Non-unitary quantum walks:

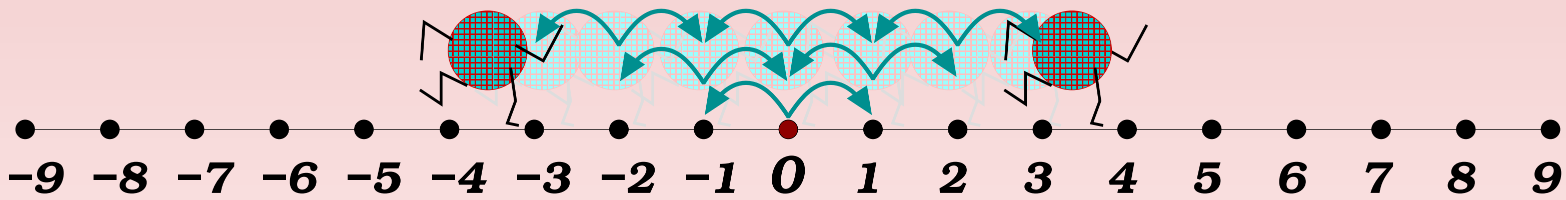
exploring the space between classical and quantum computing

Viv Kendon

Quantum Information
School of Physics
& Astronomy
University of Leeds
Leeds LS2 9JT
V.Kendon@leeds.ac.uk

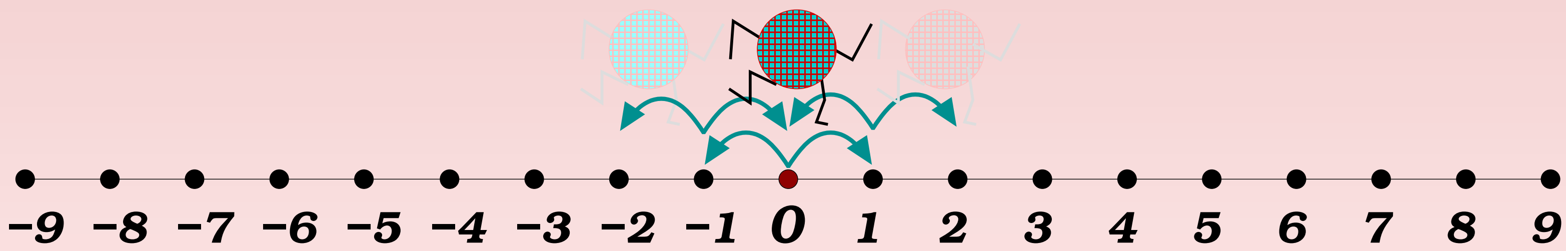
Review article on decoherence in quantum walks

`quant-ph/0606016`

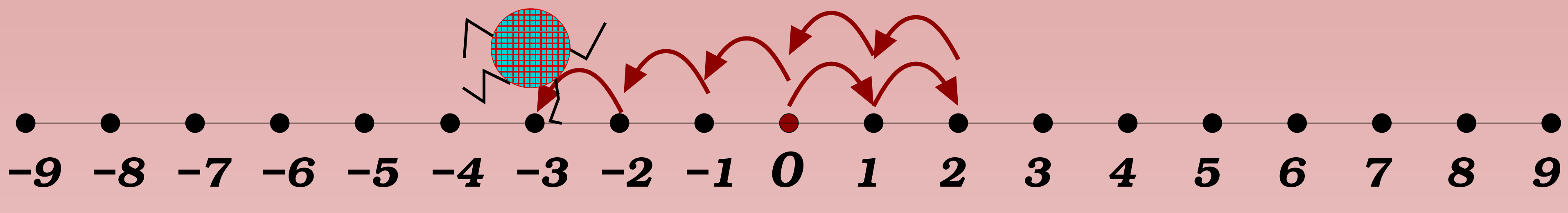


Overview

1. Pure quantum walks
2. Add decoherence: non-unitary quantum walks
3. Mixing times on cycles
4. Other examples
5. Summary



Classical Random Walk on a Line



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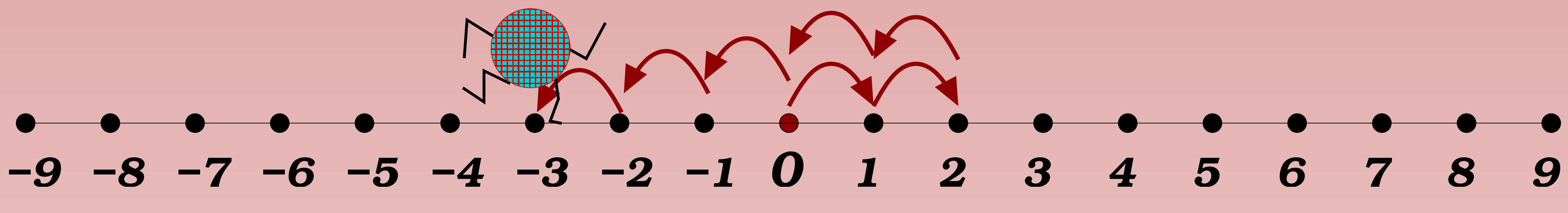

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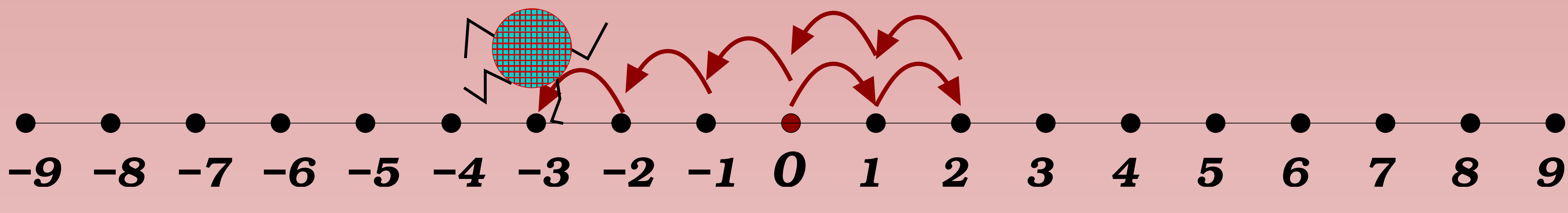

