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► ANTON SETZER, *A type theory for Mahlo universes*.

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In our thesis [5] we determined the proof theoretical strength of Martin Löf's type theory (MLTT) with W-type and one universe (there was parallel work on this by E. Griffor and M. Rathjen, see [2]). One natural task is, to design stronger type theories and determine its precise strength. In this talk, we will make a first, very small step in this direction and give a type theory which has strength $|\psi_{\Omega_1}\Omega_{M+\omega}|$ and is slightly stronger than and was motivated by the theory KPM, introduced and analyzed in [4]. We conjecture that it is stronger than the super-universe construction introduced by Palmgren in [3] and P. Dybjers general formulation of simultaneous inductive-recursive definitions [1]. We have heard, that E. Griffor is currently working on more general concepts, by which he will probably go far beyond what is achieved in this abstract.

The theory is a an extension of (MLTT) with W-type and one universe (here called V) in the version à là Tarski, by having a rule, which assigns to every function f mapping pre-universes in V to pre-universes (pre-universe means, that we have just a collection of sets) a new element of V which is a new universe, closed under f. Therefore we have the rules

$$\frac{f : (\Sigma x \in V.T(x) \to V) \to (\Sigma x \in V.T(x) \to V)}{\mathrm{Fix}(f) : V} \qquad \frac{a : T(\mathrm{Fix}(f))}{S(f,a) : V}$$

$$\frac{b : T(\mathrm{Fix}(f)) \quad c : T(S(f,a)) \to T(\mathrm{Fix}(f))}{\mathrm{Restr}(f,b,c) : \Sigma x \in T(\mathrm{Fix}(f)).T(S(f,x)) \to T(\mathrm{Fix}(f))}$$

$$\frac{b : T(\mathrm{Fix}(f)) \quad c : T(S(f,a)) \to T(\mathrm{Fix}(f))}{S(f,\mathrm{Restr}(f,b,c)0) = f(< S(f,b), \lambda x.S(f,cx) >)0 : V}$$

$$\frac{b : T(\mathrm{Fix}(f)) \quad c : T(S(f,a)) \to T(\mathrm{Fix}(f)) \quad d : T(f(< S(f,b), \lambda x.S(f,cx) >)0)}{S(f,\mathrm{Restr}(f,b,c)1d) = f(< S(f,b), \lambda x.S(f,cx) >)1d : V}$$

and rules expressing, that $\mathrm{Fix}(f)$ is a sub-universe of V like:

$$\frac{b : T(\mathrm{Fix}(f)) \quad x : T(S(f,b)) \Rightarrow c : T(\mathrm{Fix}(f))}{\sigma_f x \in b.c : T(\mathrm{Fix}(f))}$$

$$\frac{b : T(\mathrm{Fix}(f)) \quad x : T(S(f,b)) \Rightarrow c : T(\mathrm{Fix}(f))}{S(f, \sigma_f x \in b.c) = \sigma x \in S(f,b).S(f,c) : V}.$$

We determine a lower bound by giving a direct well-ordering proof up to the proof theoretic strength. Here we will extend methods in [5], by using the fixed-point operator to introduce universes, for finding big ordinals $\psi_M(a)$. The upper bound for this system can be (work in progress) determined by interpreting in a similar way as in [5] the type theory in a Kripke-Platek style theory KPM^+, with axioms which guarantee the existence of one recursive Mahlo M and of ω admissibles above M.

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▶ STAN J. SURMA, *From the closure-theoretic deductive methodology to some of its non-standard alternatives*.

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In *Uber die Rolle den transfiniten Schlussweisen in einer allgemeinen Idealtheorie* published in ***Mathematische Nachtrichten***, vol. 7 (1952), pp. 165–182, Jurgen Schmidt showed that, loosely speaking, $\mathrm{Th}(\mathrm{Cn}(\mathrm{Th})) = \mathrm{Th}$ and $\mathrm{Cn}(\mathrm{Th}(\mathrm{Cn})) = \mathrm{Cn}$ where Th is a closure system and Cn is a closure operator over the powerset $P(S)$ of a set S. As a move in the same direction I describe four methodological frameworks, each based on a single primitive term, i.e., on Cons, on Ln, on Max and on Sep.

(i) Cons derives from the intuitive idea of consistency and, by the suggested axiomatisation, it is characterised as a non-trivial (i.e., $S \notin \mathrm{Cons}$), hereditary (i.e., $Y \in \mathrm{Cons} \cap 2_X$ *implies* $X \in \mathrm{Cons}$) and regular (i.e., *any* $X \in \mathrm{Cons}$ *extends to an inclusion-maximal* $Y \in \mathrm{Cons}$) property of subsets of S;

(ii) An intuitive motivation for Ln, a Lindenbaum operator, comes from the well-known Lindenbaum extension lemma. Ln is characterised as a non-trivial, (i.e., $\mathrm{Ln}(S) = \emptyset$), extensive (i.e., $\mathrm{Ln}(X) \subseteq 2_X$), inclusive (i.e., $\mathrm{Ln}(\emptyset) \cap 2_X \subseteq \mathrm{Ln}(X)$), and antimonotonic (i.e., $X \subseteq Y$ *implies* $\mathrm{Ln}(Y) \subseteq \mathrm{Ln}(X)$) and regular (i.e., $X = Y$ *for any* $X \in \mathrm{Ln}(\emptyset)$ *and* $Y \in \mathrm{Ln}(X)$) operator from $P(S)$ to $P(P(S))$;

(iii) Max, intuitively a property of maximality, e.g., that of deductive completeness is characterised as a non-trivial (i.e., $S \notin \mathrm{Max}$), and regular (i.e., $X = Y$ *for any* $X \in \mathrm{Max}$ *and* $Y \in \mathrm{Max} \cap 2_X$) property of subsets of S; and

(iv) Sep can be motivated on intuitive grounds as logical independence. It is axiomatised here as a non-trivial (i.e., $S \notin \mathrm{Sep}(A)$), hereditary (i.e., $Y \in \mathrm{Sep}(A) \cap 2_X$ implies $X \in \mathrm{Sep}(A)$), exclusive (i.e., $X \in \mathrm{Sep}(A)$ *implies* $A \notin X$), convex (i.e., $X \in \mathrm{Sep}(A)$ *implies* $\{B : X \notin \mathrm{Sep}(B)\} \in \mathrm{Sep}(A)$) and regular (i.e., $X \in \mathrm{Sep}(A)$ implies that there is $Y \in \mathrm{Sep}(A) \cap 2_X$ such that $B \in Y$ for any B and C such that $Y \cup \{B\} \in \mathrm{Sep}(C)$) operator from S to $P(P(S))$.

It is proved that each of these frameworks can be translated into the standard, closure-theoretic framework in a one-to-one and theorem-preserving fashion. In other words, for