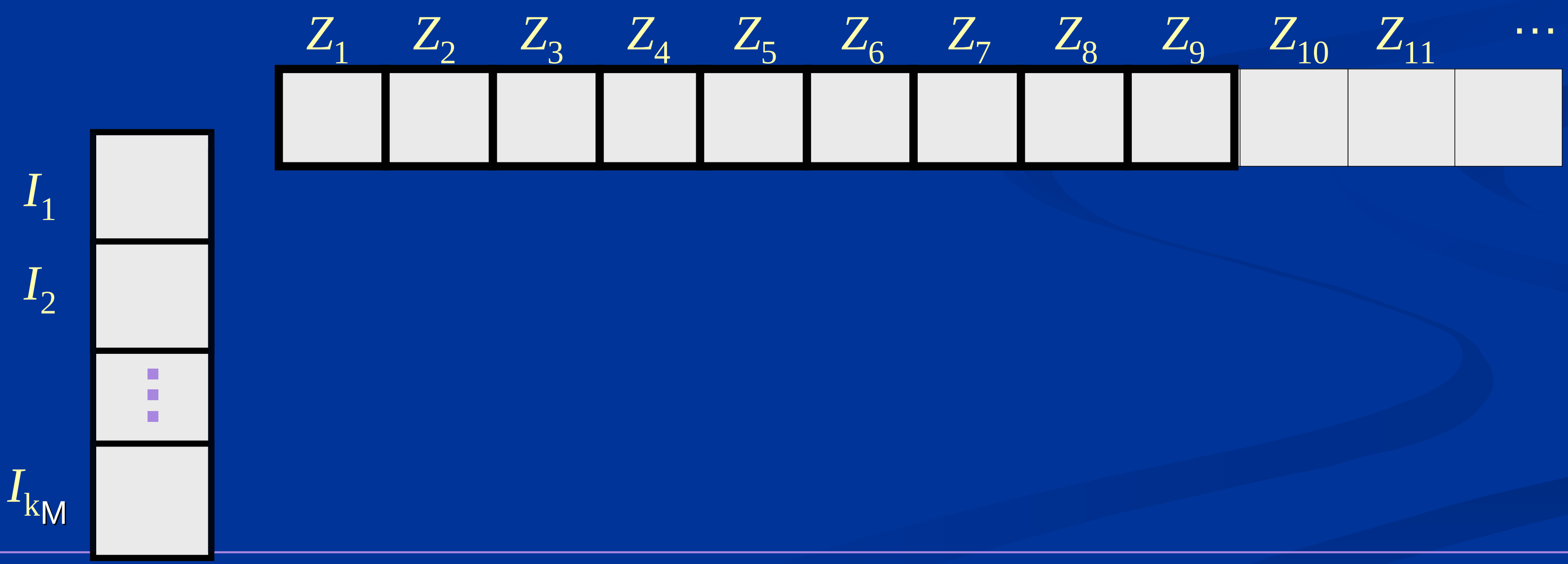
The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R ?

Registers for elements of U : $Z_1, Z_2, Z_3, \ldots$ ($U \triangleq$ the universe)

Registers for indices: $I_1, I_2, \ldots, I_{k_\mathrm{M}}$



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The Σ-machines (input)

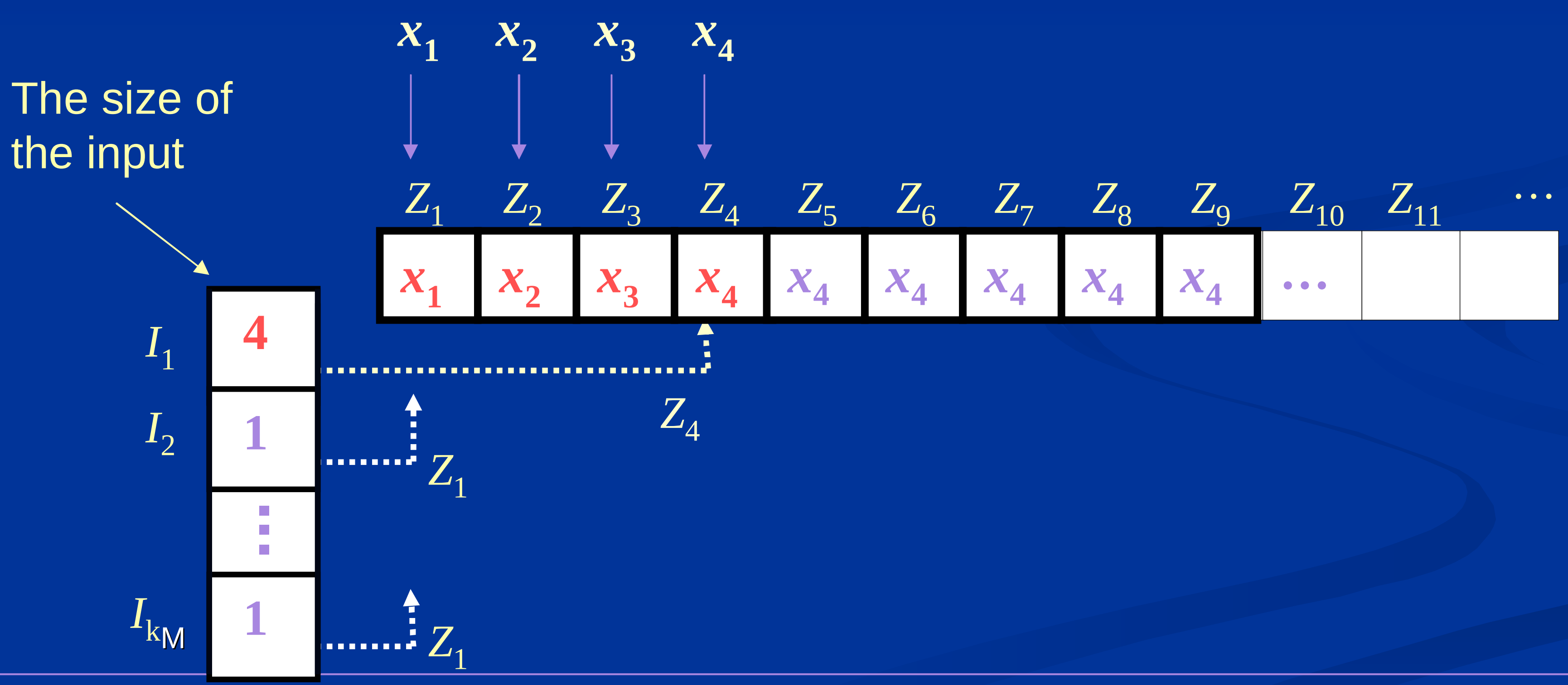
The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R ?

The input: $(Z_1, \ldots, Z_n) := (x_1, \ldots, x_n);\ I_1 := n;\ \ I_2 := 1; \ldots\ I_{k_M} := 1;$



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The Σ-machines (output)

The uniform model of computation

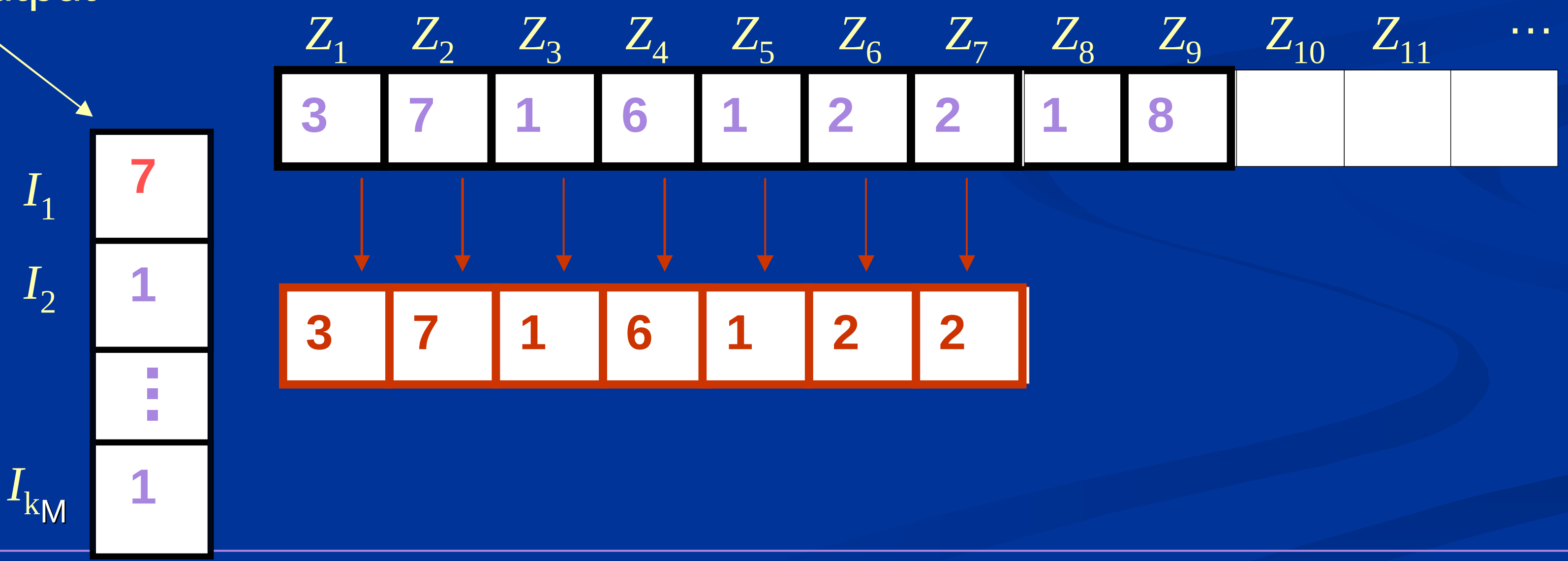
The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R ?

The output: $(Z_1, \ldots, Z_{I_1})$

The size of the output



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The Σ-machines (instructions)

The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT ∈ P for Σ_R ?

Structure: $\Sigma = (U\,;\, c_1,\ldots, c_u\,;\; f_1,\ldots, f_v\,;\, R_1,\ldots, R_w, =)$

Computation:

$l\colon\; Z_k := f_j(Z_{k_1},\ldots, Z_{k_{f_j}});$

$l\colon\; Z_k := c_j;$

Branching:

$l\colon$ if $R_j(Z_{k_1},\ldots, Z_{k_{R_j}})$ then goto l_1 else goto l_2;

$l\colon$ if $Z_k = Z_j$ then goto l_1 else goto l_2;

Copy:

$l\colon\; Z_{I_k} := Z_{I_j};$

Index computation: $I_k := 1$; $I_k := I_k + 1$; if $I_k = I_j$ then goto l_1 else goto l_2;

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The non-deterministic machines

The uniform model of computation

The complexity classes P, NP, DEC for Σ

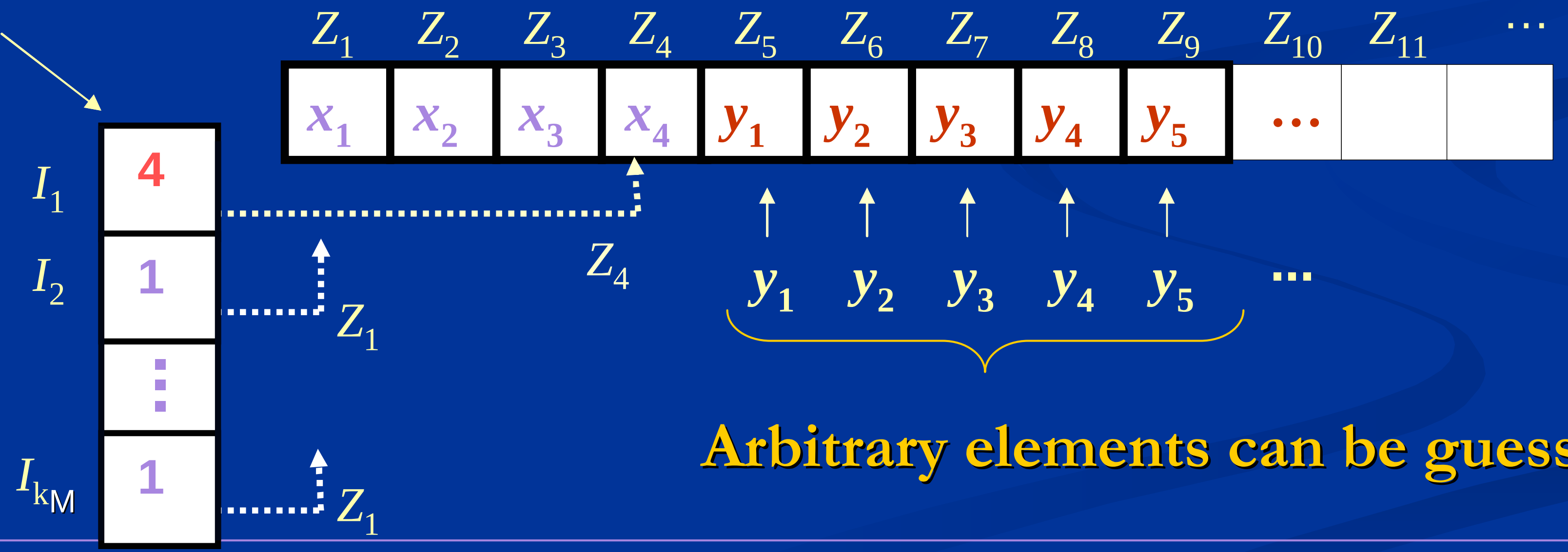
Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R ?

The guessing: $(Z_{n+1},\ldots, Z_{n+m}) := (y_1,\ldots, y_m) \in U^m$

non-deterministic!

The size of the input



Arbitrary elements can be guessed!