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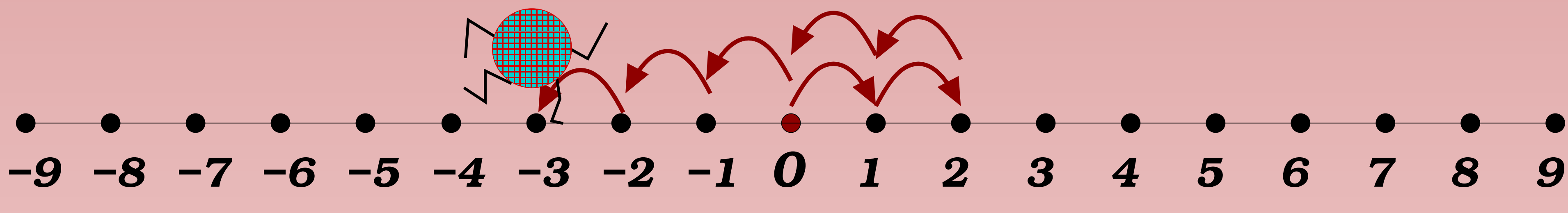

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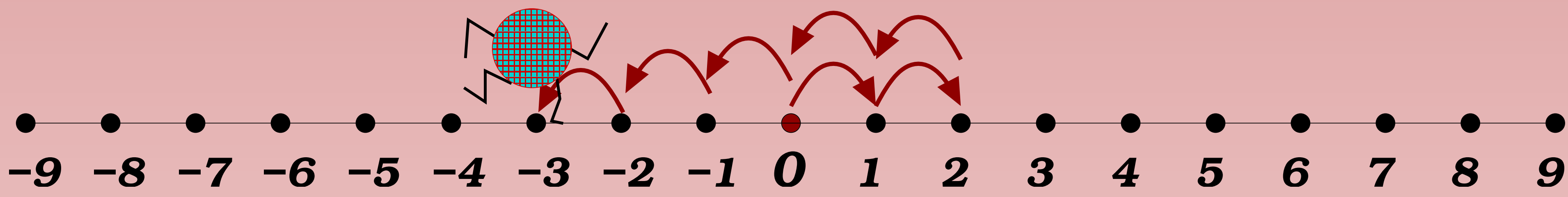

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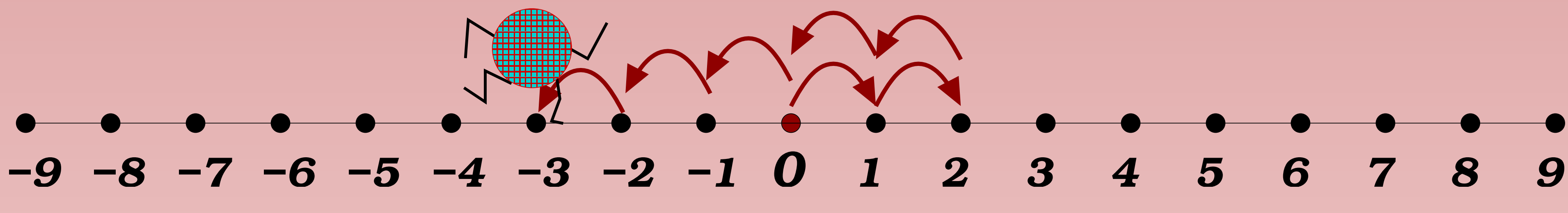

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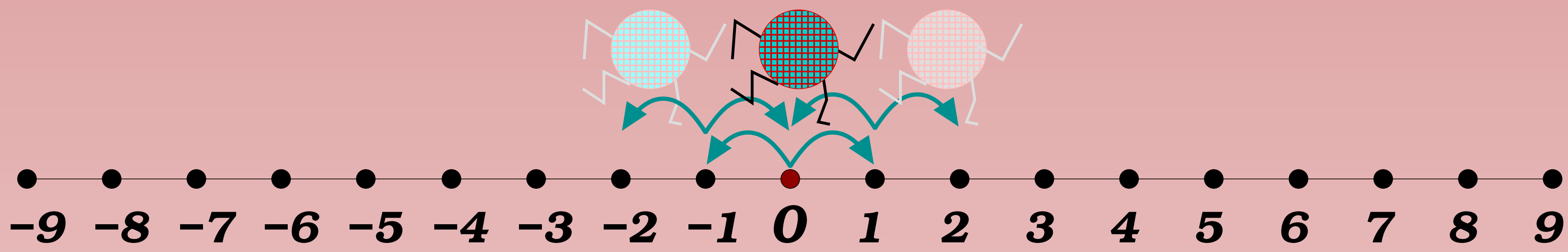
Repeat steps 1. to 5. many times $\longrightarrow$ prob. dist. $P(x, T)$, binomial

standard deviation $\langle x^2 \rangle^{1/2} = \sqrt{T}$

Quantum Walk on a Line



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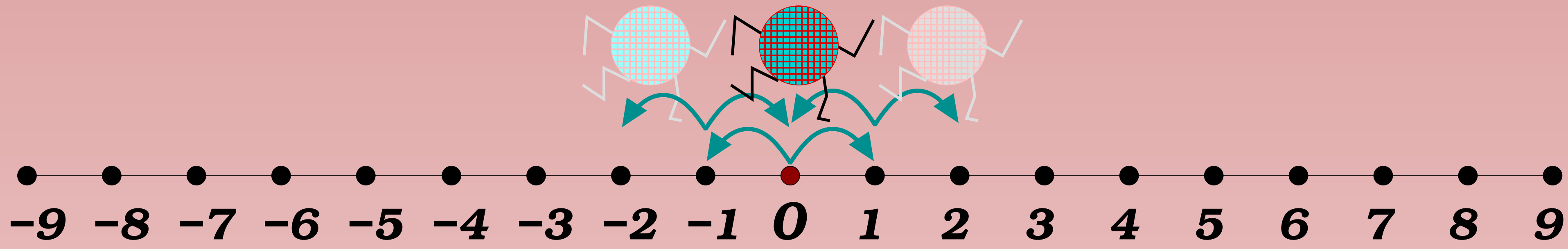

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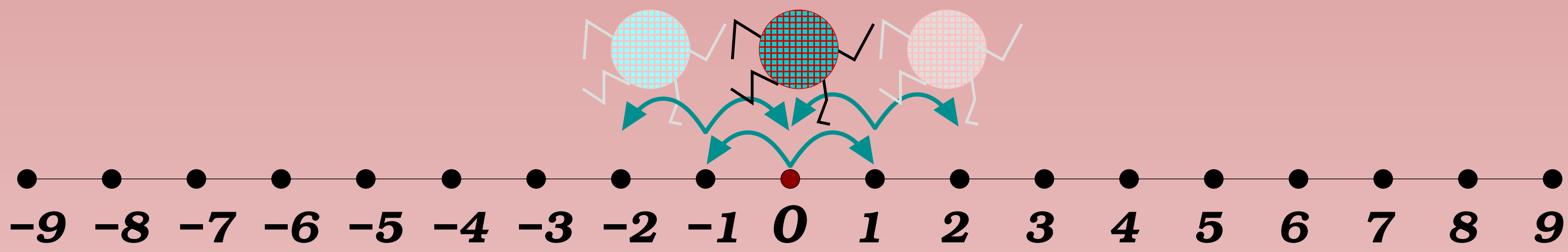

