

- ▶ ANTON SETZER, *Proof theoretical strength of Martin-Löf type theory.*
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THEOREM. *The proof theoretical strength of extensional and intensional Martin-Löf Type Theory with W -type and one universe in the formalization à la Russel is $\psi_{\Omega_1}(\Omega_{I+\omega})$.*

The proof of this theorem has two directions, here we will concentrate on the proof of an upper bound by embedding Martin-Löf Type Theory in a set theory other versions as well. KPi^+ is an extension of Kripke-Platek set theory by adding axioms demanding the existence of a recursive inaccessible I and of finitely many admissibles above it.

We will essentially interpret closed types as the the set of terms, introduced by some introductory rule, and close this set under reduction. For the interpretation of the W -type we have an inductive definition, which we carry through by iterating an operator up to the next admissible. In the case of the universe we need a fixed point of an operator, which leads to the next admissible (since the universe is closed under W -type), therefore we iterate an operator up to an admissible, closed under admissibles, namely I . In order to interpret W -types, which are built using the universe, we need finitely many admissibles above I .

We conclude, that all Π_1^1 -sentences provable in Martin-Löf Type Theory are provable in KPi^+ (we only have to restrict quantification over sets of natural numbers to elements of the least admissible containing ω). Therefore Tarski and Russel version of the Type Theory have as upperbound the strength $\psi_{\Omega_1}(\Omega_{I+\omega})$, which is “slightly” bigger than the strength of $(\Delta_2^1 - CA) + BI$, KPi and Feferman’s theory T_0 .

For the proof of the lower bound, which can be found in [2], we can only give a flavour. We use essentially, that we can achieve W -types with branching degrees, which are not elements of the universe, by using the sum of products of types, where the first component is possibly empty depending on the boolean value of a primitive recursive relation. Using this and well ordering techniques from proof theory we can show, that Martin-Löf Type Theory proves transfinite induction up to $\psi_{\Omega_1}(\Omega_{I+n})$.

[1] JAEGER, *Theories for admissible sets: A unifying approach to proof theory*, Bibliopolis, 1986.

[2] A. SETZER, *Proof theoretical strength of Martin-Löf type theory with W -type and one universe*, **Ph.D. Thesis**, Universität München, 1993.

- ▶ ALTYNBEK SHARIPBAY, *Construction of proof theory for digital circuits.*
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The proof theory on language of unary predicates of the second order is constructed for Digital Circuits (DC), which process some input signals, coming in discrete moments of time, into output signals.

In this language there are one constant “1” initial moment of time and one unary functional symbol “ \uparrow ” immediate successor, two kinds of variables: t_1, t_2, \dots, t_s —the moments of time and X_1, X_2, \dots, X_k —the string of signals in given set of their states. Terms are defined as usually. Atomic formulas is as like as $X(t) = \alpha$, which means that at the t -th moment of time the state of signal X is α , where “ $=$ ” is graphical equality. Formulas are produced from atomic by the application of logic operations and introducing quantifiers to the both kinds of variables. There are the rules of deduction for admissible connections of DC’s.

Behavioural and structural specifications for the hierarchy of gradually complicated DC’s are constructed by those formulas and rules: the names of behavioural specifications of basic DC considered as axioms, provide the first level, then applying rules of deduction to these axioms we can construct structural specifications for the second level of hierarchy and we can use the behavioural specifications of the second level as axioms to construct structural