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Representations of a Σ-machine

The uniform model of computation

The complexity classes P, NP, DEC for Σ

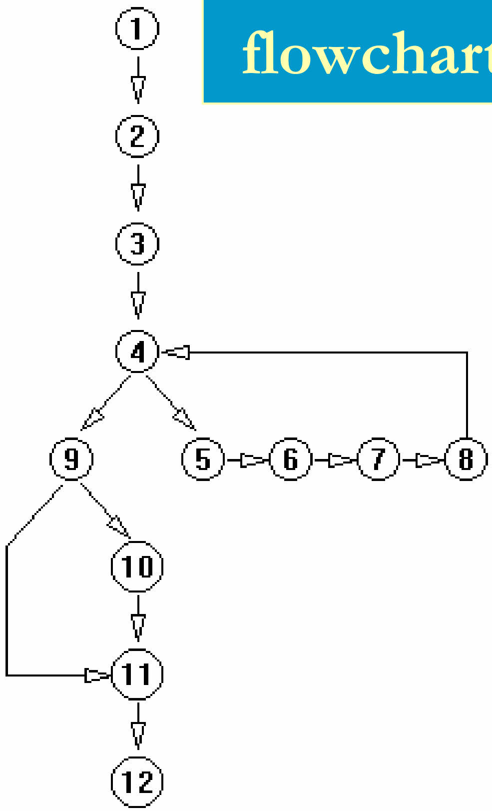
Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R?

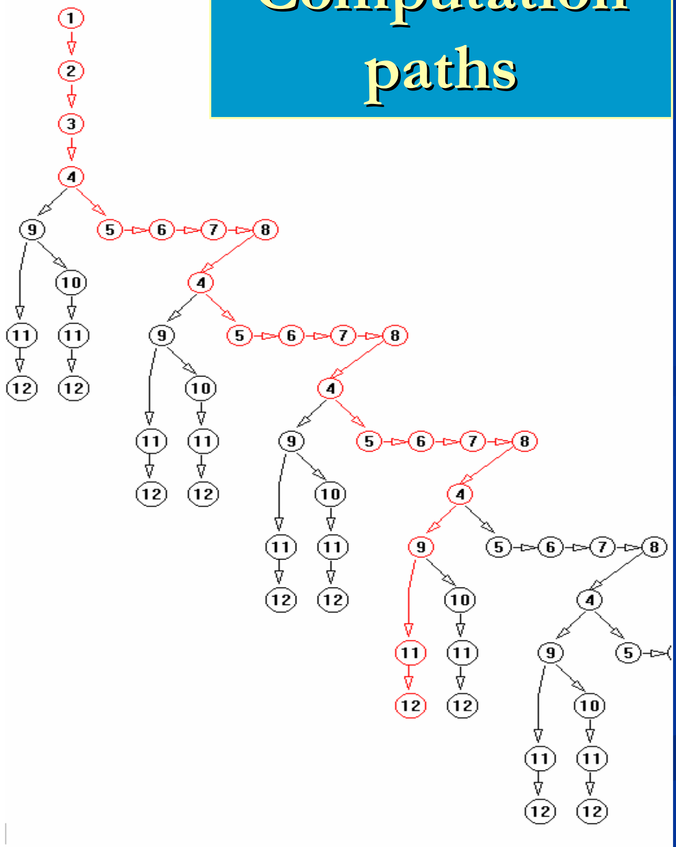
A program

1: Input: $(x_1, \ldots, x_n)$.
Guess: $(y_1, \ldots, y_n) \in \{0, 1\}$.
2: $I_2 := I_1 + 1$;
3: $Z_1 := 1 + Z_1 * Z_{I_2}$;
4: if $I_1 = I_3$ then goto 9
else goto 5;
5: $I_2 := I_2 + 1$;
6: $I_3 := I_3 + 1$;
7: $Z_1 := Z_1 + Z_{I_3} * Z_{I_2}$;
8: goto 4;
9: if $Z_1 = 1$ then goto 11
else goto 10;
10: $Z_1 := 0$;
11: $I_1 := 1$;
12: Output: Z_1.

A flowchart



Computation paths



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2. The complexity classes DEC_Σ , P_Σ , NP_Σ

Decidable, (non-) deterministically recognizable in polynomial time

Every element can be stored in one register !

One operation can be executed **in one time unit (one step) !**

⇒ Computation in **polynomial time** means

output after $p(n)$ steps for any $(x_1 ,\ldots, x_n)$

and some polynomial p.

The usual problems (for structures with two constants):

SAT_Σ The Satisfiability Problem

(Compare: the Blum-Shub-Smale model introduced in 1989,

the algebraic programming systems considered by Asveld and Tucker in 1982.)

UNI_Σ The NP-complete problem recognized by a usual universal machine

H_Σ The Halting Problem

A question

The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R ?

POIZAT (1995):

Is there a structure of finite signature with P = NP ?

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3. Why is it difficult to find an R with $P_{\Sigma_R} = NP_{\Sigma_R}$?

- The idea: Some R with $SAT_{\Sigma_R} \in P_{\Sigma_R}$
- How can a tuple be encoded by one element?
- The undecidability of SAT_{Σ_R} for some R

The idea:
A new relation *R* with $SAT_{\Sigma_R} \in P_{\Sigma_R}$

The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an *R* with P = NP for Σ_R?

How to construct *R* such that SAT $\in$ P for Σ_R ?

tuple of elements

$(x_1,\ldots, x_n, code(\psi)) \in SAT_{\Sigma_R}$?

reduction

?

$\Sigma_R \models R(CODE(x_1,\ldots, x_n, \psi))$?

one element

The first problem:

How can a tuple be encoded by one element?

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How can a tuple be encoded by one element?

The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R ?

How to construct R such that SAT $\in$ P for Σ_R ?

Solution: **We extend** $\Sigma = (U; c_1,\ldots, c_u;\ f_1,\ldots, f_v; R_1,\ldots, R_w, =)$.

$\Rightarrow$ $\Sigma_R = (A; \varepsilon, a, b, c_1,\ldots, c_u;\ \underbrace{?,\ ?,\ ?,}_{\text{operations for concatenation,...}}\ f_1',\ldots, f_v'; R_1',\ldots, R_w', R, =)$

$\Rightarrow$ $A = U^*$

$$f_j'(d_{k_1}, \ldots, d_{k_{f_j}}) = f_j(d_{k_1}, \ldots, d_{k_{f_j}});\ \ f_j'(s_{k_1}, \ldots, s_{k_{f_j}}) = \varepsilon$$

$$d_{k_1}, \ldots, d_{k_{f_j}} \in U,\ \ (s_{k_1}, \ldots, s_{k_{f_j}}) \notin U^{k_{f_j}}$$

$(x_1,\ldots, x_n, code(\psi)) \in \mathrm{SAT}_{\Sigma_R}$? $\Leftarrow$ tuple of strings

reduction $\Downarrow$ concat $\Downarrow$

$\Sigma_R \vDash R(string(x_1,\ldots, x_n, \psi))$? $\Leftarrow$ one string

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Why is it difficult to find an R with $SAT_{\Sigma_R} \in DEC_{\Sigma_R}$?

The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R?

For many relations R :

The **assumption**

of the decidability of SAT_{Σ_R} (by means of R)

implies the **decidability of H_{Σ_R}** .

⇒ $SAT_{\Sigma_R} \notin DEC_{\Sigma_R}$.

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Some R_{h} with $\mathrm{SAT}_{\mathrm{halt}} \notin \mathrm{DEC}_{\Sigma_{R_{\mathrm{h}}}}$

The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R ?

How to construct R such that SAT $\in$ P for Σ_R ?

$\Sigma_R = (\{a, b\}^*; \varepsilon;$ operations for concat. ; $R, =)$. TM set of all deterministic Turing machines.

$$R_{\mathrm{h}}(s) = \begin{cases} true & \text{if } s = r\,Code_{\mathrm{TM}}(M),\ M \in TM,\ M \text{ halts on } Code_{\mathrm{TM}}(M) \text{ after } |r| \text{ steps} \\ false & \text{otherwise} \end{cases}$$

$$\mathrm{SAT}_{\mathrm{halt}} =_{\mathrm{df}} \{code(R_{\mathrm{h}}(Y_1 \underbrace{d_1 \cdots d_{k-1}}_{Code_{\mathrm{TM}}(M)})) \mid M \in TM \ \ \&\ \Sigma_{R_{\mathrm{h}}} \models \exists Y_1 R_{\mathrm{h}}(Y_1 d_1 \cdots d_{k-1})\}$$

$\mathrm{SAT}_{\mathrm{halt}} \in \mathrm{DEC}_{\Sigma_{R_{\mathrm{h}}}}$ — **Simulation by a TM** ⇒ **$\mathrm{H}_{TM} \in \mathrm{DEC}_{TM}$** ↯

$\mathrm{H}_{TM} \notin \mathrm{DEC}_{TM} \ \Rightarrow\ \mathrm{SAT}_{\mathrm{halt}} \notin \mathrm{DEC}_{\Sigma_{R_{\mathrm{h}}}} \ \Rightarrow\ \mathrm{SAT}_{\Sigma_{R_{\mathrm{h}}}} \notin \mathrm{DEC}_{\Sigma_{R_{\mathrm{h}}}}$

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4. How to construct a relation *R* such that $\mathrm{SAT}_{\Sigma_R} \in \mathrm{P}_{\Sigma_R}$?

The idea:

$$\Sigma_R \models R(string(\mathbf{x}, \psi)) \Leftrightarrow (\mathbf{x}, code(\psi)) \in \mathrm{SAT}_{\Sigma_R}$$

R is definable $\Leftrightarrow$ SAT_{Σ_R} is decidable!

- Can we find an R such that SAT_{Σ_R} is decidable?
- Can we give a definition of R such that $\mathrm{SAT}_{\Sigma_R} \in \mathrm{P}_{\Sigma_R}$?

Is R definable such that

$$R(string(x,\psi)) \leftrightarrow \exists Y\ \psi\,(x,Y)\ ?$$

The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R ?

- Problems:
  - ψ can contain R.
  - The guesses for $Y_1,\ldots, Y_m$ can be the codes of formulae.
  - Can all possible guesses for any formula be finitely described by one quantifier-free $(\neg,\ \vee,\ \wedge)$-formula over Σ_R?
- **Solution:** Definition of R such that

  R and SAT_{Σ_R} can be described by **a tree**

  which is similar to a tree of computation paths.

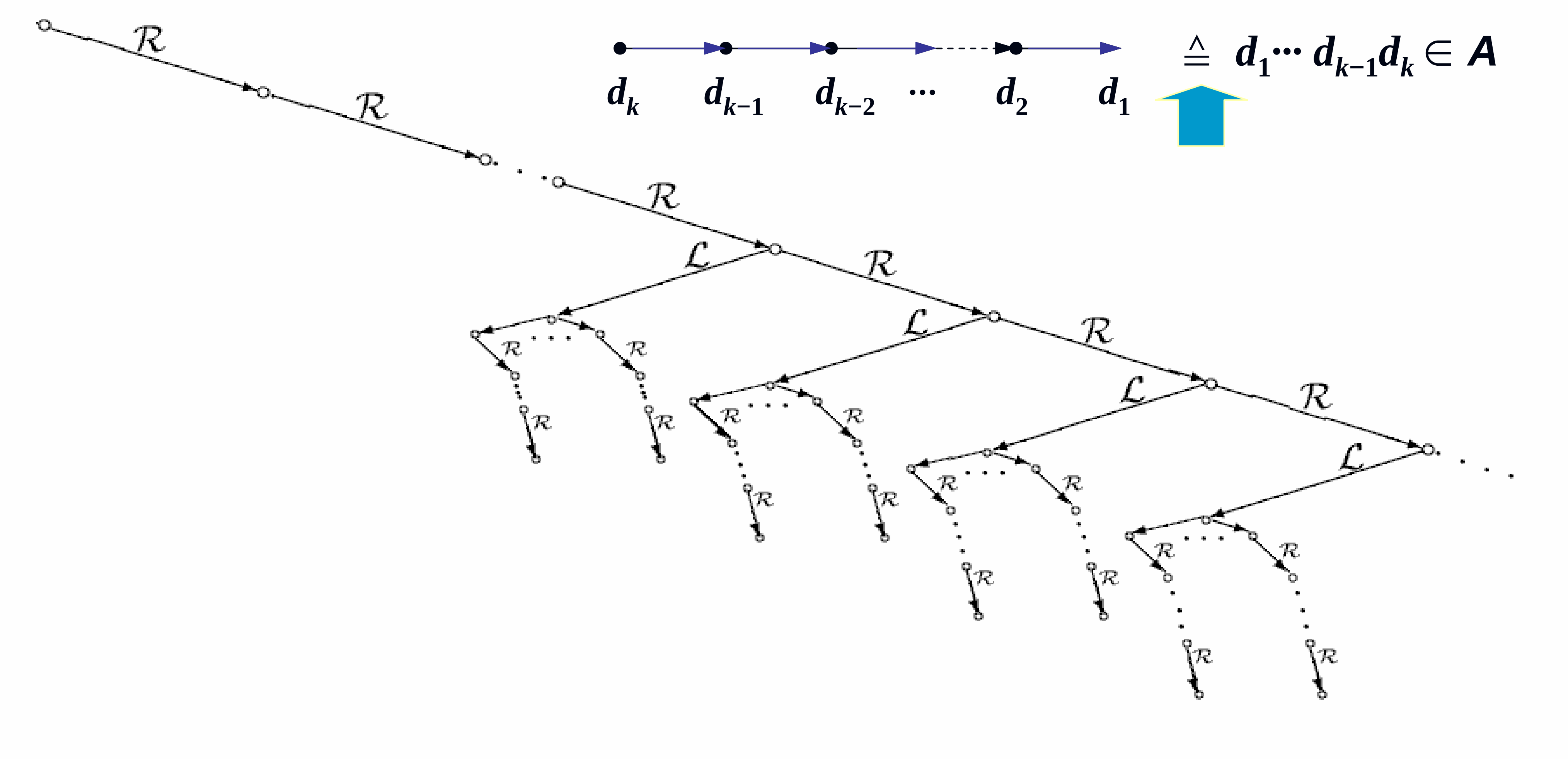
The description of SAT_{Σ_R} by a tree

The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an *R* with P = NP for Σ_R?