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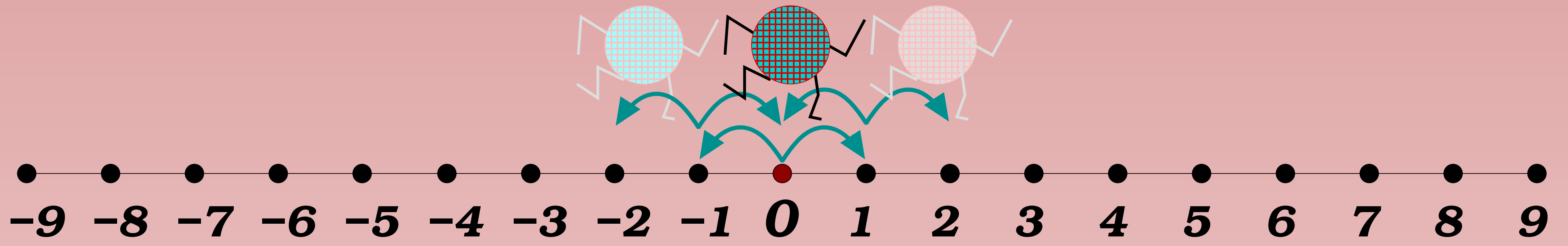

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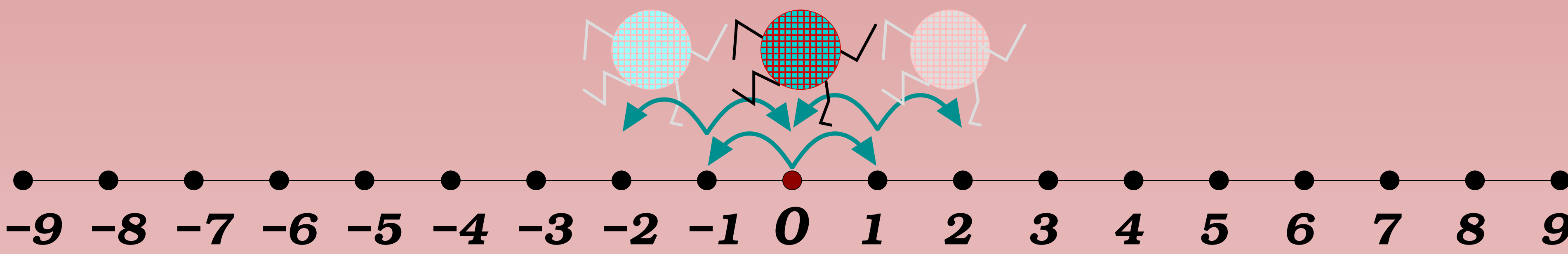

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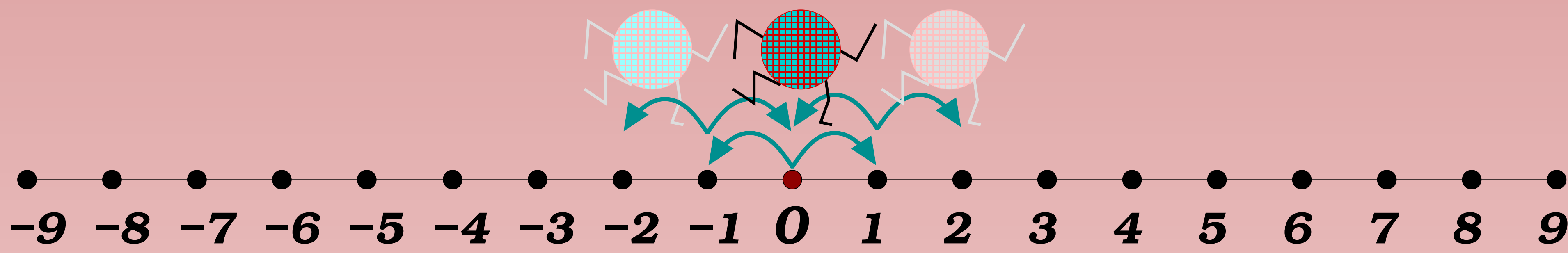

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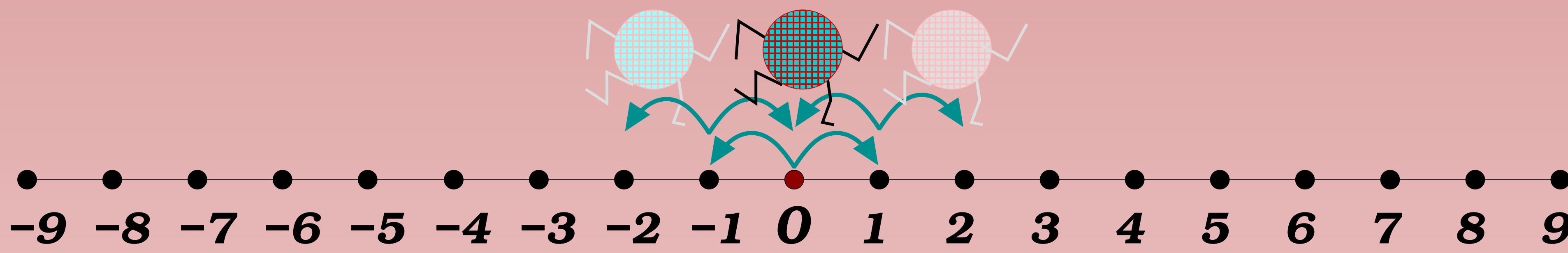

