Can general relativistic computers break the Turing barrier?

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- Can general relativistic computers break the Turing barrier?
- Are there final limits to human knowledge?
- Limitative results versus human creativity (paradigm shifts).
- Gödel's logical results in comparison/combination with Gödel's relativistic results.
- Can Hilbert's programme be carried through after all?

1 Aims, perspective

The Physical Church-Turing Thesis, PhCT, is the conjecture that whatever physical computing device (in the broader sense) or physical thought experiment will be designed by any future civilization, it will always be simulatable by a Turing machine. The PhCT was formulated and generally accepted in the 1930's. At that time a general consensus was reached declaring PhCT valid, and indeed in the succeeding decades the PhCT was an extremely useful and valuable maxim in elaborating the foundations of theoretical computer science, logic, foundation of mathematics and related areas. But since PhCT is partly a physical conjecture, we emphasize that this consensus of the 1930's was based on the physical world-view of the 1930's. Moreover, many thinkers considered PhCT as being based on mathematics + common sense. But "common sense of today" means "physics of 100 years ago". Therefore we claim that the consensus accepting PhCT in the 1930's was based on the world-view deriving from Newtonian mechanics. Einstein's equations became known to a narrow circle of specialists around 1920, but around that time the consequences of these equations were not even guessed at. The world-view of modern black hole physics was very far from being generally known until much later, until after 1980.

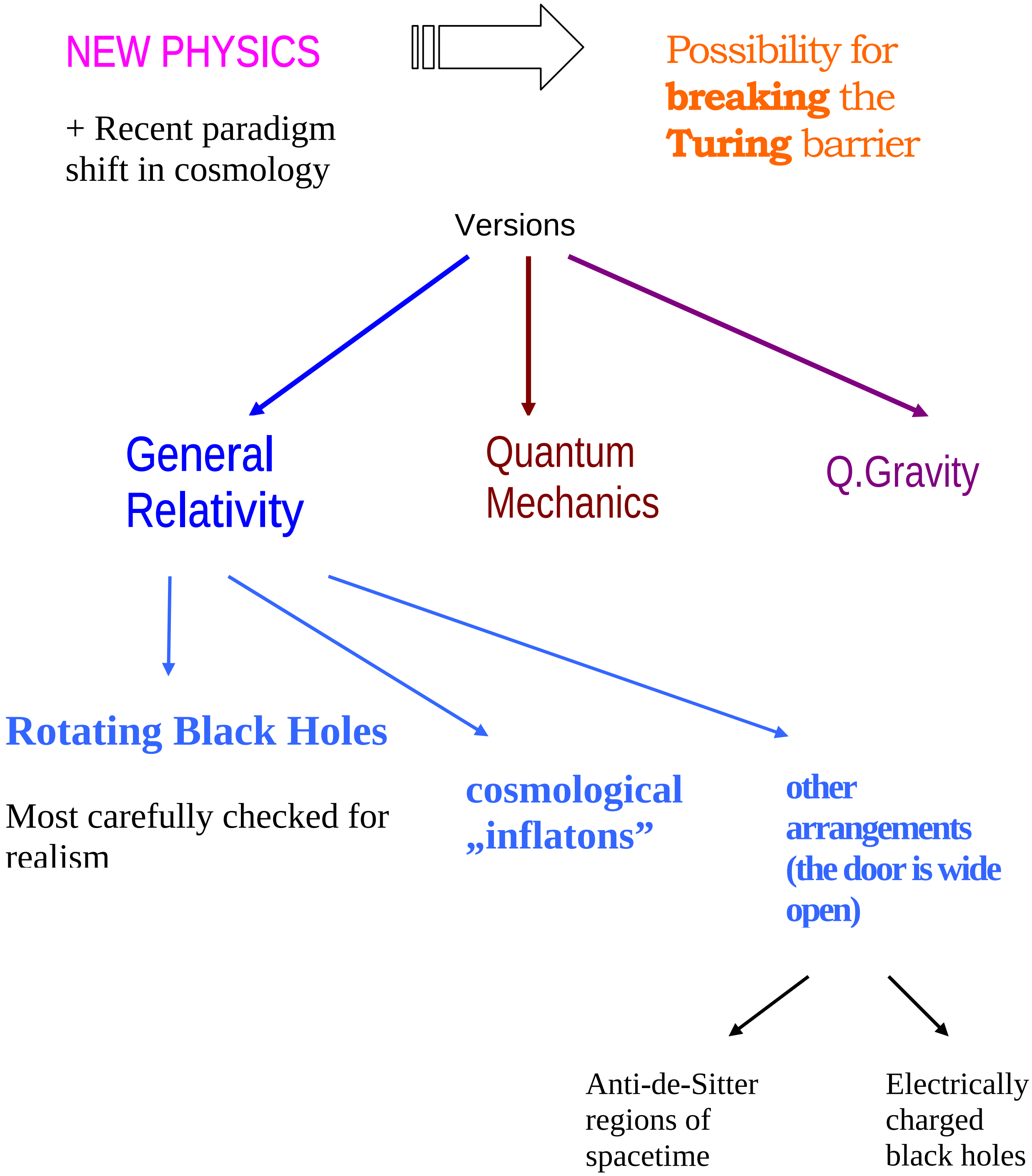
Our main point is that in the last few decades (well after 1980) there has been a major paradigm shift in our physical world-view. This started in 1970 by Hawking's and Penrose's singularity theorem firmly establishing black hole physics and putting general relativity into a new perspective. After that, discoveries and new results have been accelerating. About 10 years ago astronomers obtained firmer and firmer evidence for the existence of larger and larger more exotic black holes [18],[17] not to mention evidence supporting the assumption that the universe is not finite after all [20]. Nowadays the whole field is in a state of constant revolution. If the background foundation on which PhCT was based has changed so fundamentally, then it is desirable to re-examine the status and scope of applicability of PhCT in view of the change of our general world-picture. Cf. also [5] for a related perspective.