How to construct *R* such that SAT ∈ P for Σ_R?



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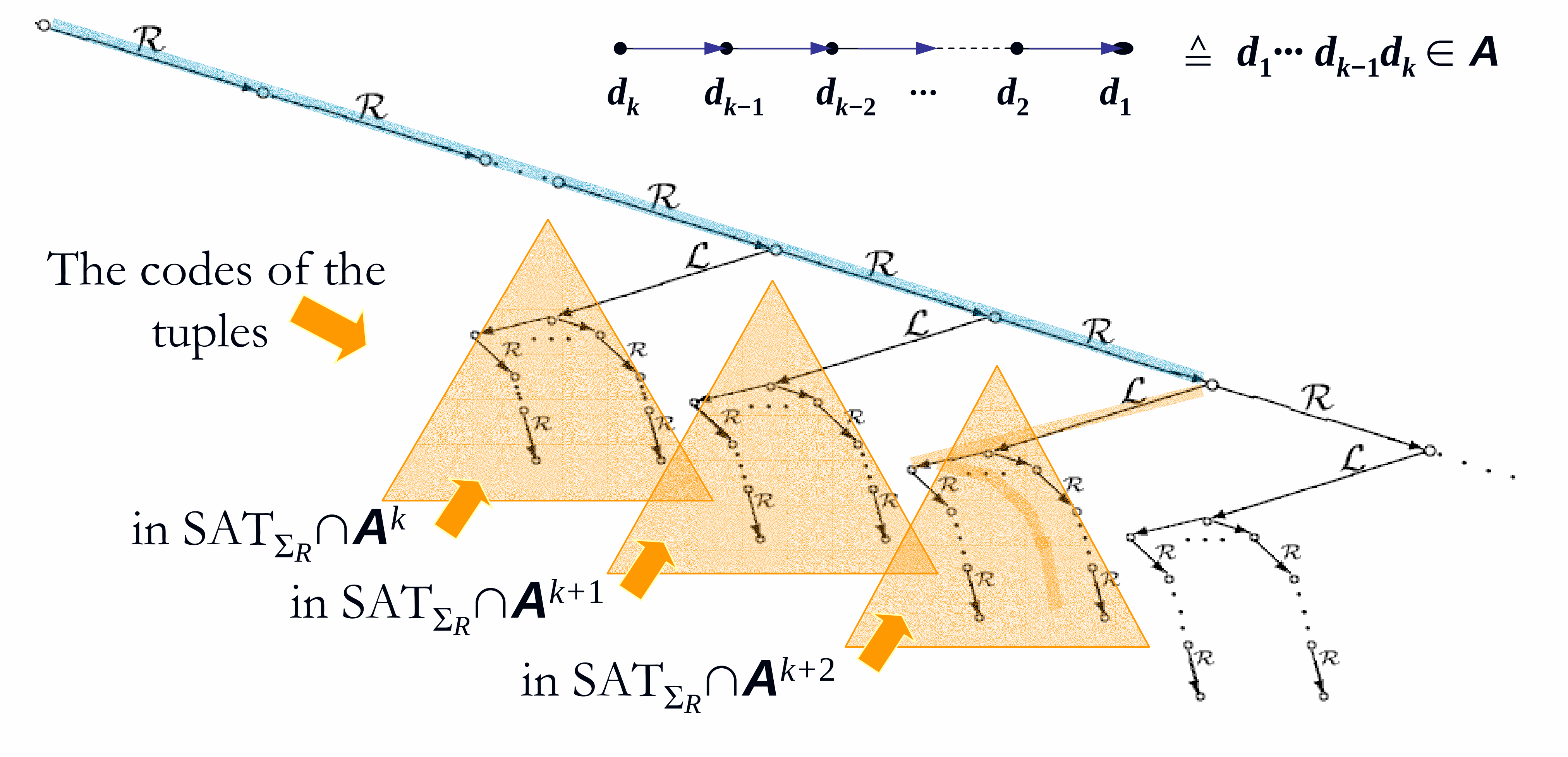
The description of SAT_{Σ_R} by a tree

The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R ?



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Some R with $\mathrm{SAT}_{\Sigma_R} \in \mathrm{DEC}_{\Sigma_R}$

The uniform model of computation

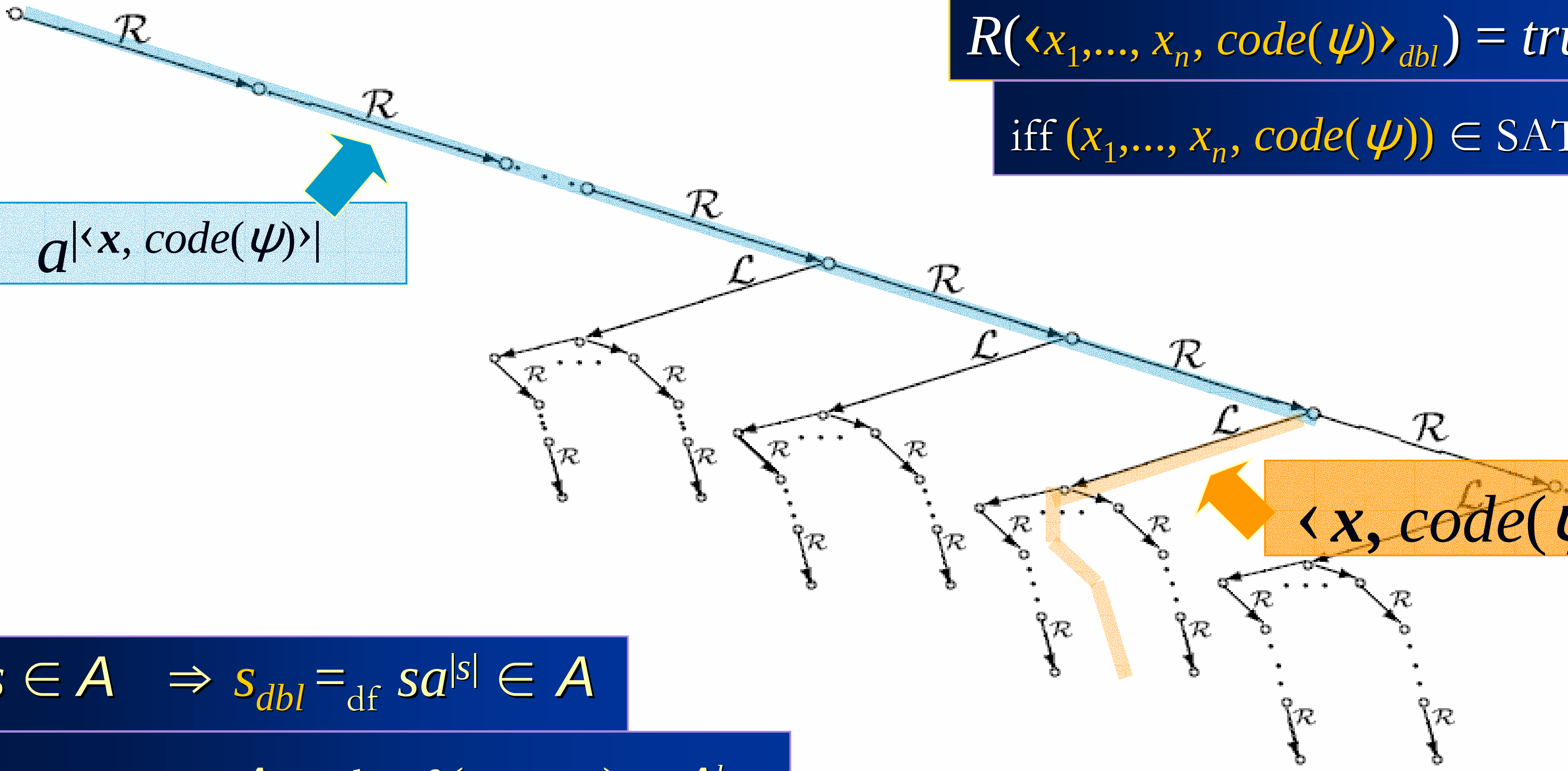
The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R ?

$R(\langle x_1,\ldots, x_n, code(\psi)\rangle_{dbl}) = true$

iff $(x_1,\ldots, x_n, code(\psi)) \in \mathrm{SAT}_{\Sigma_R}$



$a^{|\langle x,\, code(\psi)\rangle|}$

$\langle x, code(\psi)\rangle$

$s \in A \quad \Rightarrow \quad s_{dbl} =_{\mathrm{df}} sa^{|s|} \in A$

$\langle s_1,\ldots, s_k\rangle \in A$ code of $(s_1,\ldots, s_k) \in A^k$

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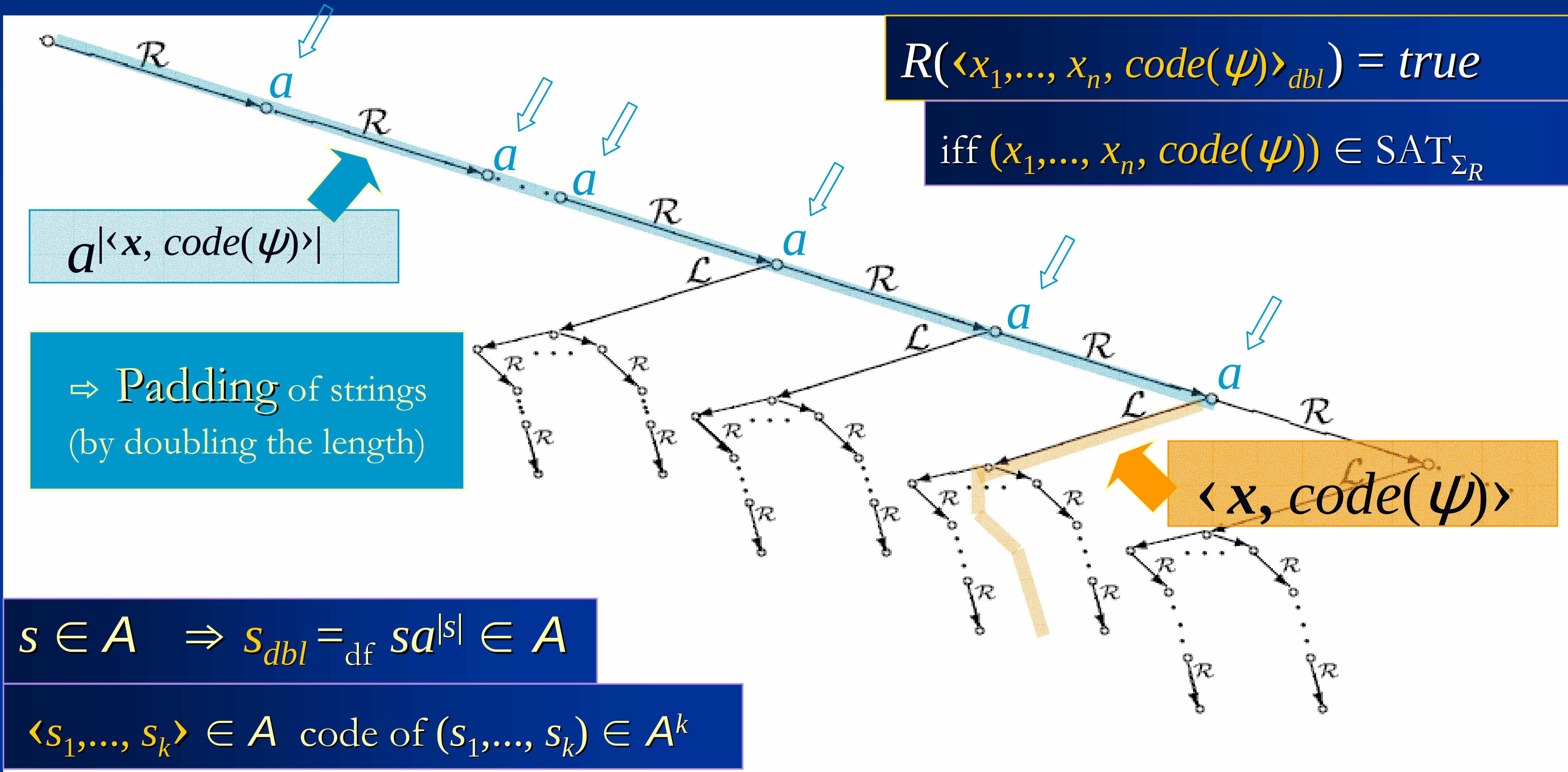
Some *R* with $SAT_{\Sigma_R} \in DEC_{\Sigma_R}$

The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an *R* with P = NP for Σ_R?

How to construct *R* such that SAT $\in$ P for Σ_R ?



$R(\langle x_1,\ldots, x_n, code(\psi)\rangle_{dbl}) = true$

iff $(x_1,\ldots, x_n, code(\psi)) \in SAT_{\Sigma_R}$

$a^{|\langle x,\, code(\psi)\rangle|}$

⇨ Padding of strings (by doubling the length)

$\langle x, code(\psi)\rangle$

$s \in A \quad \Rightarrow \quad s_{dbl} =_{df} sa^{|s|} \in A$

$\langle s_1,\ldots, s_k\rangle \in A$ code of $(s_1,\ldots, s_k) \in A^k$

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The new operations for slow (!!!) computation

The uniform model of computation

The complexity classes P, NP, DEC for Σ

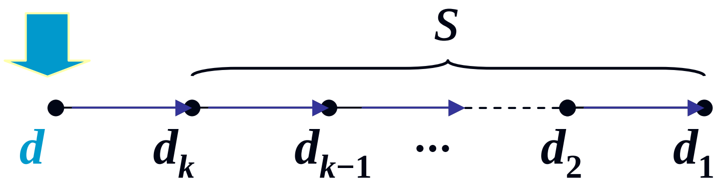
Why is it difficult to find an R with P = NP for Σ_R ?