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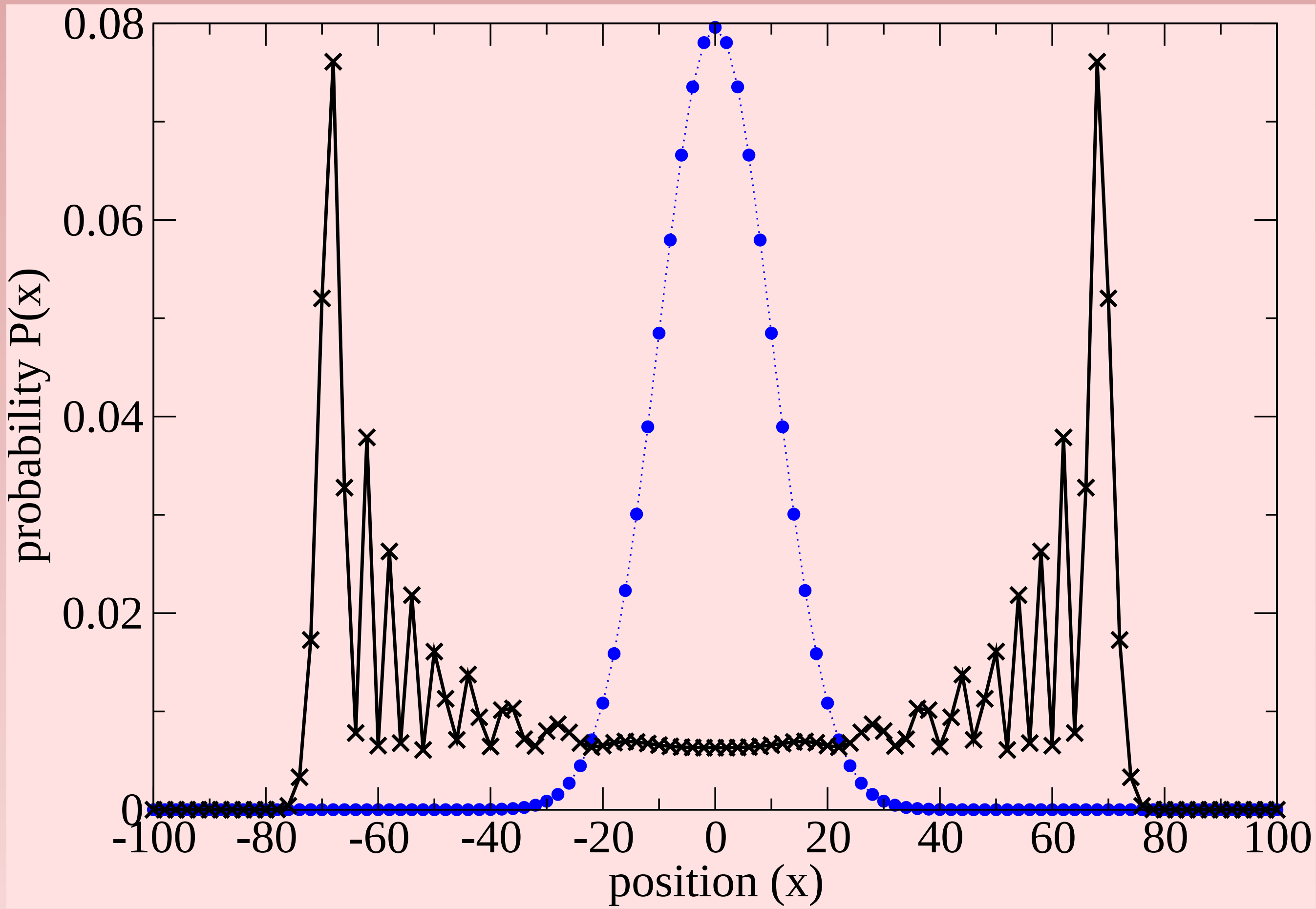
Quantum vs Classical on a Line



quantum spread $\propto T$ compared with classical $\sqrt{T}$

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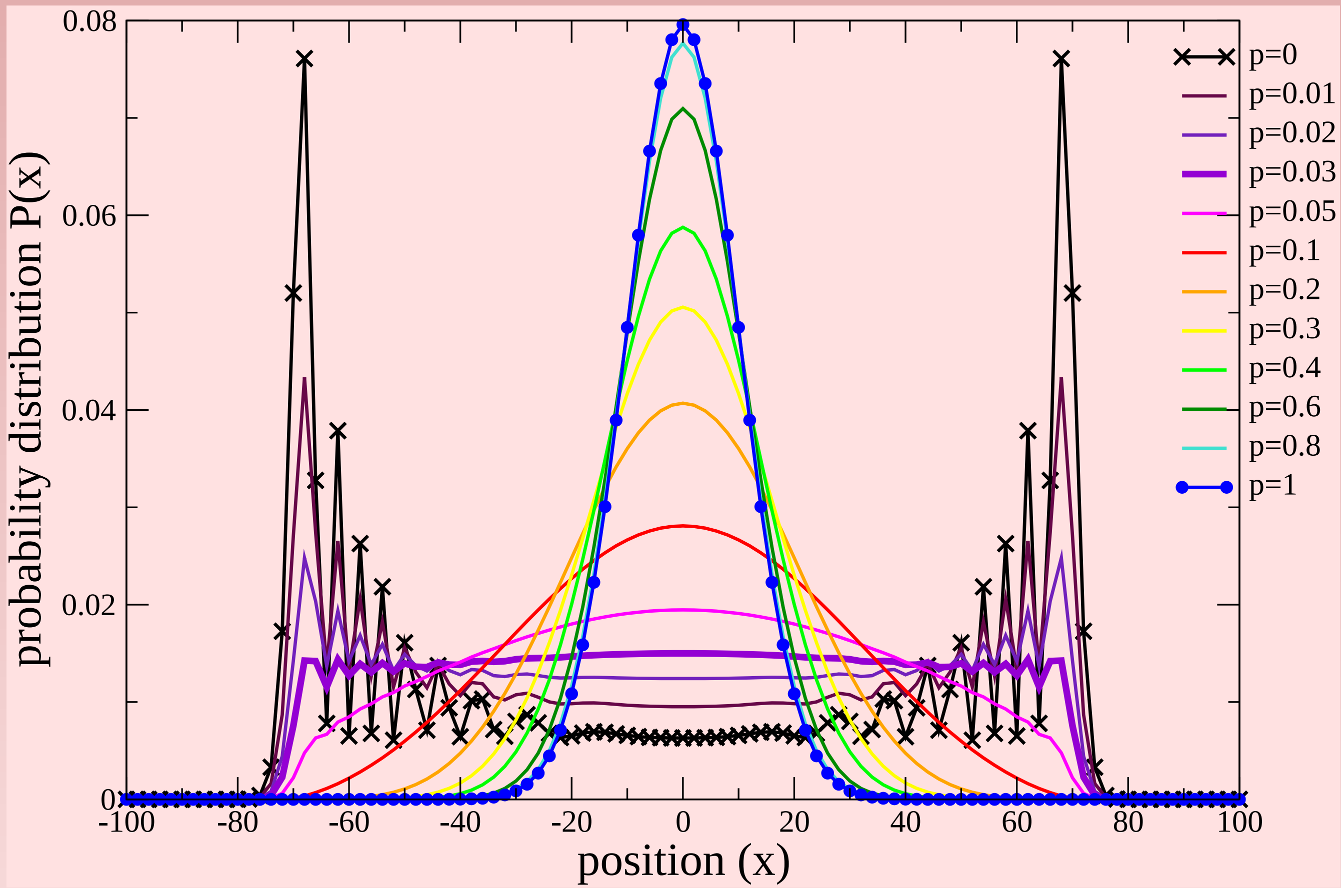
Add decoherence (measure with prob p at each step):