A special feature of the Newtonian world-view is the assumption of an absolute time scale. Indeed, this absolute time has its mark on the Turing machine as a model for computer. As a contrast, in general relativity there is no absolute time. Kurt Gödel was particularly interested in the exotic behavior of time in general relativity (GR). Gödel [8] was the first to prove that there are models of GR to which one cannot add a partial order satisfying some natural properties of a "global time". In particular, in GR various observers at various points of spacetime in different states of motion might experience time radically differently. Therefore we might be able to speed up the time of one observer, say C (for "computer"), relatively to the other observer, say P (for "programmer"). Thus P may observe C computing very fast. The difference between general relativity and special relativity is (roughly) that in general relativity this speed-up effect can reach, in some sense, infinity assuming certain conditions are satisfied. Of course, it is not easy to ensure that this speed-up effect happens in such a way that we could utilize it for implementing some non-computable functions.

In [7], [15] we prove that it is consistent with Einstein's general relativity theory that by certain kinds of relativistic experiments, future generations might find the answers to non-computable questions like the halting problem of Turing machines or the consistency of Zermelo Fraenkel set theory (the foundation of mathematics, abbreviated as ZFC set theory from now on). For brevity, we call such thought experiments *relativistic computers.* Moreover, the spacetime structure we assume to exist in these experiments is based in [7],[15] on huge slowly rotating black holes the existence of which is made more and more likely (almost certain) by recent astronomical observations [18],[17].

We are careful to avoid basing the beyond-Turing power of our computer on "side-effects" of the idealizations in our mathematical model/theory of the physical world. For example, we avoid relying on infinitely small objects (e.g. pointlike test particles, or pointlike bodies), infinitely elastic balls, infinitely (or arbitrarily) precise measurements, or anything like these. In other words, we make efforts to avoid taking advantage of the idealizations which were made when GR was set up. Discussing physical realizability and realism of our design for a computer is one of the main issues studied in [15].

The next three pages summarize the ideas said so far.


NEW PHYSICS
+ Recent paradigm shift in cosmology
Possibility for breaking the Turing barrier
Versions
General Relativity
Quantum Mechanics
Q.Gravity
Rotating Black Holes
Most carefully checked for realism
cosmological „inflatons”
other arrangements (the door is wide open)
Anti-de-Sitter regions of spacetime
Electrically charged black holes

Church Thesis was formulated in the pre-relativistic (Newtonian) worldview

Turing Machine concept incorporates "ABSOLUTE TIME"

⇓

Believable that after General Relativity (GR) breaking Turing barrier becomes conceivable.