How to construct R such that SAT $\in$ P for Σ_R ?

$\Sigma \;\; = (U ; c_1,\ldots, c_u;\; f_1,\ldots, f_v; R_1,\ldots, R_w, =),$

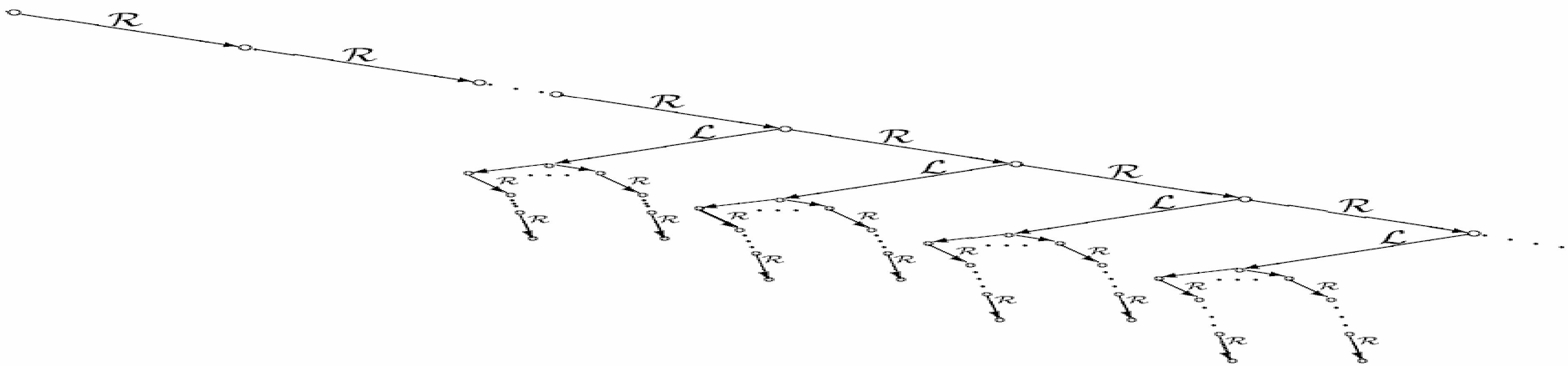
$\Sigma_R = (A;\; \varepsilon, a, b, c_1,\ldots, c_u;\; add, sub_l, sub_r, f_1',\ldots, f_v'; R_1',\ldots, R_w', R, =)$

$add(s, d) = sd \qquad sub_l(sd) = s \qquad sub_r(sd) = d \qquad s \in A, d \in U$



$s = d_1 \cdots d_{k-1} d_k$

$add(s, d) = d_1 \cdots d_k d$



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Similar trees

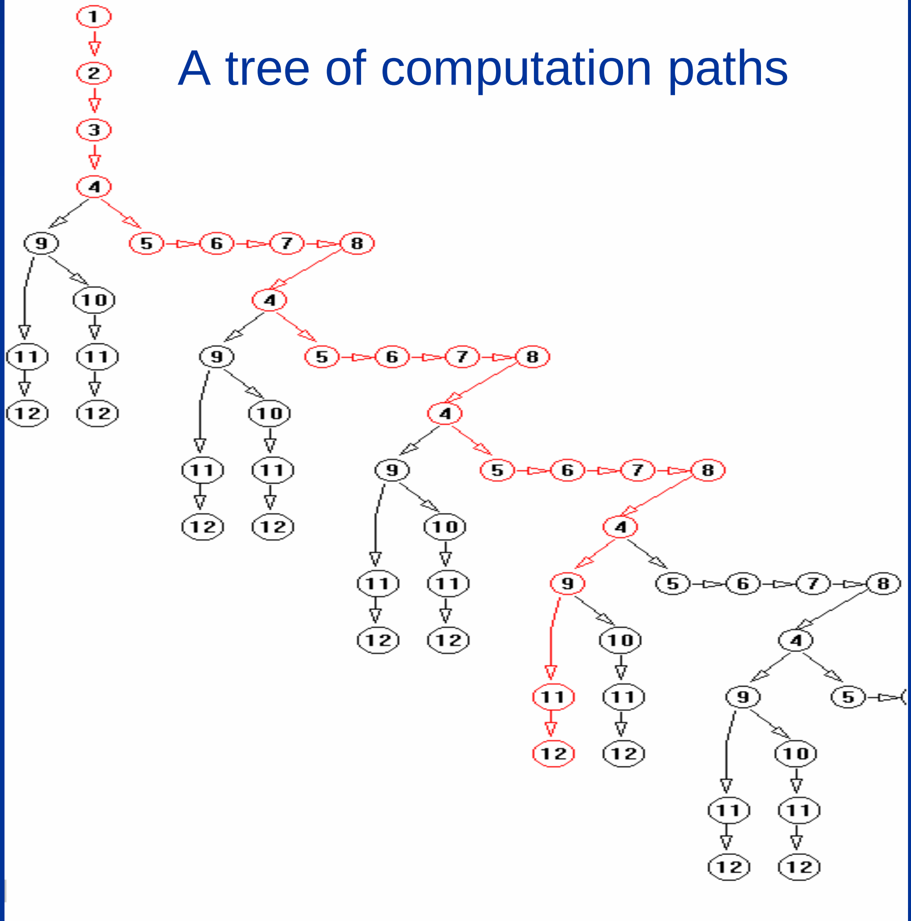
The uniform model of computation

The complexity classes P, NP, DEC for Σ

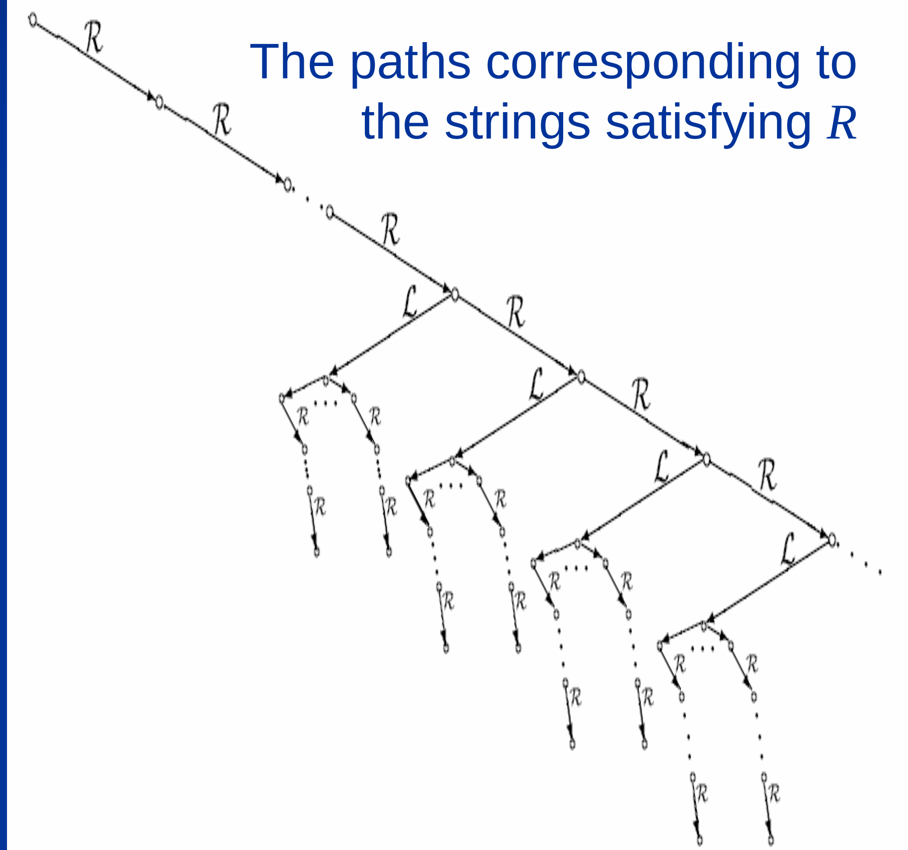
Why is it difficult to find an R with P = NP for Σ_R ?

How to construct R such that SAT $\in$ P for Σ_R ?

A tree of computation paths



The paths corresponding to the strings satisfying R



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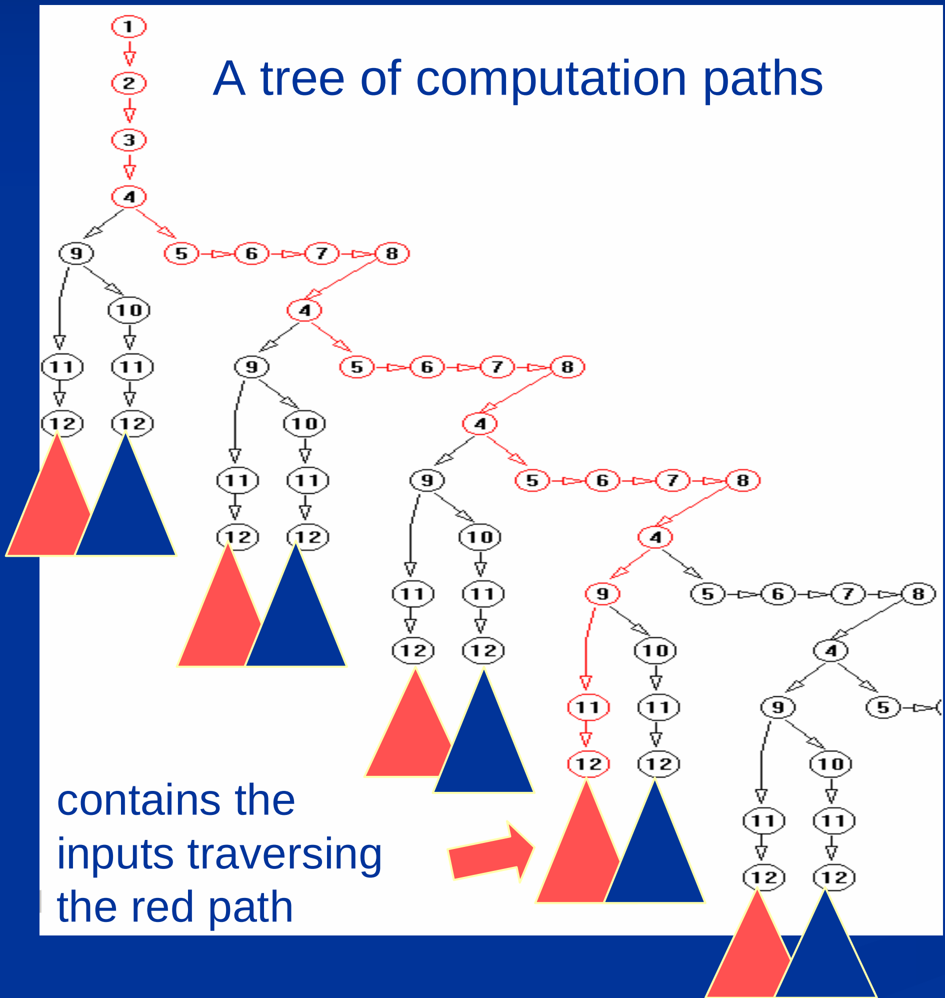
Similar trees

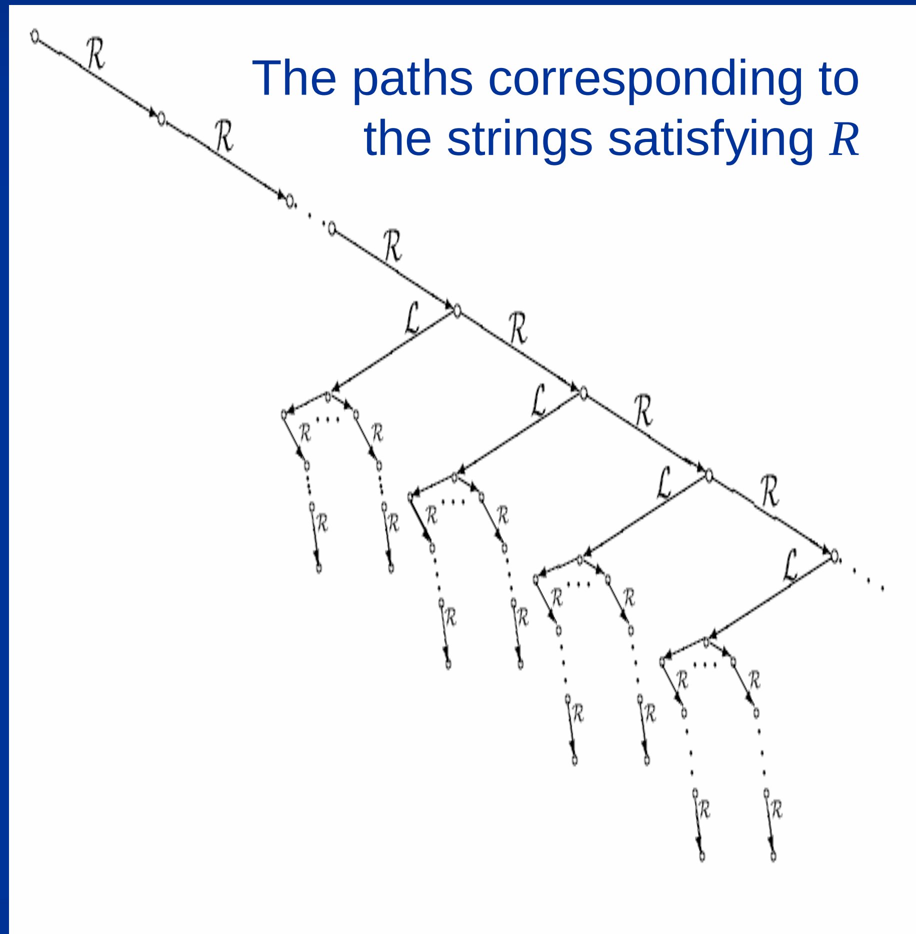
The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R ?