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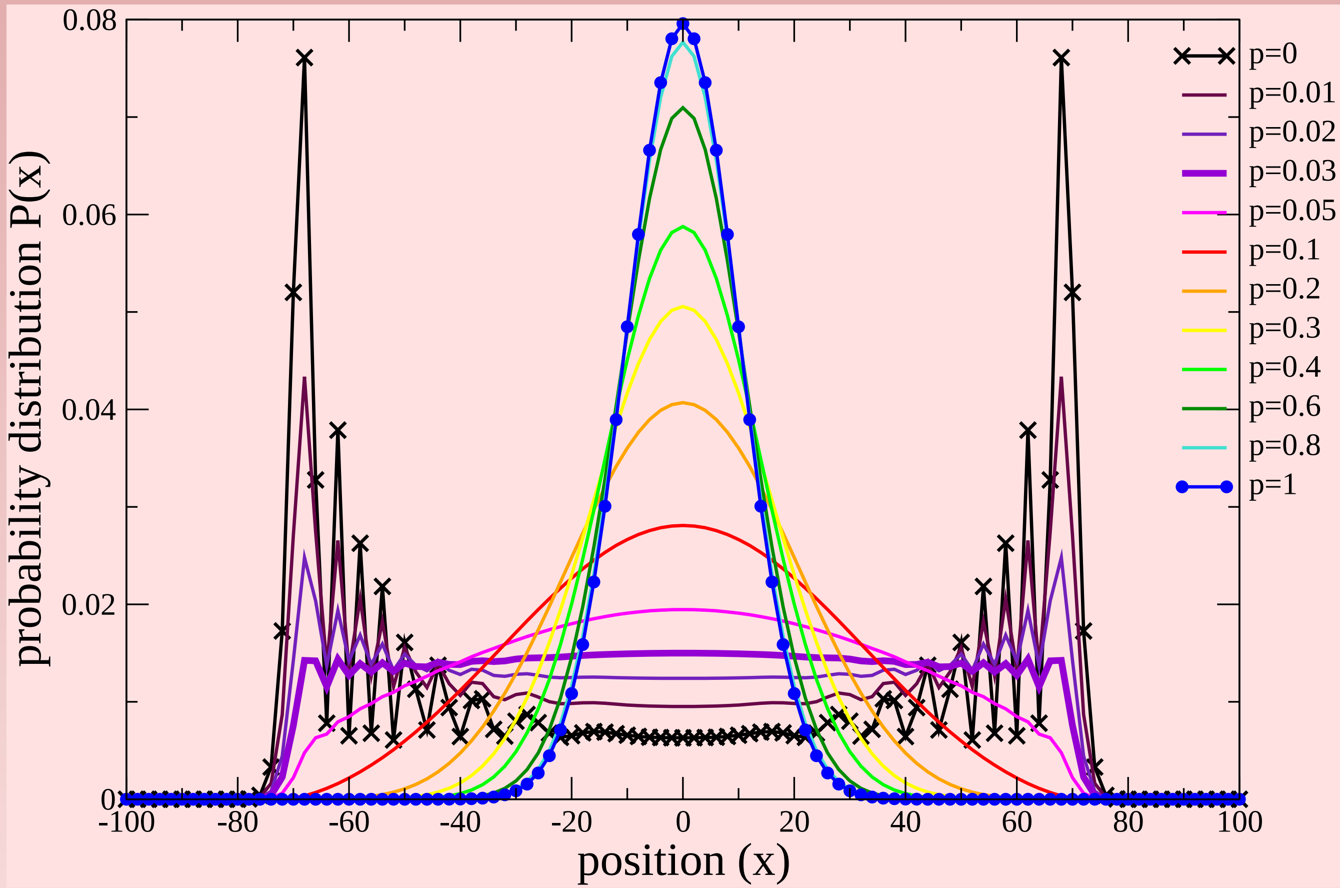

Top hat distribution for just the right amount of noise! [quant-ph/0209005]

Continuous Time Quantum Walk

Childs *et al.* give an approximate solution to the “glued trees” problem using a continuous time walk:

$\mathbf{A}$ – adjacency matrix of the graph ($A_{jk} = 1$ iff $\exists$ an edge between sites j and k)

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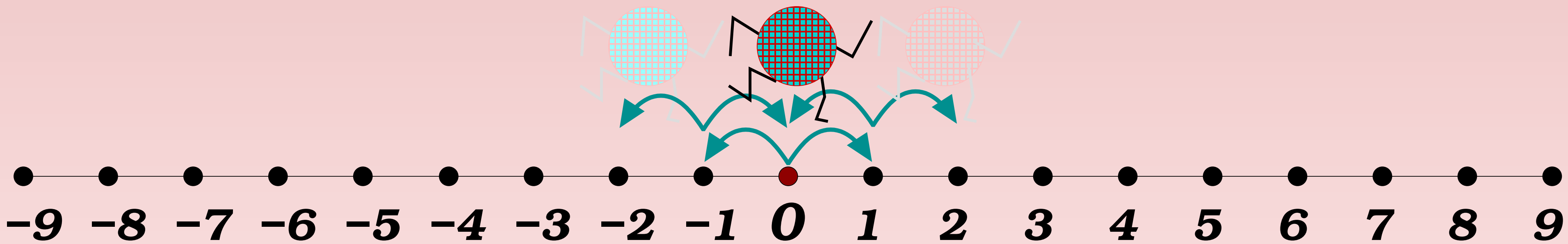
Walk is simply $e^{-i\mathbf{H}t}$ followed by measurement at suitable time t

Making an algorithm involves significant detail (oracle, colouring...)



Algorithms with quantum walks

"Glued trees" problem:

Find your way from

"Entrance" to "Exit"

