Corrections to my article “Ordinal Systems”

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Unfortunately Markus Michelbrink, Hannover, Prof. H. Schwichtenberg, Munich, and myself have detected some mistakes in the article
of which one is a crucial cut-and-paste error in the definition of ordinal systems.

A corrected version is available via my home page (see below). The list of corrections follows.

Abstract, line 6 – 7
Replace
“transfinitely iterated fixed point theories [IDα]” by
“theories of transfinitely iterated inductive definitions [IDα].”

Def. 1.1 (a)
For clarification insert the following text after the paragraph defining what a class is:
“A binary relation $\prec$ is a class. For binary relations $\prec$ we define $r \prec s := \pi(r, s) \in \prec$, where $\pi$ is a standard primitive recursive pairing function on the natural numbers having the usual properties.”
Add further before “Transfinite induction over $(A, \prec)$ is in PRA reducible to transfinite induction over $(A_i, \prec_i) (i = 1, \ldots, n), \ldots” the following sentence:
“Let $A$ be a class, $\prec$ be a binary relation, both depending on unary free predicates $A_i$ and binary free predicates $\prec_i (i = 1, \ldots, n).$”

Def. 1.1 (b)
This part has to be rewritten as follows:
“Assume $B$ is a class and $\prec$ is a binary relation, both depending on unary free predicates $A_i$ and binary free predicates $\prec_i (i = 1, \ldots, m)$. $(B, \prec)$ is an elementary construction from $(A_1, \prec_1), \ldots, (A_m, \prec_m)$, if the following holds: the formulas defining $B, \prec$ are formulas of the language of PRA with bounded quantifiers only (i.e. quantifiers of the form $\forall x < t, \exists x < t$); PRA$^+$ proves that, if $(A_i, \prec_i)$ are linear orderings $(i = 1, \ldots, m)$, so is $(B, \prec)$; transfinite induction over $(B, \prec)$ is PRA-reducible to transfinite induction over $(A_i, \prec_i)$.”

Def. 1.1 (j)
“$\bigcup \{X \subseteq [A] \mid (X, \prec)$ well-ordered $\}$” instead of
“$\bigcap \{X \subseteq [A] \mid (X, \prec)$ well-ordered $\}$."

Def. 2.1 (d)
Replace in (OS 1) and (OS 3) “Arg” by “T”.

Sect. 2.2, line 9/10
Replace
“(more precisely the formula $\forall n \in \mathbb{N}(n \leq n \land \forall x \in T(x < n \leftrightarrow \exists y < n, x = 1))$ is provable in PRA),”
by
“(more precisely the formula $\forall n \in \mathbb{N}(n \leq n \land \forall x \in T(x < n \leftrightarrow \exists y < n, x = 1))$ is provable in PRA),”

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http://www.math.uu.se/~setzer
Lem. 22 (a), line 1/2  Omit “Let $b \in T$.”

Lem. 22 (a), line 4  Replace

“\( \forall \alpha, \beta < \gamma (\alpha < \beta \rightarrow a_{\alpha} \prec a_{\beta}) \)” by

“\( \forall \alpha, \beta < \gamma (\alpha < \beta \rightarrow a_{\alpha} \prec a_{\beta}) \)”.

Proof of Lem 22, (a), line 10  Replace

“\( k(b) \prec b \prec a \)” by

“\( k(b) \prec b \prec a \)”.

Lem. 28 (a)  Replace

“\( \forall a \in T', k(a) \subseteq T \)” by

“\( \forall a \in T', k(a) \subseteq T \)”.

Lem 28 (b) line 1  Replace

“\( \forall a \in T'(f[k(a)] = k^0(f(a)) \land f[k(a)] = 1'(f(a))) \)” by

“\( \forall a \in T'(f[k(a)] = k^0(f(a)) \land f[k(a)] = 1(f(a))) \)”.