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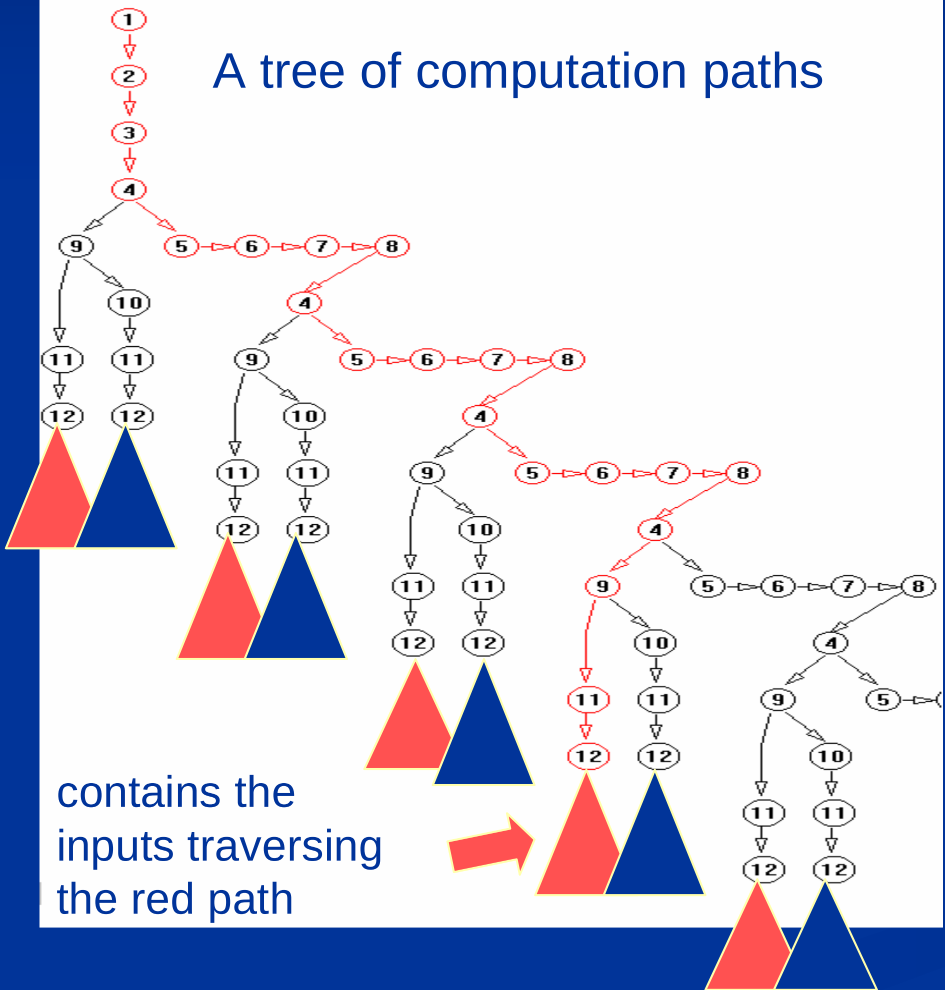
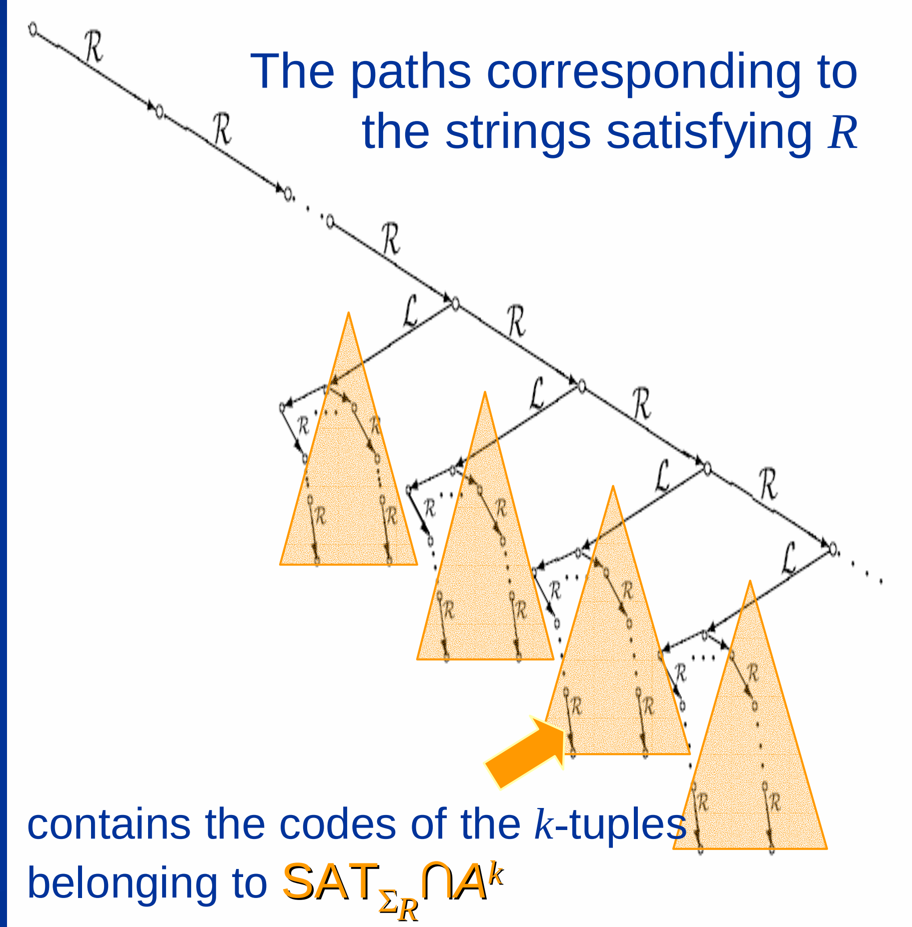
Similar trees

- The uniform model of computation
- The complexity classes P, NP, DEC for Σ
- Why is it difficult to find an R with P = NP for Σ_R?
- How to construct R such that SAT $\in$ P for Σ_R ?





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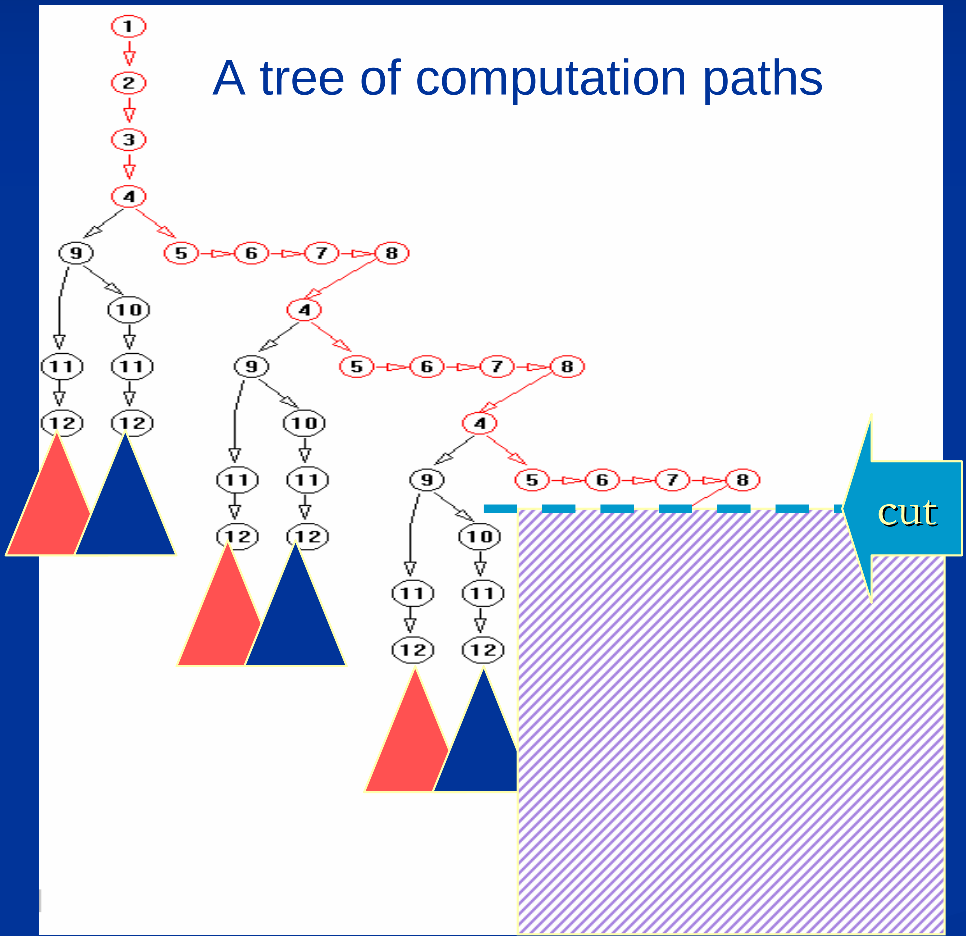
Trees and polynomial time

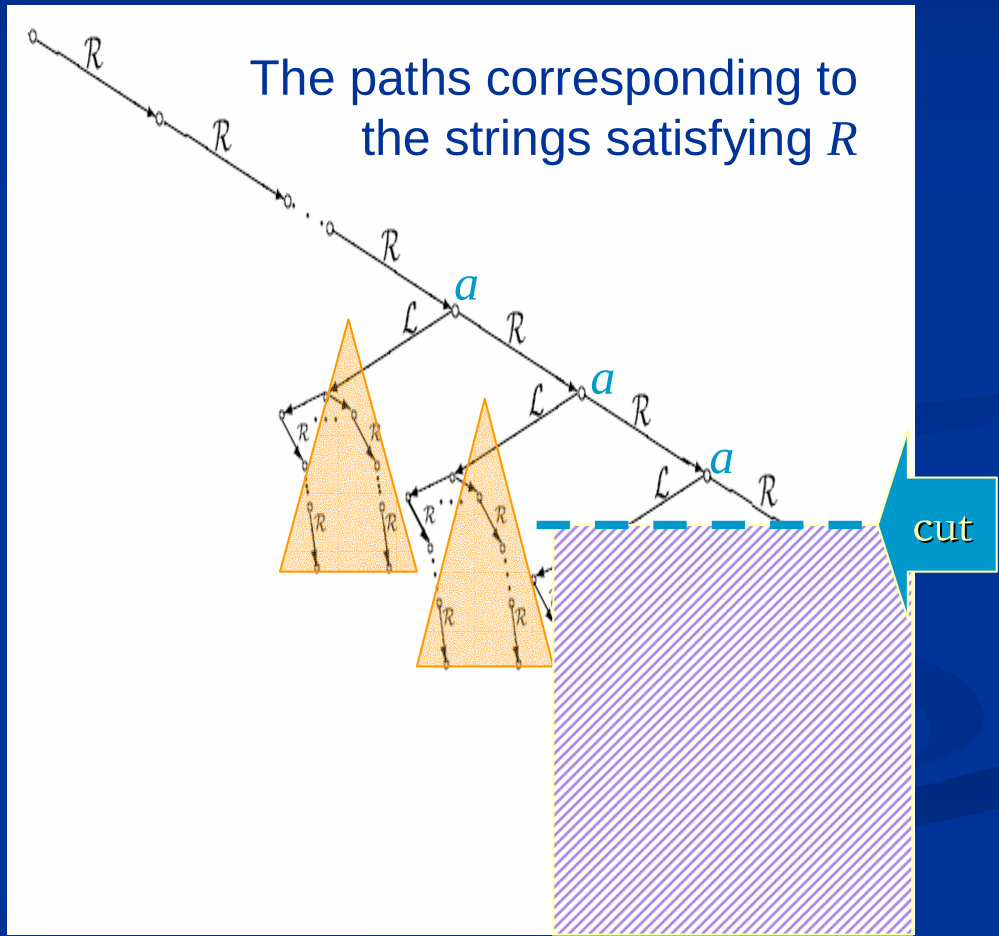
The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R ?





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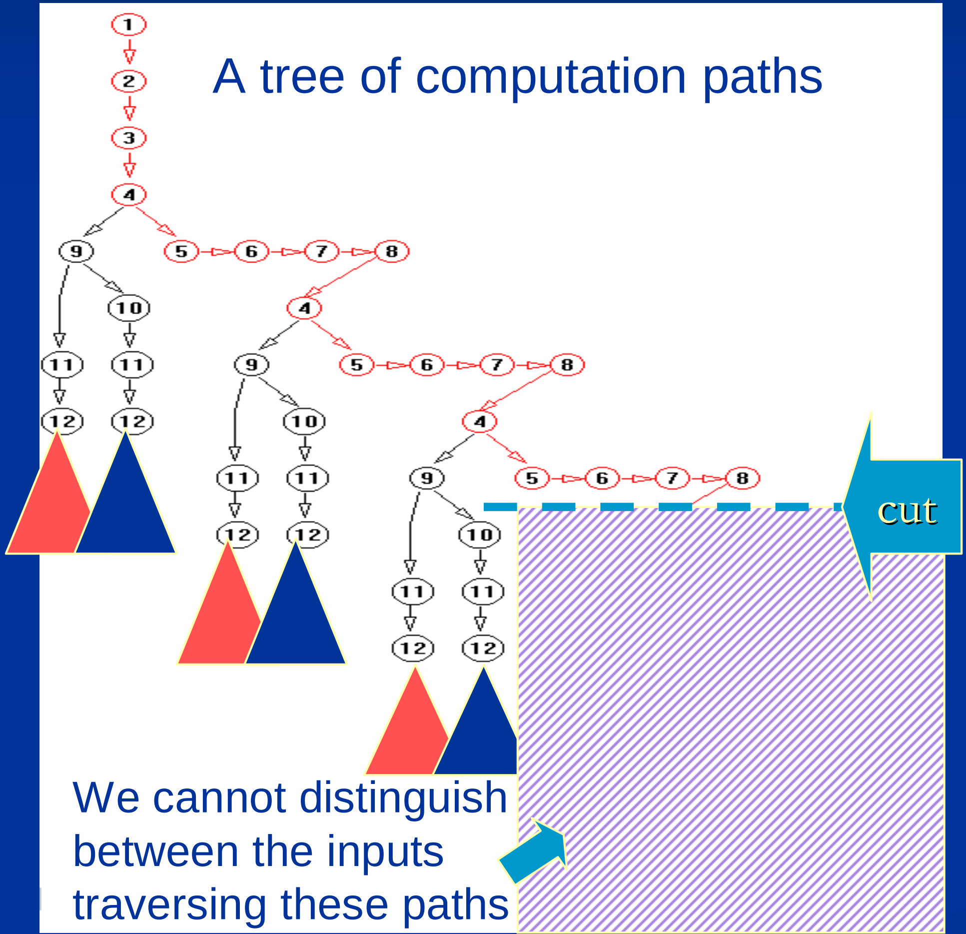
Trees and polynomial time