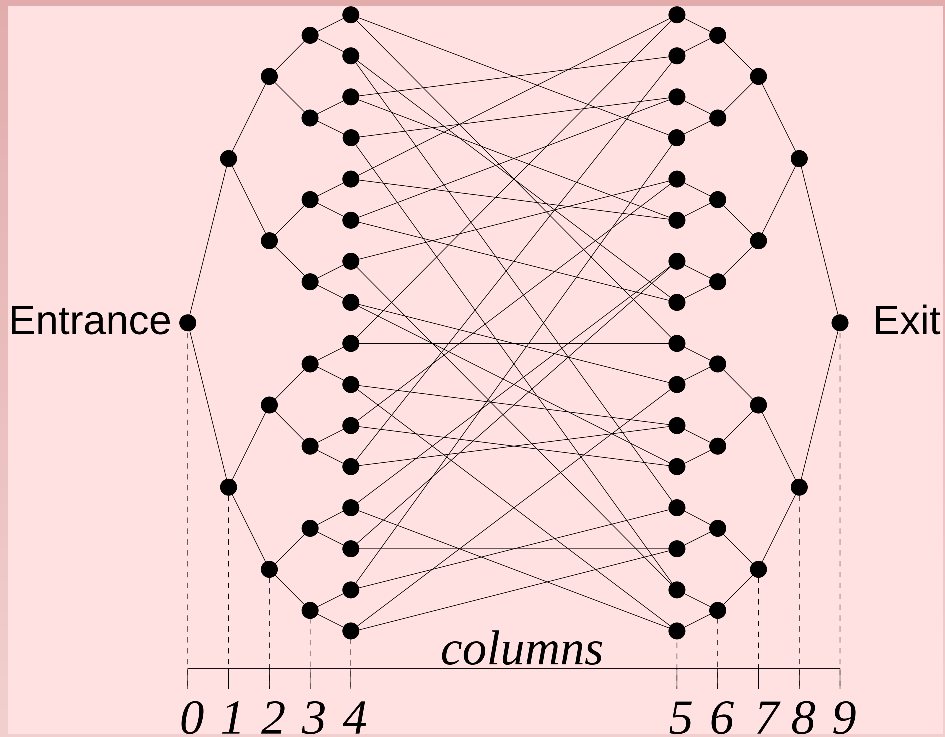
`Childs, Cleve,`

`Deotto, Farhi,`

`Gutmann, Spielman,`

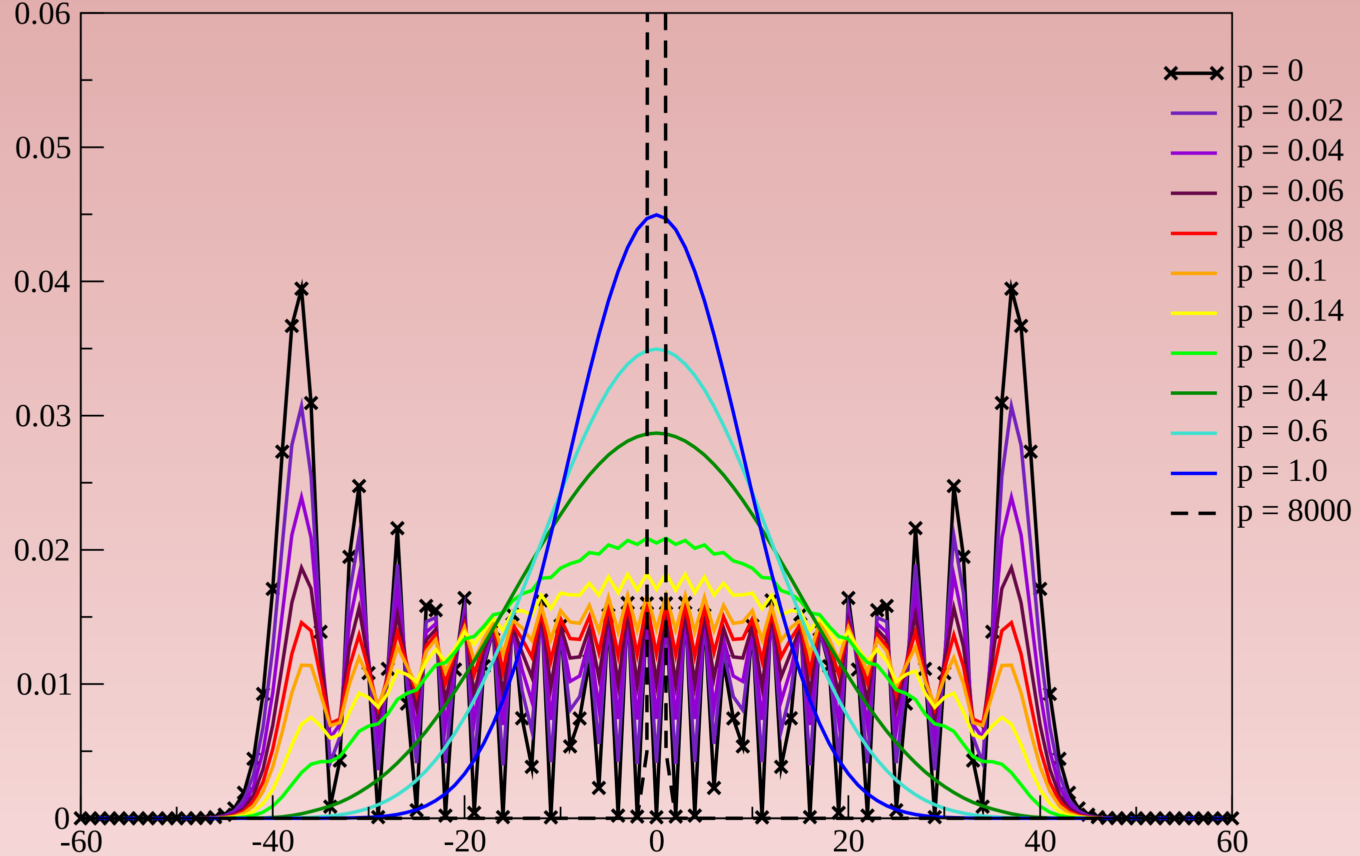
`quant-ph/0209131`

`(STOC 2003)`



Proof in principle that quantum walk algorithms can give exponential speed up

Decohering Continuous-time Walk on a Line



Not such a clear “top-hat” as discrete-time...

Decoherence Model for Quantum Walks

Discrete time quantum walk ($\mathbf{S}$ = shift, $\mathbf{C}$ = coin toss):

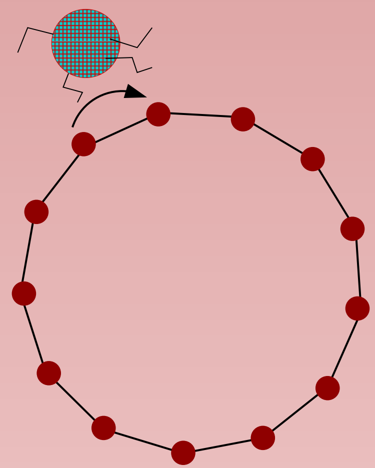
$$\boldsymbol{\rho}(t+1) = (1-p)\mathbf{S}\mathbf{C}\boldsymbol{\rho}(t)\mathbf{C}^\dagger\mathbf{S}^\dagger + p\sum_i \mathbb{P}_i\mathbf{S}\mathbf{C}\boldsymbol{\rho}(t)\mathbf{C}^\dagger\mathbf{S}^\dagger\mathbb{P}_i^\dagger$$

Continuous time quantum walk:

$$\frac{d\boldsymbol{\rho}(t)}{dt} = -i\gamma[\mathbf{A}, \boldsymbol{\rho}] - p\boldsymbol{\rho} + p\sum_i \mathbb{P}_i\boldsymbol{\rho}\mathbb{P}_i^\dagger$$

rate of decoherence: p per unit time

Mixing times on finite graphs



Quantum walks don't mix – *unitary, reversible*

Time averaged distributions

$$\overline{P(x,p,T)} = \frac{1}{T}\sum_{t=0}^{T} P(x,p,t)$$

always mix:

$$M_\epsilon = \min\left\{T|\forall t > T : ||\overline{P(x,p,t)} - P_u||_{\text{tv}} < \epsilon\right\}$$

where P_u is the limiting distribution.