GR ⇒ you can manipulate time (just like space)

Kurt Gödel was fascinated with this feature.

Clockwork Universe ⟺ **“Feedback Universe”**

“Mechanical”
“Newtonian world-view”

Living Universe
Last 10 years of cosmology
Lee Smolin

Barry Cooper

Emergence

Cybernetics
System Theory

Jiri Wiedermann
Jan van Leeuwen

AI, Internet,
Open Systems

Tangible,
hard experimental data

Revolution in cosmology

2 An intuitive idea for how relativistic computers work

In this part of the talk we would like to illuminate the ideas of how relativistic computers work, without going into the mathematical details. The mathematical details are elaborated, among others, in [7], [9], [15]. To make our narrative more tangible, here we use the example of huge slowly rotating black holes for our construction of relativistic computers. But we emphasize that there are many more kinds of spacetimes suitable for carrying out essentially the same construction (these are called Malament-Hogarth spacetimes in the physics literature). So, relativistic computers are not tied to rotating black holes, there are other general relativistic phenomena on which they can be based. An example is anti-de Sitter spacetime which attracts more and more attention in explaining recent discoveries in experimental cosmology. We chose rotating black holes because they provide a tangible example for illustrating the kind of reasoning underlying general relativistic approaches to breaking the "Turing barrier". Astronomical evidence for their existence makes them an even more attractive choice for our didactic purposes.

New freedom (handle on the problem):

We can manipulate time in GR.
(Already in SR. But that is not enough.)

Fourth dimension: “moving in time”

Boldly: Assume for a second (only!!!) that time travel exists.

Time travel → Beyond Turing computer.

Less boldly: Instead of time travel use **huge rotating black holes** (observed).

Let us start out from the so-called Gravitational Time Dilation effect (GTD). The GTD is a theorem of relativity which says that gravity makes time run slow. More sloppily: gravity slows time down. Clocks that are deep within gravitational fields run slower than ones that are farther out. We will have to explain what this means, but before explaining it we would like to mention that GTD is not only a theorem of general relativity. This theorem, GTD, can be already proved in (an easily understandable logic-based version of) special relativity in such a way that we simulate gravity by acceleration [11], [13]. So one advantage of GTD is that actually why it is true can be traced down by using only the simple methods of special relativity. Another advantage of GTD is that it has been tested several times, and these experiments are well known. Roughly, GTD can be interpreted by the following thought experiment. Choose a high enough tower on the Earth, put precise enough (say, atomic) clocks at the bottom of the tower and the top of the tower, then wait enough time, and compare the readings of the two clocks. Then the clock on the top will run faster (show more elapsed time) than the one in the basement, at each time one carries out this experiment. Figure 2 represents how GTD can be proved in special relativity using an accelerated spaceship for creating artificial gravity and checking its effects on clocks at the two ends of the spaceship. Detailed purely logical formulation and proofidea is found in [12].

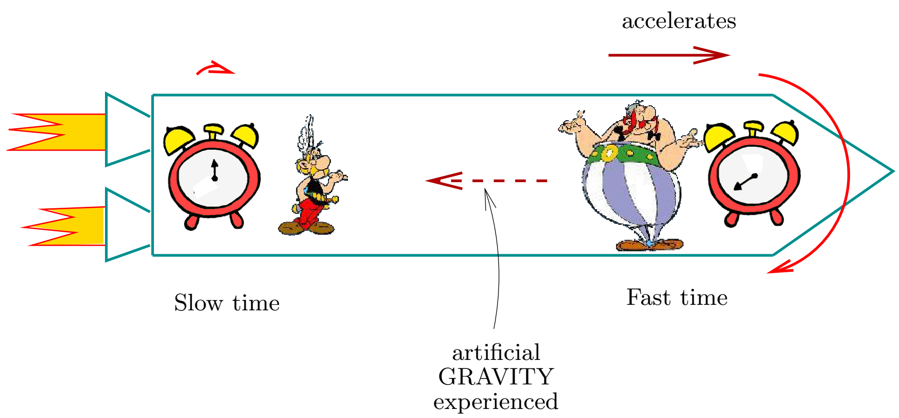


Fig. 2. GTD is a theorem of Special Relativity (SR) (easily proved in first-order logic version of SR).

