

CS_275 Automata and Formal Language Theory

Course Notes

Additional Material

(This material is no longer taught and not exam relevant)

Part II: The Recognition Problem (II)

Sect II.3.: Finite State Automata (13)

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[http://www.cs.swan.ac.uk/~csetzer/lectures/
automataFormalLanguage/current/index.html](http://www.cs.swan.ac.uk/~csetzer/lectures/automataFormalLanguage/current/index.html)

II.3.1. String Recognition (13.1)

II.3.2. Nondeterministic Finite State Automata (13.2)

II.3.3. Examples of Automata (13.3)

II.3.4. Automata with Empty Move Transitions (13.4)

II.3.5. Deterministic Finite State Automata (13.6)

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NFA with Empty Moves are Equivalent to NFAs

We show that for every NFA A with empty moves we can find an NFA A' without empty moves s.t. $L(A') = L(A)$.

This is done as follows:

Let $A = (Q, q_0, F, T, \longrightarrow)$.

We define

$$A' = (Q, q_0, F', T, \longrightarrow')$$

where

- ▶ $q \xrightarrow{a}' q'$ iff $q \xrightarrow{a}^* q'$,
- ▶ $F' := \{q \in Q \mid \exists q' \in F. q \xrightarrow{\epsilon}^* q'\}$.

NFA with Empty Moves are Equivalent to NFAs

So the transitions of A' are obtained by allowing first finitely many empty moves and then one proper transition and again finitely many empty moves.

At the end we might need to make finitely many empty moves before reaching the accepting state, therefore the set of accepting states is the set of states from which we can reach an accepting state of A using empty moves.

Correctness of the Translation

- ▶ One can now easily see that for $w \neq \epsilon$ we have

$$q \xrightarrow{w}^* q' \text{ iff } q \xrightarrow{w}^{\prime*} q'$$

- ▶ Note that $q \xrightarrow{\epsilon}^{\prime*} q'$ iff $q' = q$.

Correctness of the Translation

► We get

$$\begin{aligned}
 L(A') &= \{w \in T^* \mid \exists q' \in F'. q \xrightarrow{w} q'\} \\
 &= \{w \in T^* \mid \exists q' \in Q. \exists q'' \in F. \\
 &\quad ((w = \epsilon \wedge q' = q) \vee (w \neq \epsilon \wedge q \xrightarrow{w} q')) \\
 &\quad \wedge q' \xrightarrow{\epsilon} q''\} \\
 &= \{w \in T^* \mid \exists q' \in F. q \xrightarrow{w} q'\} \\
 &= L(A)
 \end{aligned}$$

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Proof Idea of Theorem II.3.5.1.

- ▶ We will define a new automaton $A' = (Q', q'_0, F', T, \longrightarrow')$.
- ▶ Q' is the set of all subsets of Q , i.e. $\mathcal{P}(Q)$.
Note that since Q' is finite $\mathcal{P}(Q)$ will be finite too.
 - ▶ Having reached state $\{q_1, \dots, q_k\}$ means that $\{q_1, \dots, q_k\}$ are the set of states we could have reached in A by making different choices, but following the same word.
- ▶ $q'_0 := \{q_0\}$.
 - ▶ Initially the states we have reached are the elements of $\{q_0\}$.

Proof Idea of Theorem II.3.5.1.

- ▶ Let $B \subseteq Q$.

$$B \xrightarrow{a} C$$

where

$$C = \{q \in Q \mid \exists q' \in B. q' \xrightarrow{a} q\}$$

- ▶ If we could have reached any of the states in B , then after reading a in addition, we could have reached any of the states we can reach from a state in B by an a -transition.

Proof Idea for Theorem II.3.5.1.

- ▶ The accepting states are the set of states containing at least one accepting state.
 - ▶ If having read word w we can reach the states $\{q_1, \dots, q_k\}$, then the word w can be accepted, if one of q_1, \dots, q_k is an accepting state.

Resulting DFA

automaton	A'
states	$\mathcal{P}(Q)$
terminals	T
start	$\{q_0\}$
final	$B \in \mathcal{P}(Q)$ s.t. $B \cap F \neq \emptyset$
transitions	$B \xrightarrow{a} C$ where $C = \{q' \mid \exists q \in B. q \xrightarrow{a} q'\}$.

Simplification

- ▶ Usually only some states of A' are reachable.
- ▶ We can omit all unreachable states and get an equivalent automaton.
- ▶ We can construct the reachable states of A' by starting with $\{q_0\}$, and constructing from there systematically all transitions and the states reached.
- ▶ Furthermore, there will be a state \emptyset .
 - ▶ When we have reached that state we have consumed a word for which there is no complete run of A .
 - ▶ $\emptyset \notin F'$, $\emptyset \xrightarrow{a} \emptyset$.
 - ▶ So \emptyset is a sink, a state from which we cannot escape, and which doesn't accept anything.
 - ▶ If we omit \emptyset , we obtain a DFA with the same language.

Proof of Theorem II.3.5.1.

- ▶ Consider the DFA as given above.
- ▶ Define for $B \subseteq Q$, $w \in T^*$, $\delta(B, w) \subseteq Q$ by

$$\delta(B, w) := \{q \in Q \mid \exists q' \in B. q' \xrightarrow{w} q\}$$

- ▶ So we have

$$B \xrightarrow{a} \delta(B, a)$$

Proof of Theorem II.3.5.1.

- ▶ We show for $B \subseteq Q$ that

$$B \xrightarrow{w} / * \delta(B, w)$$

by induction on the length of w :

- ▶ Case $w = \epsilon$:

$$\begin{aligned} \delta(B, \epsilon) &= \{q \in Q \mid \exists q' \in B. q' \xrightarrow{\epsilon} * q\} \\ &= \{q \in Q. \mid \exists q' \in B. q' = q\} \\ &= B \end{aligned}$$

and

$$B \xrightarrow{\epsilon} / * B = \delta(B, \epsilon)$$

Proof of Theorem II.3.5.1.

- ▶ Case $w = w'a$:
By IH

$$B \xrightarrow{w'}^* \delta(B, w')$$

Furthermore

$$\begin{aligned} \delta(B, w') &\xrightarrow{a} \{q' \mid \exists q \in \delta(B, w'). q \xrightarrow{a} q'\} \\ &= \{q' \mid \exists q'' \in B. \exists q \in Q. q'' \xrightarrow{w'}^* q \wedge q \xrightarrow{a} q'\} \\ &= \{q' \mid \exists q'' \in B. q'' \xrightarrow{w'a}^* q'\} \\ &= \delta(B, w'a) \end{aligned}$$

Therefore

$$B \xrightarrow{w'a}^* \delta(B, w'a)$$

Proof of Theorem II.3.5.1.

We obtain now

$$\begin{aligned}
 L(A') &= \{w \in T^* \mid \exists B \in F'. \{q_0\} \xrightarrow{w}{}^* B\} \\
 &= \{w \in T^* \mid \delta(\{q_0\}, w) \cap F \neq \emptyset\} \\
 &= \{w \in T^* \mid \exists q \in F. q \in \delta(\{q_0\}, w)\} \\
 &= \{w \in T^* \mid \exists q \in F. q_0 \xrightarrow{w}{}^* q\} \\
 &= L(A)
 \end{aligned}$$