d.c.e. degrees. Arslanov, Lempp and Shore [1] proved the existence of non-isolating c.e. degrees other than $\mathbf{0}$ and $\mathbf{0}'$. A further result of LaForte [4] (also proved independently by Ding and Qian [3]) shows that the isolating c.e. degrees (and isolated d.c.e. degrees) are dense in the upper-semi lattice of c.e. degrees. We show that such a density result cannot be obtained for the non-isolating c.e. degrees, and examine the best possible density result which can be proved.

[1] M. M. Arslanov, S. Lempp, and R. A. Shore, *On isolating r.e. and isolated d.r.e. degrees*, to appear, 1995.

[2] S. B. Cooper and X. Yi, *Isolated d.r.e. degrees*, to appear, 1995.

[3] D. Ding and L. Qian, *Isolated d.r.e. degrees are dense in r.e. degree structure*, to appear.

[4] G. L. LaForte, *The isolated d.r.e degrees are dense in the r.e. degrees*, ***Mathematical Logic Quarterly***, vol. 42 (1996), no. 2.

[5] R. I. Soare, ***Recursively enumerable sets and degrees***, Springer-Verlag, Berlin, 1987.

► ANTON SETZER, *Defining the least universe in Martin-Löf's type theory*.

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In our thesis [3] (see as well [4]) we determined the proof theoretical strength of Martin Löf's type theory ($\mathrm{ML_1W}$) with W-type and one universe (there was parallel work on this by E. Griffor and M. Rathjen, see [1]) to be the collapse of the ωth admissible above the first recursive inaccessible. The intuition behind this result is that the universe is a fixed point under the formation of rules for it, and the definition depends on the inductive definition of the W-type. If we had that the universe is the least such fixed point, guaranteed by elimination rules as defined in [2] or [1], then, since the W-type corresponds to the step to the next admissible, the universe corresponds to one inductive definition closed under the step to the next admissible, which, since the inductive definition is strong enough, corresponds to one recursive inaccessible. However, in the definition of $\mathrm{ML_1W}$, we have no elimination rules and therefore only a fixed point. Note that axioms guaranteeing fixed points are usually far weaker than axioms for least fixed points. However elimination rules are not necessary, because in $\mathrm{ML_1W}$, we can define a least universe in the following sense:

THEOREM. *There exists a type* $\phi(x)$ *such that* $\{x : \mathrm{U} \mid \phi(x)\}$ *is the least universe closed under the formation rules for* U:

(a) $\{x : \mathrm{U} \mid \phi(x)\}$ *is closed under the formation rules*: *We can prove in* $\mathrm{ML_1W}$ $\phi(\mathbf{n}_0)$, $\forall a : \mathrm{U}.\forall b : (\mathrm{T}(a) \to \mathrm{U}).\phi(a) \to ((\forall x : \mathrm{T}(a).\phi(bx)) \to \phi(\sigma x : a.(bx)))$, *similarly for the other constructors.*

(b) $\{x : \mathrm{U} \mid \phi(x)\}$ *is the least such universe*: *If* $x : \mathrm{U} \Longrightarrow C(x)$ *type,* $C(\mathbf{n}_0)$, $\forall x : \mathrm{U}.\forall y : (\mathrm{T}(x) \to \mathrm{U}).C(x) \to ((\forall z : \mathrm{T}(x).C(yz)) \to C(\sigma z : x.yz))$, *similarly for the other constructors, then* $\forall x : \mathrm{U}.\phi(x) \to C(x)$.

This strengthens the intuition about why we achieve this strength, and is a step towards simplified well ordering proofs for $\mathrm{ML_1W}$.