The next picture, Figure 3, represents the same GTD effect as before, but now using a tall tower on the Earth experiencing the same kind of gravity as in the spaceship. Gravity causes the clock on the top ticking faster. Therefore computers there also compute faster. Assume the programmer in the basement would like to use this GTD effect to speed up his computer. So he sends the computer to the top of the tower. Then he gets some speed-up effect, but this is too little. The next two pictures, Figure 4 and Figure 5, are about the theoretical possibility of increasing this speed-up effect.

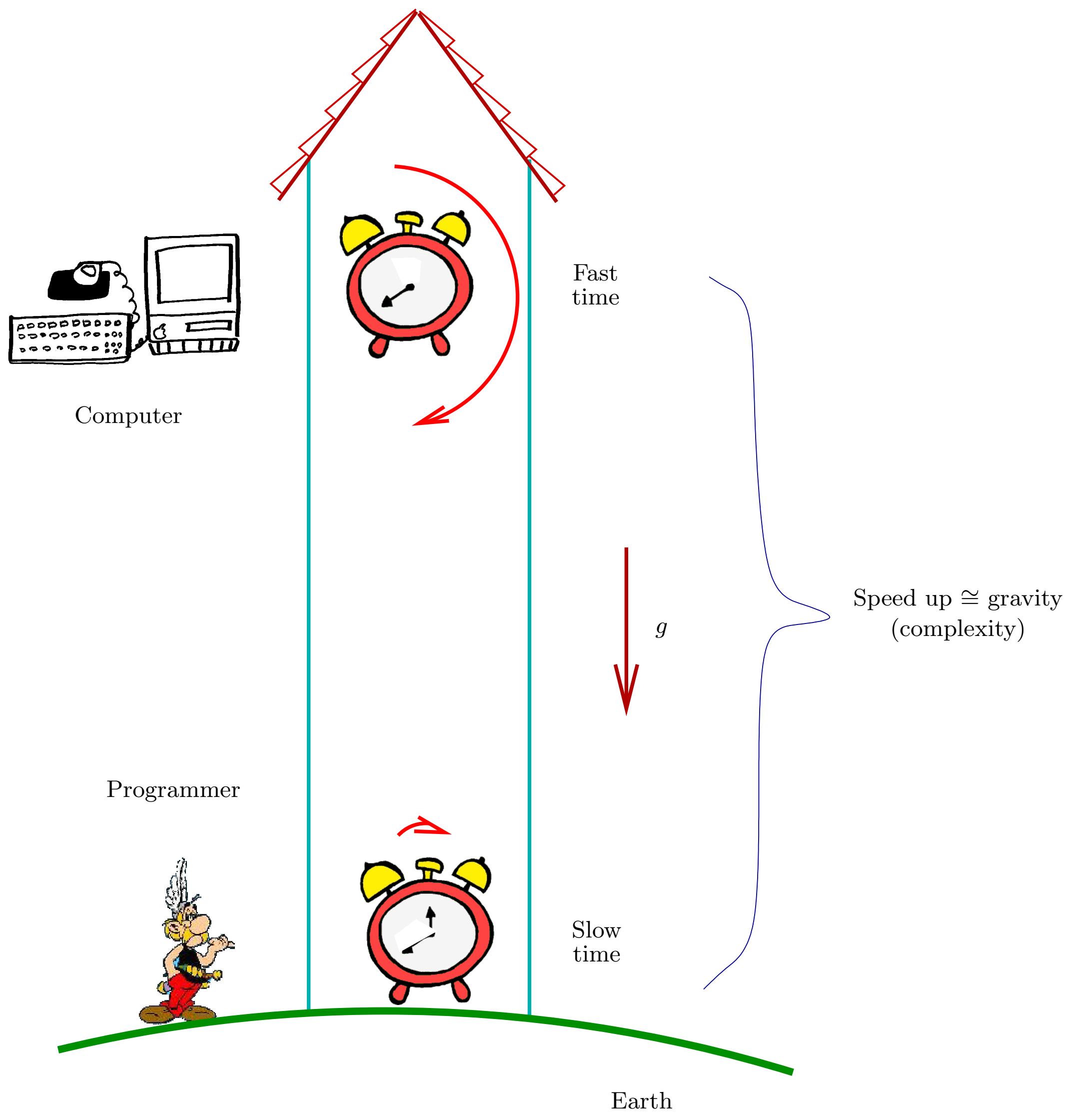


Fig. 3. TIME WARP (Tower Paradox, effects of gravity on time). Clocks higher in a gravitational well tick faster.