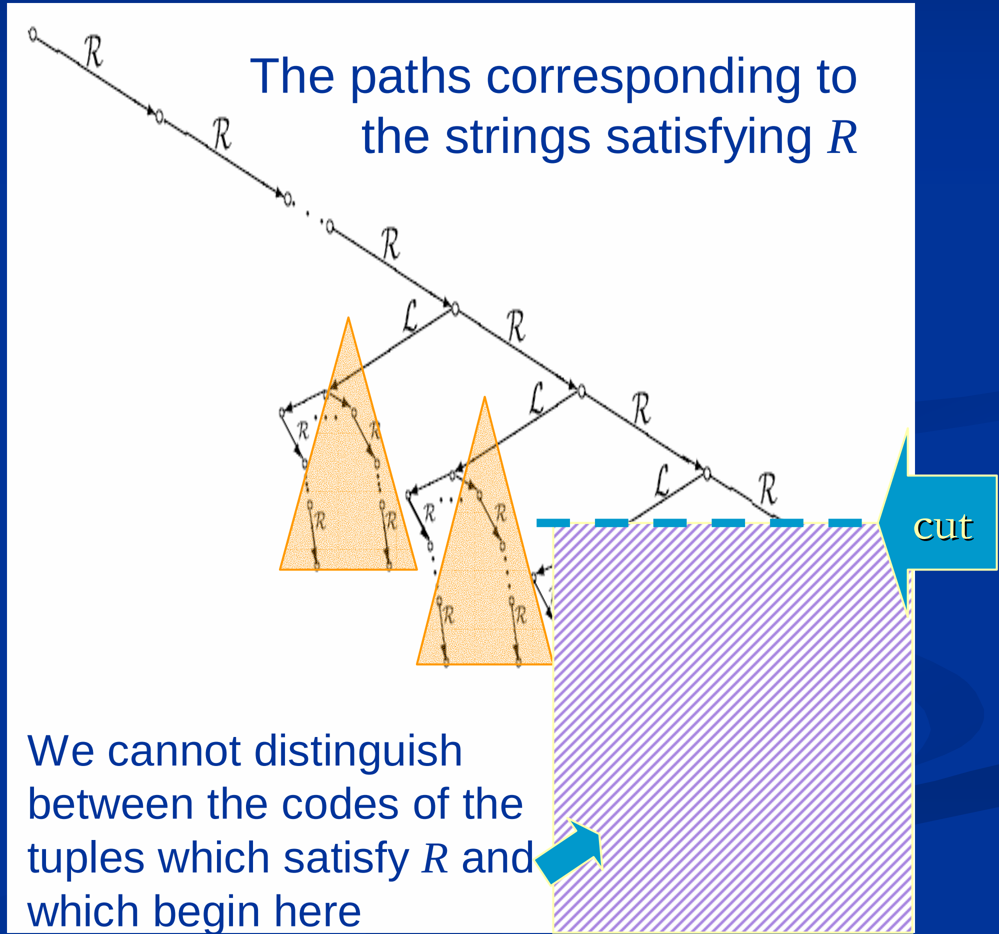
The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R ?





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The recursive definition is possible

The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R ?

R and SAT_{Σ_R} can be described by a tree which is similar to a tree of computation paths.

➡ Reduction of the tuples $(x_1,\ldots, x_n, code(\psi))$ to small strings.

➡ Replacement of arbitrary guesses by small guesses.

➡ Restriction of the quantifier domains.

➡ Recursive definition of R .

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Let us consider some details in the next talk.

Thank you for your attention!

Christine Gaßner

I thank

Gerald van den Boogaart,
Volkmar Liebscher,
Rainer Schimming,
Michael Schürmann

of the

Department of Mathematics and Computer Science,
Ernst Moritz Arndt University of Greifswald

for discussion on the presentation.

Appendix

DEC_Σ , P_Σ , NP_Σ

Decidable, (non-) deterministically recognizable in polynomial time

The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R ?

The size of an input $(x_1,\ldots, x_n)$: ***n*.**

⇒ **Every $u \in U$ can be stored in one register.**

The execution of any instruction: **one time unit** (one step).

⇒ **The execution of one operation = one time unit.**

Computation in polynomial time: **output after $p(n)$ steps**

for any input $(x_1,\ldots, x_n)$ and some polynomial p.

Non-deterministic / deterministic acceptance: output a (or halt).

Deterministic rejection: output b (or no halt). ($a, b \in U$, $a \neq b$)

$P_\Sigma \subseteq NP_\Sigma$, $P_\Sigma \subseteq DEC_\Sigma$ ⇒ $NP_\Sigma \not\subseteq DEC_\Sigma \Rightarrow P_\Sigma \neq NP_\Sigma$

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NP-complete problems, Halting Problem

The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R ?

The Satisfiability Problem

$$\mathbf{SAT}_\Sigma = \{(\mathbf{x}, code(\psi)) \in U^\infty \mid \psi \in L_\Sigma \ \& \ \Sigma \models \exists \mathbf{Y}\, \psi(\mathbf{x}, \mathbf{Y})\}$$

The NP- complete problem recognized by a usual universal machine

$$\mathbf{UNI}_\Sigma = \{(x_1,\ldots, x_n, Code(M), a,\ldots, a) \in U^{n+k+t} \mid M \in NM_\Sigma \ \& \ M \text{ accepts } (x_1,\ldots, x_n) \text{ within } t \text{ steps}\} \subseteq U^\infty$$

The Halting Problem

$$\mathbf{H}_\Sigma = \{(\mathbf{x}, Code(M)) \in U^\infty \mid M \in M_\Sigma \ \& \ M \text{ halts on } \mathbf{x}\}$$

$U \triangleq$ the universe, $L_\Sigma \triangleq$ formulae, $NM_\Sigma \triangleq$ non-det. machines, $M_\Sigma \triangleq$ det. machines

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SAT_Σ

Satisfiability Problem

The uniform model of computation

The complexity classes P, NP, DEC for Σ

Why is it difficult to find an R with P = NP for Σ_R?

How to construct R such that SAT $\in$ P for Σ_R ?

$\Sigma \quad = (U\,;\, a, b, c_3,\ldots, c_u\,;\; f_1,\ldots, f_v\,;\, R_1,\ldots, R_w, =)$

L_Σ set of quantifier- free $(\neg,\ \vee,\ \wedge)$-formulae

- $(=, \neq, R_1,\ldots, R_w, \neg R_1,\ldots, \neg R_w)$-literals
- $(a, b, c_3,\ldots, c_u,\ f_1,\ldots, f_v)$-terms.

$$\psi \in L_\Sigma \;\Rightarrow\; code(\psi) \in \{a, b\}^\infty \subseteq U^\infty =_{\mathrm{df}} \bigcup_{k\geq 1} U^k$$

$$\mathbf{SAT}_\Sigma = \{(x_1,\ldots, x_n, code(\psi)) \in U^\infty \mid \quad \psi \in L_\Sigma$$

$$\&\quad \Sigma \models \exists(Y_1,\ldots, Y_m)\ \psi(x_1,\ldots, x_n, Y_1,\ldots, Y_m)\}$$

$\mathrm{SAT}_\Sigma \in \mathrm{NP}_\Sigma$, SAT_Σ is NP_Σ-complete. ⇨ $\mathrm{SAT}_\Sigma \in \mathrm{P}_\Sigma \Rightarrow \mathrm{P}_\Sigma = \mathrm{NP}_\Sigma$

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UNI_Σ and H_Σ

Recognition by universal machine, Halting Problem

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NM_Σ set of all non-deterministic Σ-machines.

$$\mathrm{UNI}_\Sigma = \{(x_1,\ldots, x_n, Code(M), a,\ldots, a) \in U^{n+k+t} \mid M \in NM_\Sigma \text{ \& } M \text{ accepts } (x_1,\ldots, x_n) \text{ within } t \text{ steps}\}$$

$\mathrm{UNI}_\Sigma \in \mathrm{NP}_\Sigma$, UNI_Σ is NP_Σ-complete. ⇨ $\mathrm{UNI}_\Sigma \in \mathrm{P}_\Sigma \Rightarrow \mathrm{P}_\Sigma = \mathrm{NP}_\Sigma$

M_Σ set of all deterministic Σ-machines.

$$\mathrm{H}_\Sigma = \{(x_1,\ldots, x_n, Code(M)) \mid (x_1,\ldots, x_n) \in U^n \text{ \& } M \in M_\Sigma \text{ \& } M \text{ halts on } (x_1,\ldots, x_n)\}$$

⇨ $\mathrm{H}_\Sigma \notin \mathrm{DEC}_\Sigma$

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Some historical remarks

P versus NP for special structures

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GÖDEL, TURING, CHURCH, KLEENE, COOK, KARP (1931, 1936, 1971, 1972):

The classical theory of computation

ENGELER, FRIEDMAN, MANSFIELD (1967, 1971):

Computation over structures for functions of fixed arities

ASVELD, TUCKER (1980, 1982):

Deterministic and binary non-deterministic algebraic programming systems and an abstract Satisfiability Problem

BLUM, SHUB, SMALE (1989):

The uniform model of computation over the real numbers

CUCKER (1990):

Investigation of computation paths

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MEER (1992, 1993):

P $\neq$ NP for ($\mathbb{R}$; 0, 1; +, $-$; =) and for ($\mathbb{R}$; 0, 1; sin, cos, $\cdot$, +, $-$; =)

KOIRAN (1994):

DNP = NP for ($\mathbb{R}$; 0, 1; +, $-$; $\leq$)

DN $\triangleq$ digitally (binary) non-deterministic: $y_1,\ldots, y_m \in \{0 , 1\}$

POIZAT (1995):

The question: Is there a structure of finite signature with P = NP?

PRUNESCU (2001, Greifswald):

Talk on Poizat‘s idea to define an additional relation for a structure over $\{0,1\}^*$ which implies P = NP

MAINHARDT (2001, Greifswald):

A structure of infinite signature with P = NP

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Some R_h with $SAT_{halt} \notin DEC_{\Sigma_{R_h}}$ (The idea of the proof)

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$R_h(s)$ iff $s = r\, Code_{TM}(M)$, $M \in TM$, M halts on $Code_{TM}(M)$ after $|r|$ steps

Assume: P_0 decides SAT_{halt}

$code(R_h(Y_1\, d_1 \cdots d_{k-1}))$

P_0

$Code_{TM}(M) = d_1 \cdots d_{k-1}$

M_0

Acceptance iff M halts on $Code_{TM}(M)$

$\Rightarrow$ Contradiction since $H_{TM} \notin DEC_{TM}$

Simulation of P_0 by a TM M_0
Simulation of any test $R_h(r\, Code_{TM}(M_s))$ by simulation of $|r|$ steps of M_s

$H_{TM} \notin DEC_{TM} \Rightarrow SAT_{halt} \notin DEC_{\Sigma_{R_h}}$

$SAT_{\Sigma_{R_h}} \notin DEC_{\Sigma_{R_h}}$

$P_{\Sigma_{R_h}} \neq NP_{\Sigma_{R_h}}$

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The description of SAT_{Σ_R} by a tree

for $\Sigma_R = (\{a, b\}^*; \varepsilon; add_a, add_b, sub_a, sub_b; R, =)$

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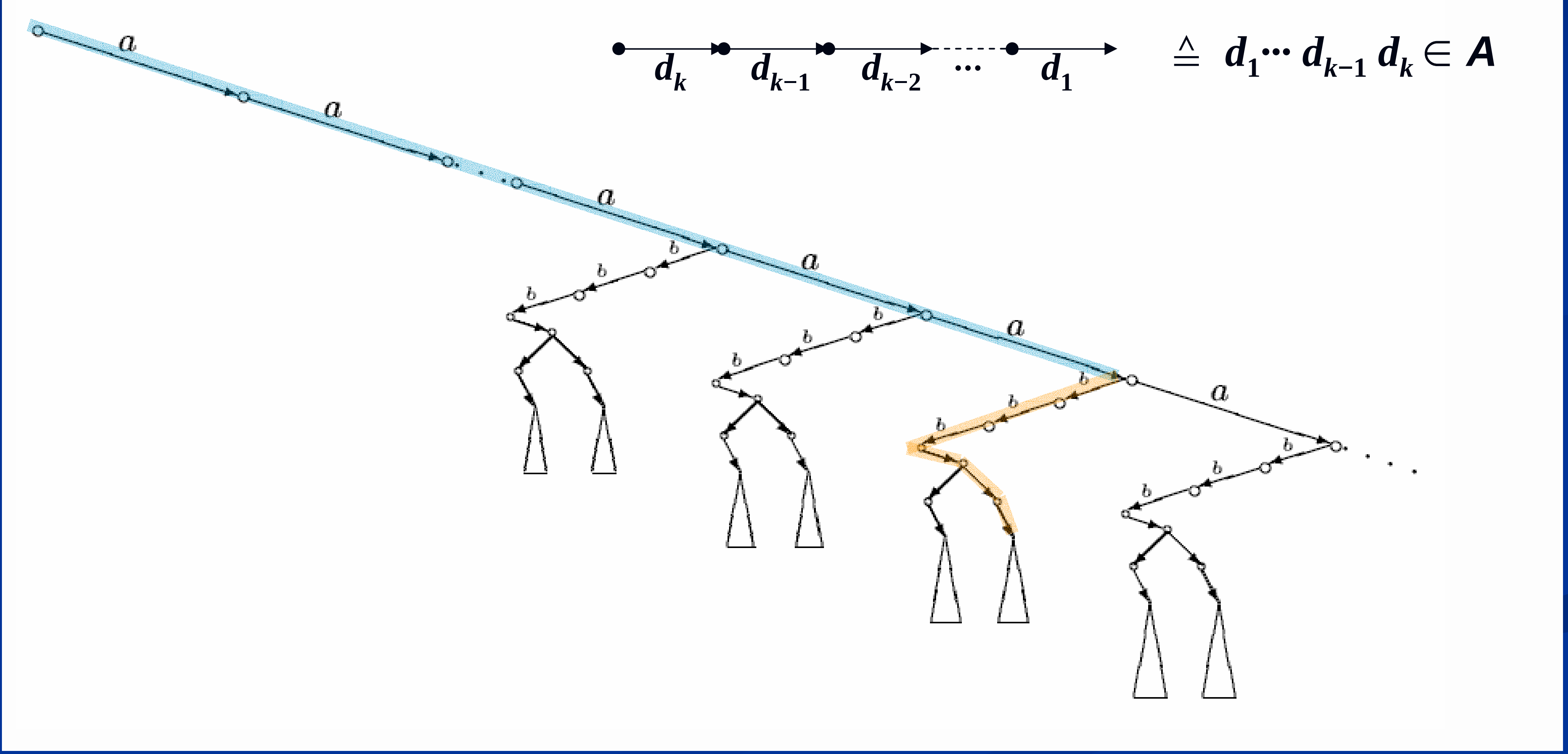
How to construct R such that SAT ∈ P for Σ_R?