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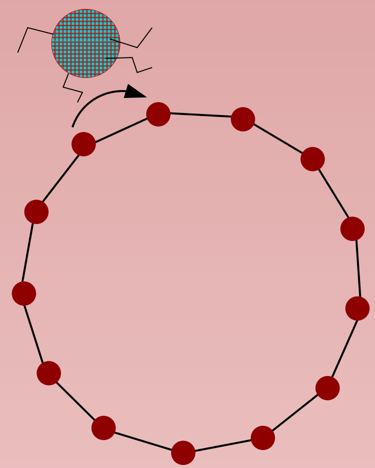

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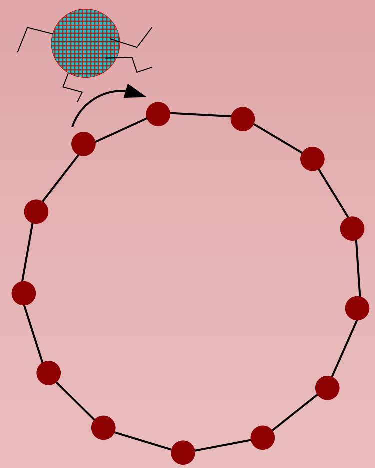

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At most quadratic speed up of mixing time over classical

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BUT – P_u *not always the uniform distribution* (classical: always uniform)

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and causes **faster** mixing than pure quantum! `[quant-ph/0209005]`

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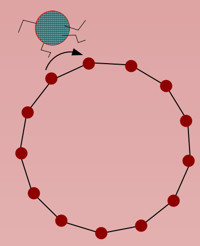

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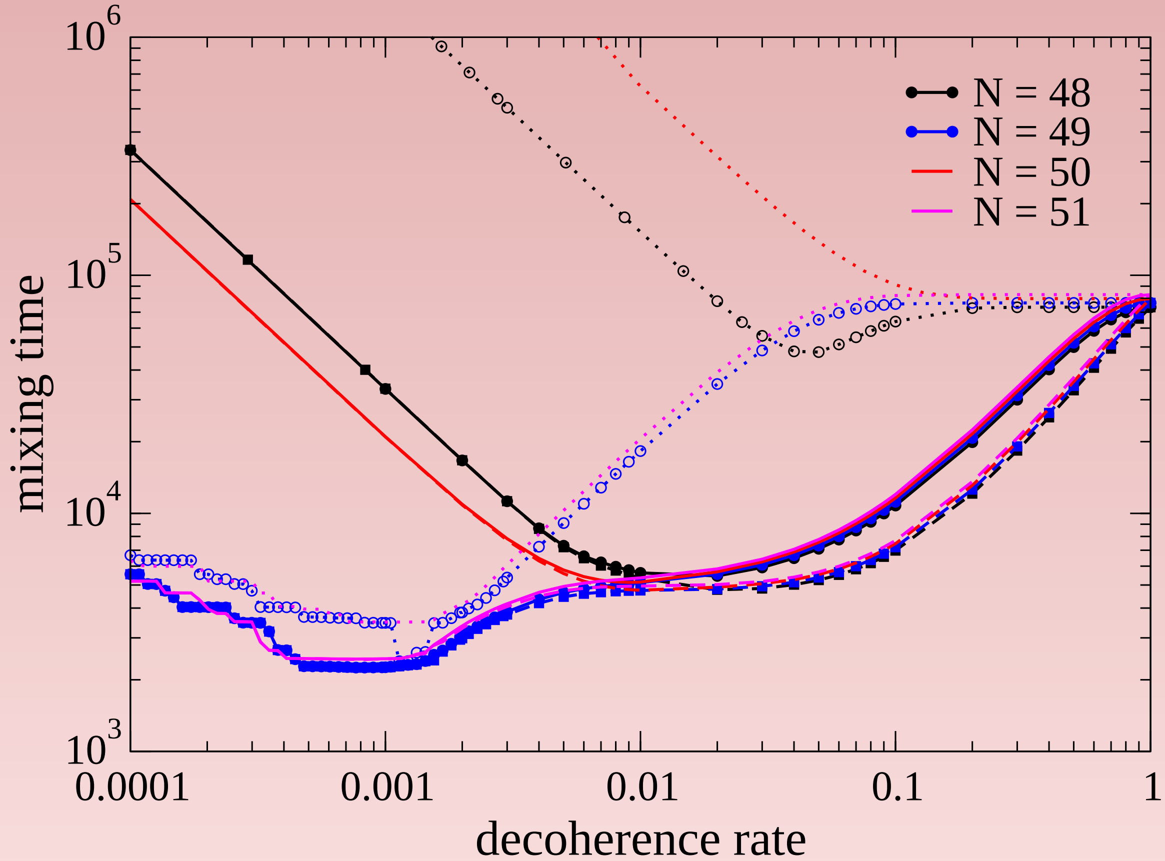
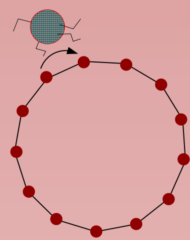