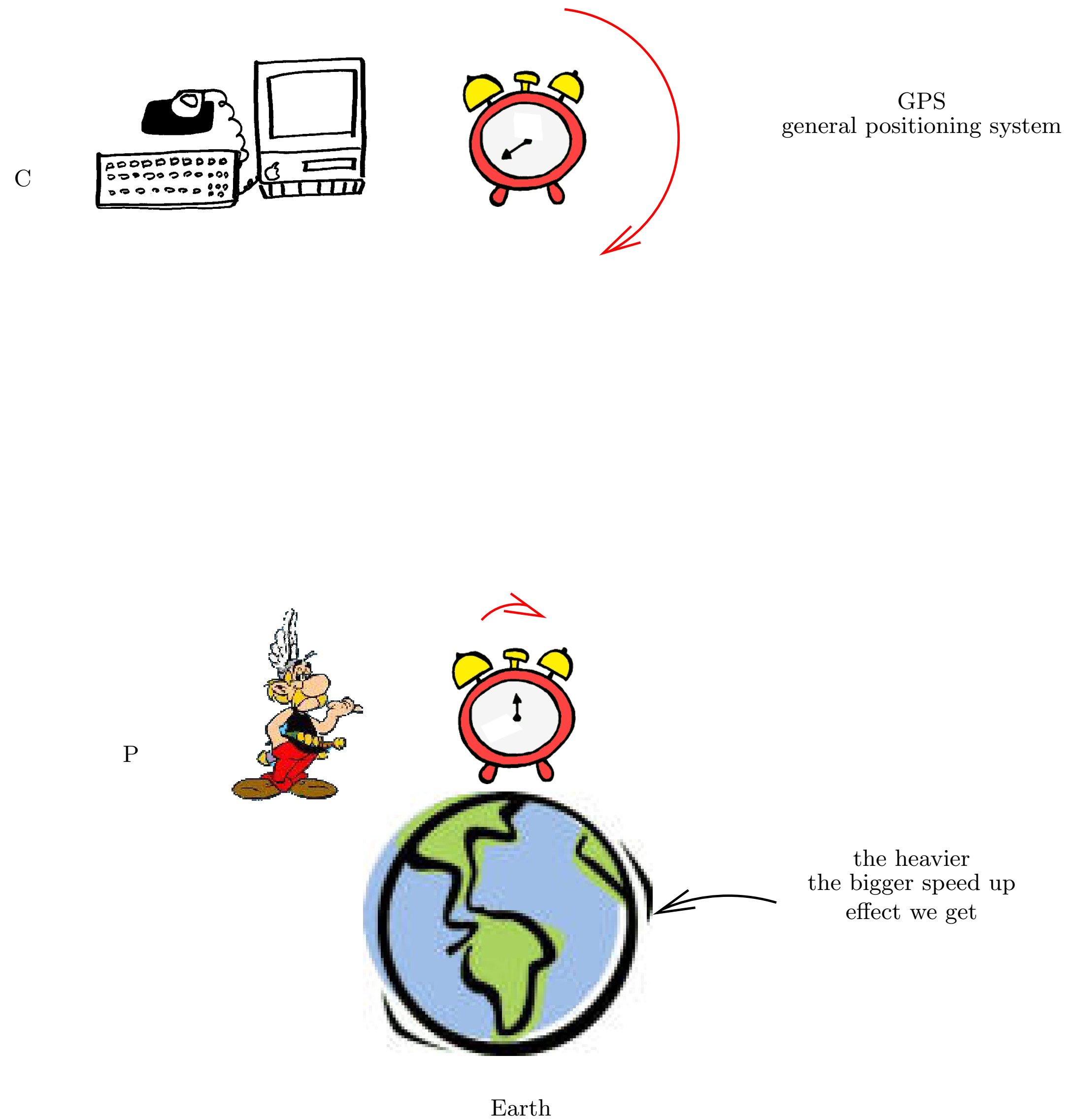


Fig. 4. Thought experiment for fast computation: The programmer "throws" his slave-computer to a high orbit. Communicates via radio.