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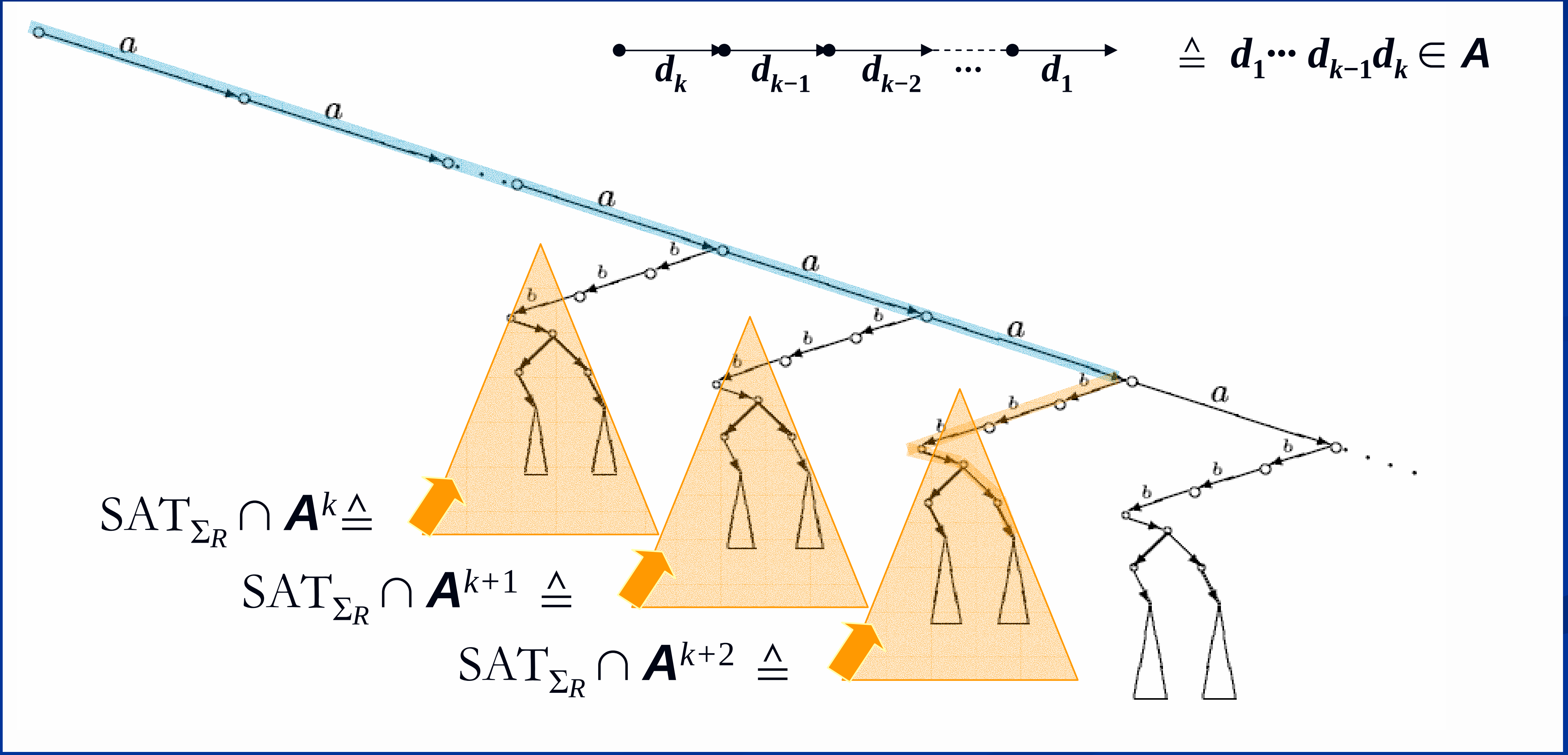

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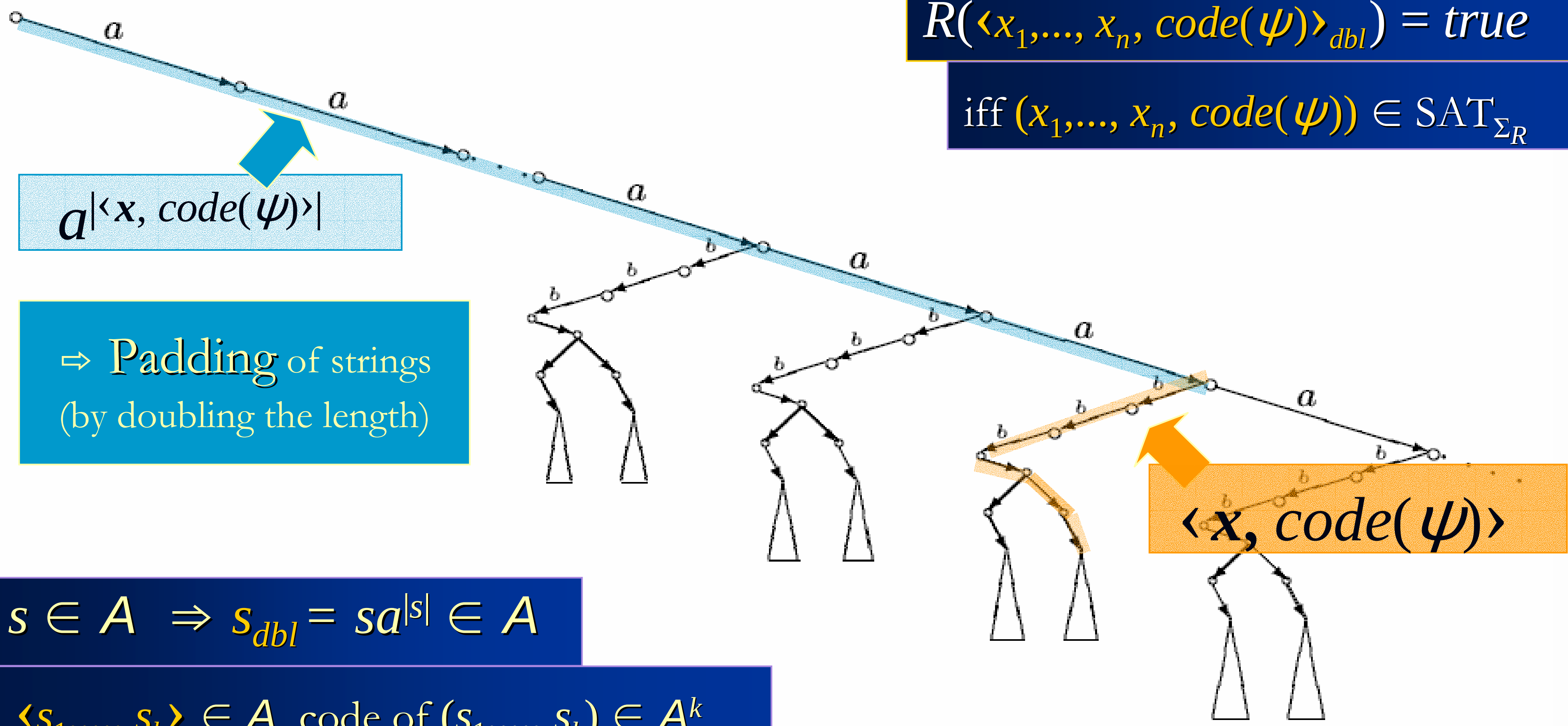

$R(\langle x_1,\ldots, x_n, code(\psi)\rangle_{dbl}) = true$

iff $(x_1,\ldots, x_n, code(\psi)) \in SAT_{\Sigma_R}$

$a^{|\langle x,\, code(\psi)\rangle|}$

⇨ Padding of strings (by doubling the length)

$\langle x, code(\psi)\rangle$

$s \in A \;\Rightarrow\; s_{dbl} = sa^{|s|} \in A$

$\langle s_1,\ldots, s_k\rangle \in A$ code of $(s_1,\ldots, s_k) \in A^k$

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Trees

for

$\Sigma_R = (\{a, b\}^*; \varepsilon; add_a, add_b, sub_a, sub_b; R, =)$

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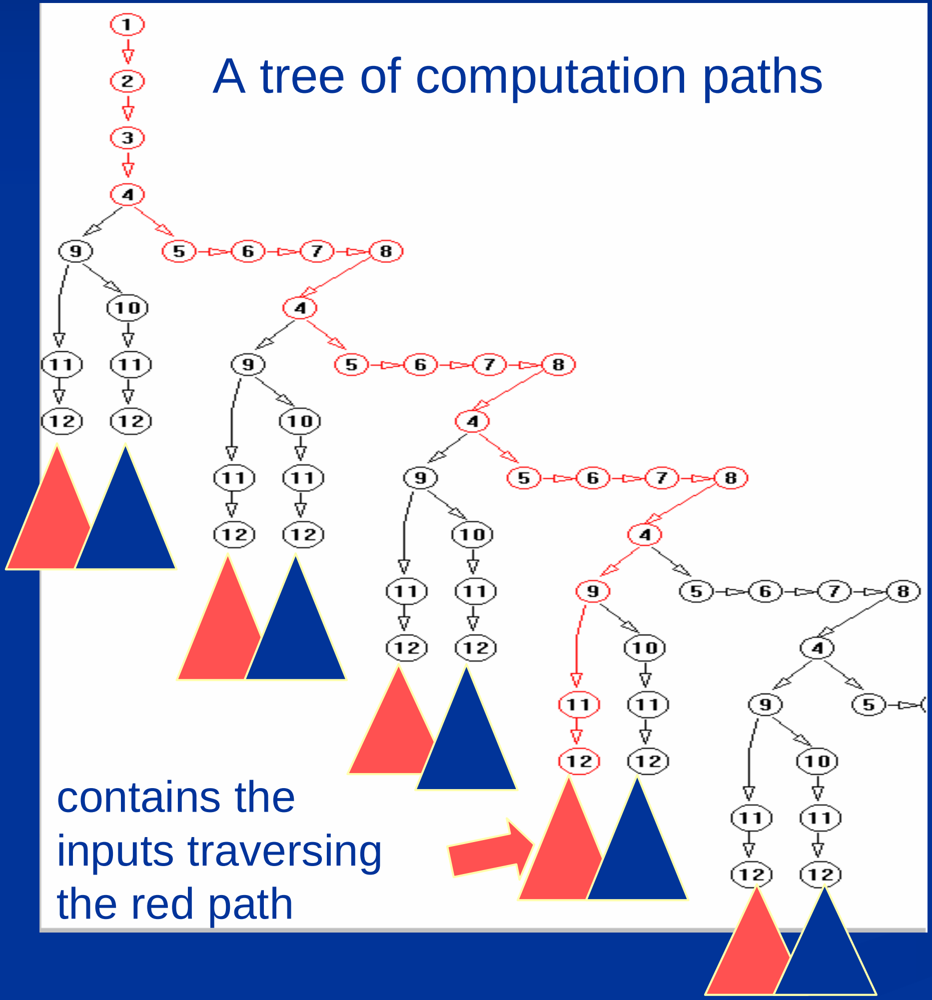

