d.c.e. degrees. Arslanov, Lempp and Shore [1] proved the existence of non-isolating c.e. degrees other than 0 and 0’. A further result of LaForte [4] (also proved independently by Ding and Qian [3]) shows that the isolating c.e. degrees (and isolated d.c.e. degrees) are dense in the upper-semi lattice of c.e. degrees. We show that such a density result cannot be obtained for the non-isolating c.e. degrees, and examine the best possible density result which can be proved.

[3] D. DING and L. QIAN, Isolated d.r.e. degrees are dense in r.e. degree structure, to appear.

ANTON SETZER, Defining the least universe in Martin-Löf’s type theory.
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In our thesis [3] (see as well [4]) we determined the proof theoretical strength of Martin Löf’s type theory (ML, W) with W-type and one universe (there was parallel work on this by E. Griffor and M. Rathjen, see [1]) to be the collapse of the nth admissible above the first recursive inaccessible. The intuition behind this result is that the universe is a fixed point under the formation of rules for it, and the definition depends on the inductive definition of the W-type. If we had that the universe is the least such fixed point, guaranteed by elimination rules as defined in [2] or [1], then, since the W-type corresponds to the step to the next admissible, the universe corresponds to one inductive definition closed under the step to the next admissible. which, since the inductive definition is strong enough, corresponds to one recursive inaccessible. However, in the definition of ML, W, we have no elimination rules and therefore only a fixed point. Note that axioms guaranteeing fixed points are usually far weaker than axioms for least fixed points. However elimination rules are not necessary, because in ML, W, we can define a least universe in the following sense:

**Theorem.** There exists a type ϕ(x) such that \{x : U | ϕ(x)\} is the least universe closed under the formation rules for U:

(a) \{x : U | ϕ(x)\} is closed under the formation rules: We can prove in ML, W ϕ(n_0), \forall a : U : \forall b : (T(a) → U).ϕ(a) → ((\forall x : T(a).ϕ(bx)) → ϕ(σx : a.(bx))), similarly for the other constructors.

(b) \{x : U | ϕ(x)\} is the least such universe: If x : U → C(x) type, C(n_0), \forall x : U \forall y : T(x) → U).C(x) → (\forall z : T(x).C(yz)) → C(σz : x,yz), similarly for the other constructors, then \forall x : U.ϕ(x) → C(x).

This strengthens the intuition about why we achieve this strength, and is a step towards simplified well ordering proofs for ML, W.

For the proof of the theorem, we define W_0 := W_X : U.T(x), for every w : W_0 the type Subtree(w) of subtrees of w (including a name root for w itself) and ϕ(v) iff there exists a tree w : W_0 and a function label from the subtrees of w to U such that I(U, label(root), v) and for any w’ : Subtree(w) we have one of the following conditions:

- I(U, label(w’), n_0)
- or I(U, label(w’), σx : a.bx) for some a : U, b : T(a) → U such that for some w”, w’’(x) : Subtree(w), which are immediate subtrees of w’, we have I(U, label(w’’), a)
Let intuitionistic propositional language be enriched by finite list $\bar{\varphi} = \{\varphi_1, \ldots, \varphi_n\}$ of additional propositional constants. A formula without occurrences of $\varphi_i$ is said to be pure. By Int we denote the intuitionistic propositional logic (in pure language). A formula of the form $\varphi_i \leftrightarrow B$, where $B$ does not contain $\varphi_i$ is called explicit expression for $\varphi_i$.

By $\bar{\varphi}$-logic we mean any set of formulas of enriched language including $\text{Int}$ and closed under modus ponens and substitution rules.

A $\bar{\varphi}$-logic $L$ is said to be conservative over $\text{Int}$ if for every pure formula $B$: $B \in L \Rightarrow B \in \text{Int}$.

**Definition 1 (P. Novikov).** A $\bar{\varphi}$-logic $L$ is called Novikov complete if for every formula $A \notin L$ the $\bar{\varphi}$-logic $L + A$ is not conservative over $\text{Int}$. ($L + A$ means the least $\bar{\varphi}$-logic including $L \cup \{A\}$.)

**Definition 2.** Constants $\varphi_1, \ldots, \varphi_n$ are independent in $\bar{\varphi}$-logic $L$ if, for every $i \leq n$, for every formula $B$ without $\varphi_i$ the $\bar{\varphi}$-logic $L + \varphi_i \leftrightarrow B$ is not conservative over $\text{Int}$ (i.e., it is impossible to enlarge $L$ by explicit expression). In the case $|\bar{\varphi}| = 1$ it is natural to say about new constant.

The Novikov’s problem may be formulated as follows: to give explicit examples of Novikov complete $\bar{\varphi}$-logics with independent constants; to describe the family of all Novikov complete $\bar{\varphi}$-logics.

Main results:

- we present the “growth technique” for constructing examples of Novikov complete $\bar{\varphi}$-logics:
  - the $\bar{\varphi}$-logic corresponding to given finite growth is finitely axiomatizable:
  - for $|\bar{\varphi}| \geq 2$ the family of all Novikov complete $\bar{\varphi}$-logics is of the power of the continuum:
  - the algorithmic problem “whether the $\bar{\varphi}$-logic of the form $\text{Int} + A$ is conservative over $\text{Int}$” is undecidable:
    - in contrast with preceding results, for $|\bar{\varphi}| = 1$ the family of all Novikov complete $\varphi$-logics is explicitly described as $L^1, L^2, L^3, L^5, \ldots, L^{2n+1}, \ldots, L^\infty$.

Here $L^1$ contains $\neg \varphi$. $L^2$ contains $\varphi$, and each of the others determines new constant; all of $L^n$ are decidable and, as a corollary, the conservativeness problem is decidable.