For the proof of the theorem, we define $W_0 := Wx : \mathrm{U}.\mathrm{T}(x)$, for every $w : W_0$ the type $\mathrm{Subtree}(w)$ of subtrees of w (including a name root for w itself) and $\phi(v)$ iff there exists a tree $w : W_0$ and a function label from the subtrees of w to U such that $\mathrm{I}(\mathrm{U}, \mathrm{label}(\mathrm{root}), v)$ and for any $w' : \mathrm{Subtree}(w)$ we have one of the following conditions:

- $\mathrm{I}(\mathrm{U}, \mathrm{label}(w'), \mathbf{n}_0)$
- or $\mathrm{I}(\mathrm{U}, \mathrm{label}(w'), \sigma x : a.bx)$ for some $a : \mathrm{U}$, $b : \mathrm{T}(a) \to \mathrm{U}$ such that for some $w'', w'''(x) : \mathrm{Subtree}(w)$, which are immediate subtrees of w', we have $\mathrm{I}(\mathrm{U}, \mathrm{label}(w''), a)$

and $\forall x : \mathrm{T}(a).\mathrm{I}(\mathrm{U}, \mathrm{label}(w'''(x)), bx)$

- or an analogous condition for the other type constructions, under which U is closed.

[1] E. GRIFFOR and M. RATHJEN, *The strength of Martin-Löf type theories*, ***Archive for Mathematical Logic***, vol. 33 (1994), pp. 347–385.

[2] E. PALMGREN, *Type-theoretic interpretation of iterated, strictly positive inductive definitions*, ***Archive for Mathematical Logic***, vol. 32 (1992), pp. 75–99.

[3] A. SETZER, *Proof theoretical strength of Martin-Löf type theory with W-type and one universe*, ***Ph.D. thesis***, University of Munich, 1993.

[4] ———, *Wellordering proofs for Martin-Löf's type theory*, submitted, 1997.

► A. D. YASHIN, *New intuitionistic logical constants and Novikov completeness*.
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Let intuitionistic propositional language be enriched by finite list $\bar{\varphi} = \{\varphi_1, \dots, \varphi_n\}$ of additional propositional constants. A formula without occurrences of φ_i is said to be *pure*. By Int we denote the intuitionistic propositional logic (in pure language). A formula of the form $\varphi_i \leftrightarrow B$, where B does not contain φ_i is called *explicit expression* for φ_i.

By $\bar{\varphi}$*-logic* we mean any set of formulas of enriched language including Int and closed under modus ponens and substitution rules.

A $\bar{\varphi}$-logic L is said to be *conservative* over Int if for every pure formula B: $B \in L \Longrightarrow B \in$ Int.

DEFINITION 1 (P. Novikov). A $\bar{\varphi}$-logic L is called *Novikov complete* if for every formula $A \notin L$ the $\bar{\varphi}$-logic $L + A$ is not conservative over Int. ($L + A$ means the least $\bar{\varphi}$-logic including $L \cup \{A\}$.)

DEFINITION 2. Constants $\varphi_1, \dots, \varphi_n$ are *independent* in $\bar{\varphi}$-logic L if, for every $i \leq n$, for every formula B without φ_i the $\bar{\varphi}$-logic $L + \varphi_i \leftrightarrow B$ is not conservative over Int (i.e., it is *impossible to enlarge* L by explicit expression). In the case $|\bar{\varphi}| = 1$ it is natural to say about *new* constant.

The *Novikov's problem* may be formulated as follows: to give explicit examples of Novikov complete $\bar{\varphi}$-logics with independent constants; to describe the family of all Novikov complete $\bar{\varphi}$-logics.

Main results:

- we present the "growth technique" for constructing examples of Novikov complete $\bar{\varphi}$-logics;
- the $\bar{\varphi}$-logic corresponding to given finite growth is finitely axiomatizable;
- for $|\bar{\varphi}| \geq 2$ the family of all Novikov complete $\bar{\varphi}$-logics is of the power of the continuum;
- the algorithmic problem "*whether the $\bar{\varphi}$-logic of the form Int + A is conservative over Int*" is undecidable;
- in contrast with preceding results, for $|\bar{\varphi}| = 1$ the family of all Novikov complete φ-logics is explicitly described as

$$L^1, L^2, L^3, L^5, \dots, L^{2n+1}, \dots, L^{\infty}.$$

Here L^1 contains $\neg\varphi$, L^2 contains φ, and each of the others determines new constant; all of L^α are decidable and, as a corollary, the conservativeness problem is decidable.