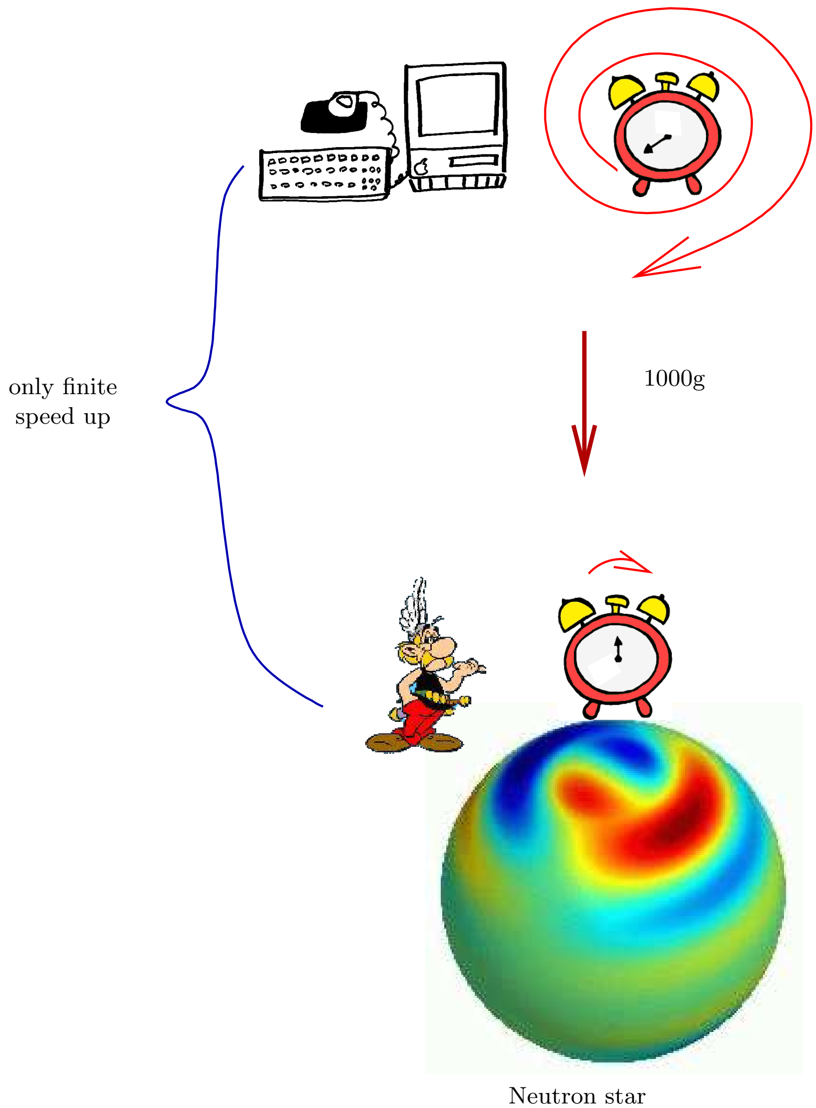


Fig. 5. By using a neutron star we still get only a finite speed-up.

How could we use GTD for designing computers that compute more than Turing Machines can? In the above outlined situation, by using the gravity of the Earth, it is difficult to make practical use of GTD. However, instead of the Earth or of a neutron star, we could choose a huge black hole, cf. Figure 6. A black hole is a region of spacetime with so big "gravitational pull" that even light cannot escape from this region. There are several types of black holes, an excellent source is Taylor and Wheeler [19]. For our demonstration of the main ideas here, we will use huge, slowly rotating black holes. (These are called slow-Kerr in the physics literature.) These black holes have two so-called *event horizons*, these are bubble-like surfaces one inside the other, from which even light cannot escape (because of the gravitational pull of the black hole). See Figures 6–8. As we approach the outer event horizon from far away outside the black hole, the gravitational "pull" of the black hole approaches infinity as we get closer and closer to the event horizon. This is rather different from the Newtonian case, where the gravitational pull also increases but remains finite even on the event horizon.[1] For a while from now on "event horizon" means "outer event horizon".

[1] The event horizon also exists in the Newtonian case, namely, in the Newtonian case, too, the event horizon is the "place" where the escape velocity is the speed of light (hence even light cannot escape to infinity from inside this event horizon "bubble").