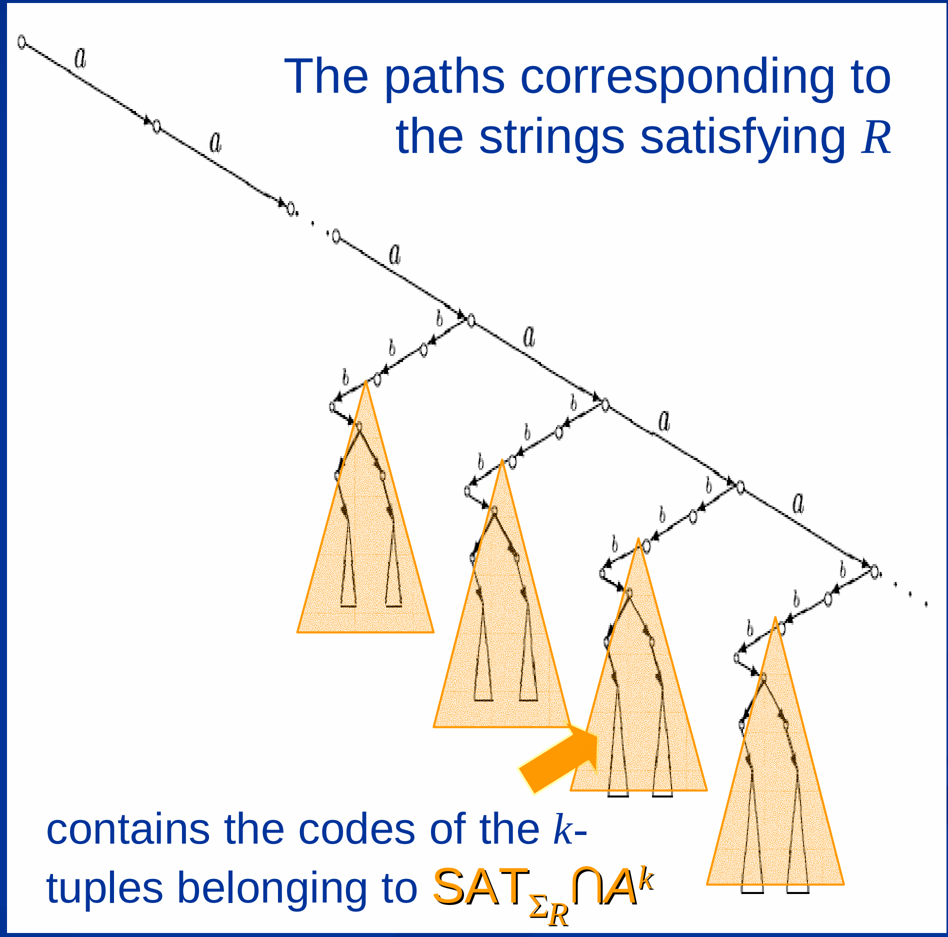

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Trees and polynomial time

for $\Sigma_R = (\{a, b\}^*; \varepsilon; add_a, add_b, sub_a, sub_b; R, =)$

- The uniform model of computation
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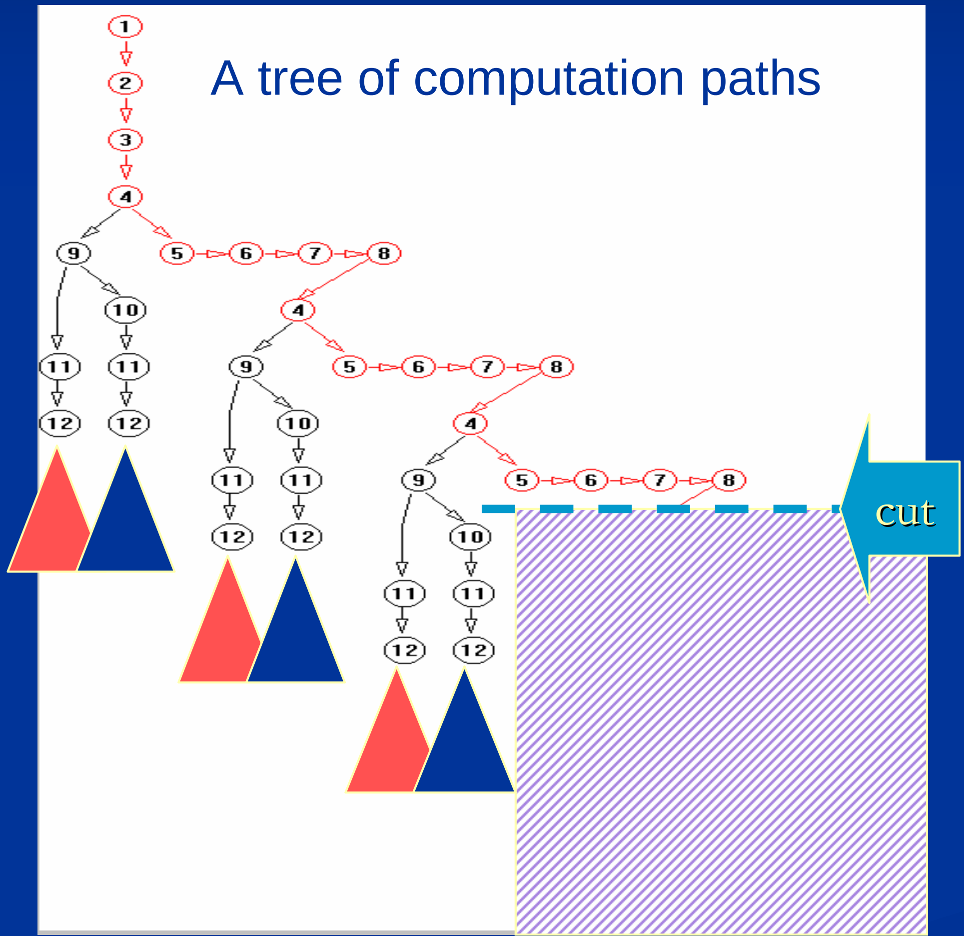

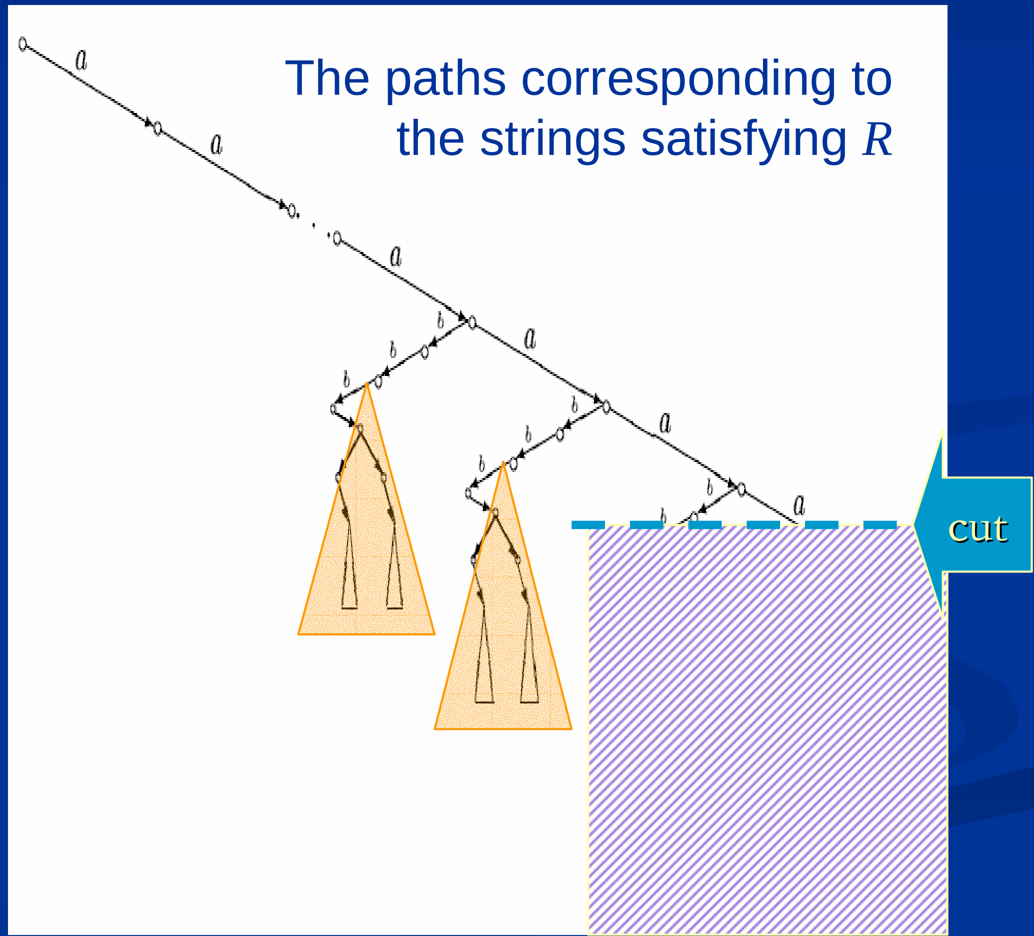

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Trees and polynomial time

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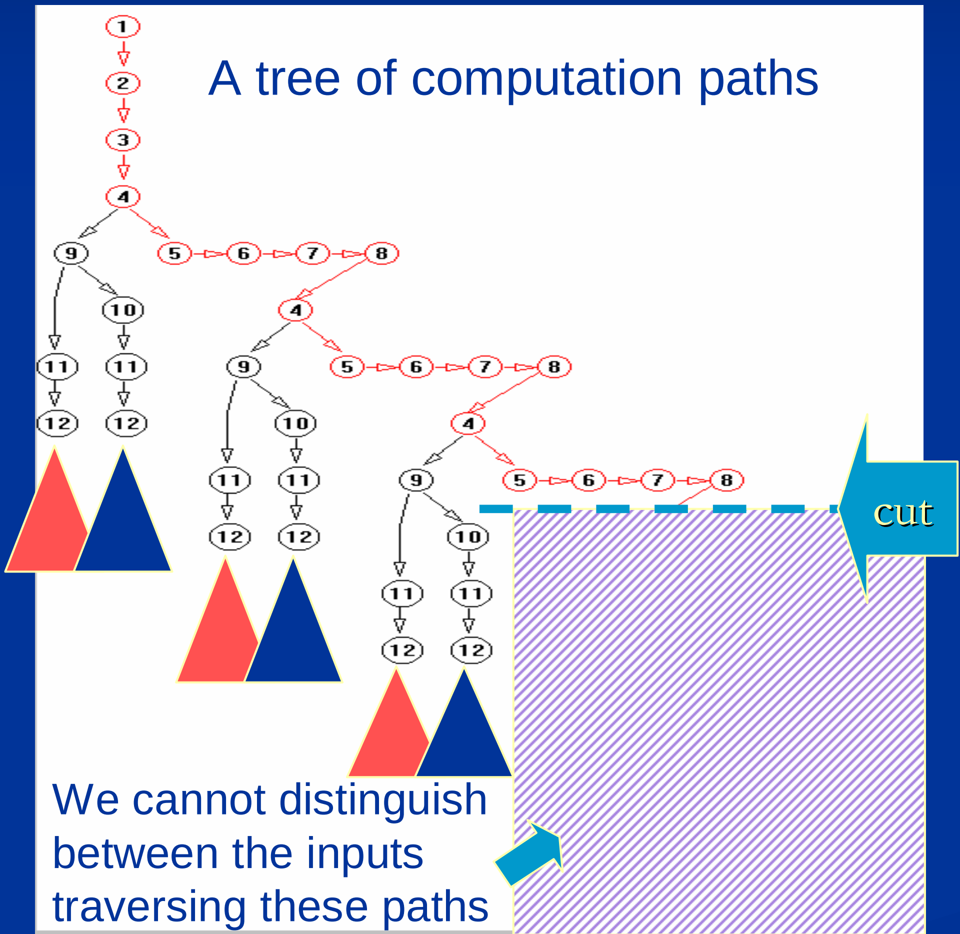

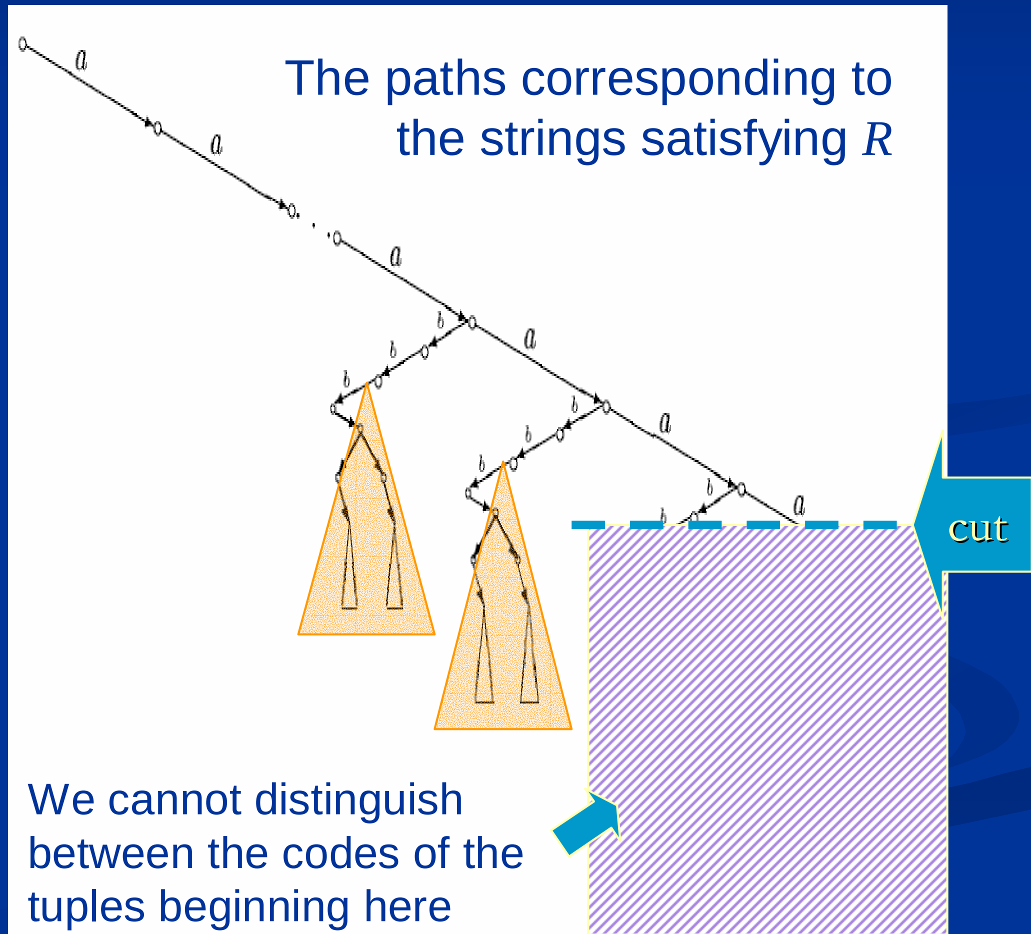

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