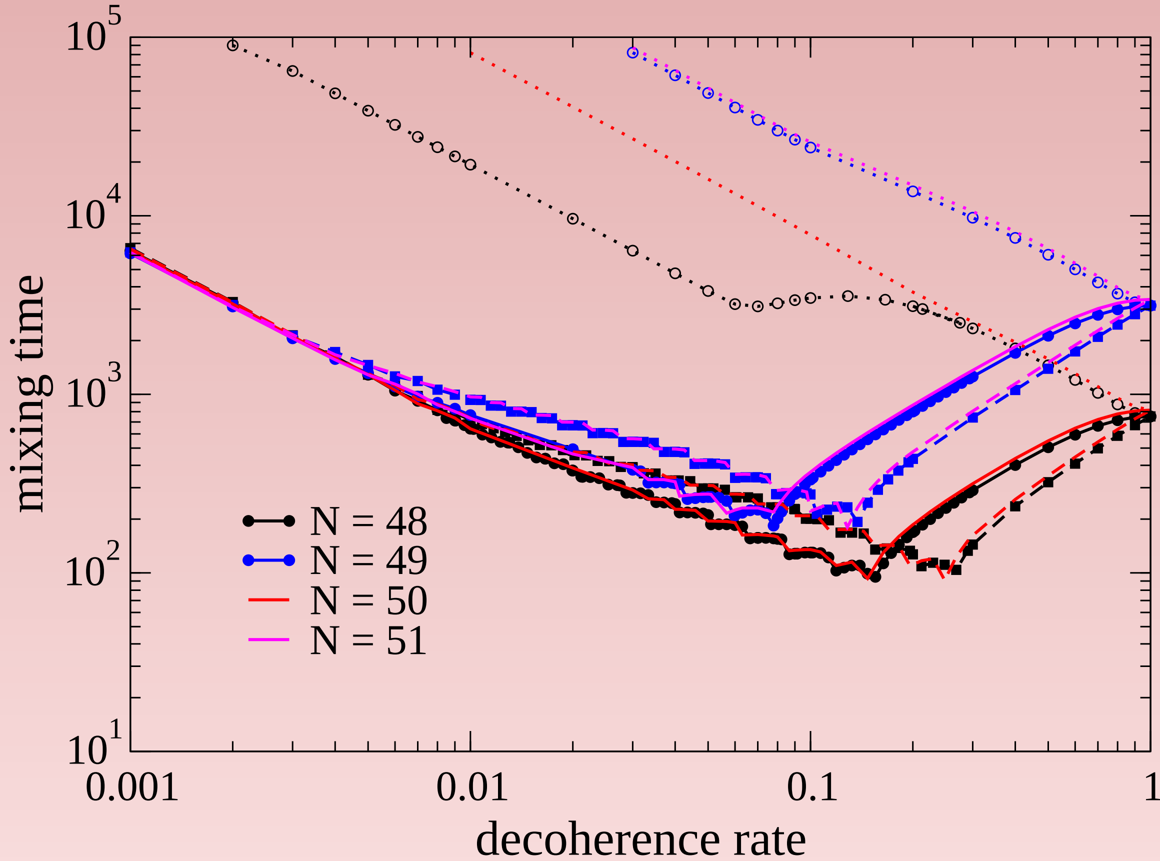
mixing time for $\overline{P(x,T)}$

(dotted lines) coin decoherence does not produce extra speed-up

(dashed lines) particle decoherence gives quantum speed-up

...even without time-averaging





mixing time for $P(x, t)$

sharp minimum between "underdamped" and "overdamped"

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- calculate entanglement between coin and position (in progress)

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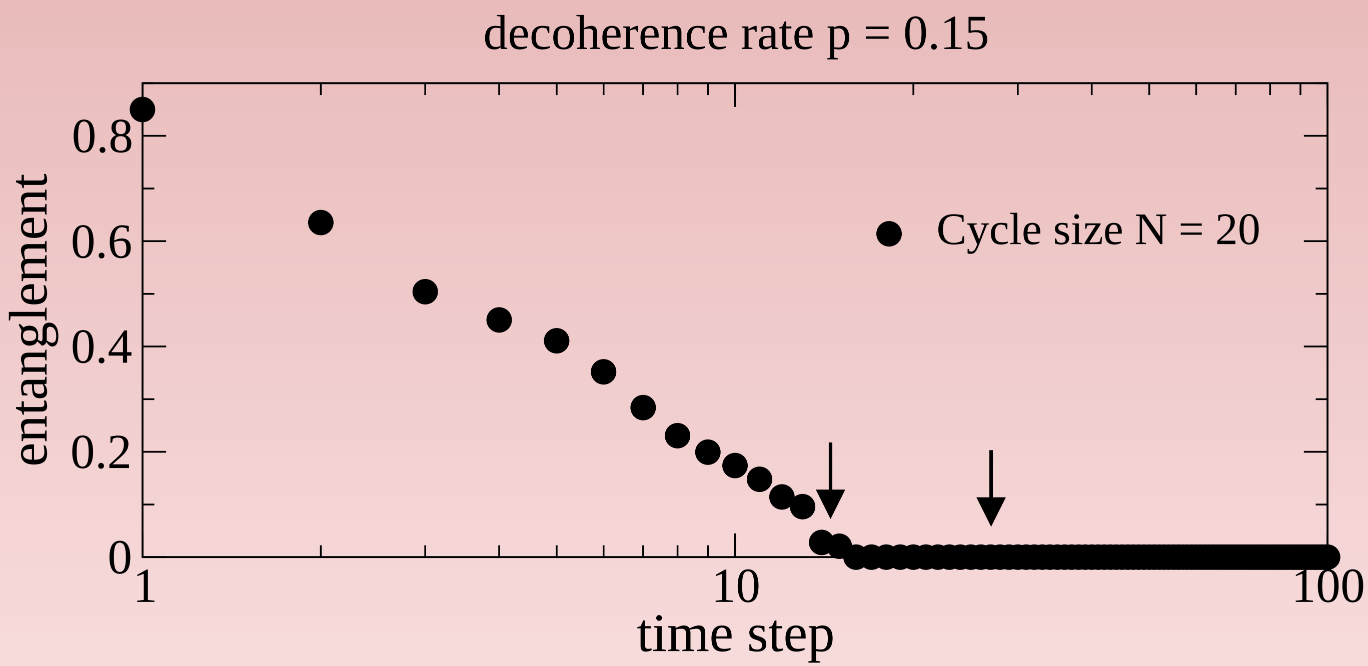
  – initial results say **NO**



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  Takes $\overline{M}_\epsilon$ from $O(N/\epsilon)$ to $O(N \log \epsilon^{-1})$
- More general study and proofs from Richter, quant-ph/0606202
- Added randomness improves other quantum protocols too:

  e.g., Quantum Key Distribution, Kraus, Gisin, Renner, PRL 95, 080501 (2005)

Summary & Open Questions

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- Another quantum view of added randomness is entanglement with environment

  (note: LOTS of work on randomness in quantum communications)

- Is it possible to take advantage when building quantum computers?

  (usually one fights to remove all the noise...)

- At least, scaing of errors/accuracy is very important in practical applications.

Acknowledgements and Funders

I have had interesting and helpful discussions of quantum walks with many, in particular:

Dorit Aharonov (Hebrew U)

Sougato Bose (UCL), Todd Brun (USC)

Richard Cleve, Andris Ambainis (U Waterloo/PI)

Julia Kempe (LRI Paris Orsay)

Andrew Childs (Caltech), Ed Farhi (MIT)

Mark Hillery (Hunter Col City U NY)

Peter Høyer, Barry Sanders, John Watrous (U Calgary)

Peter Knight, Ivens Carneiro†, Ben Tregenna,

Will Flanagan, Rik Maile, Xibai Xu, Meng Loo (Imperial)

Cris Moore (New Mexico), Alex Russell (Connecticut)

Eugenio Roldán (U València), John Sipe (U Toronto)

Fred Strauch (NIST), Mario Szegedy (Rutgers)

Tino Tamon (Clarkson U)

† *We learnt with great sadness of his accidental death in April 2006.*

Funding:

EPSRC

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