CS_275 Automata and Formal Language Theory

Course Notes

Part II: The Recognition Problem (II)

Chapter II.5.: Properties of Context Free Grammars (14)

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II.5.1. Derivation Trees for Context-Free Grammars (14.1)

Derivation Trees or Parse Trees

- Context free Grammars (abbreviated as **CFG** in the following) allow to apply to a non-terminal at position without needing the context.
- Therefore we can expand the non-terminals independently of each other.
- This allows us to define derivation trees (also called parse trees).

Example

Consider the grammar

grammar	G
terminals	a, b
nonterminals	S
start symbol	S
productions	$S \longrightarrow aSb$ $S \longrightarrow ab$

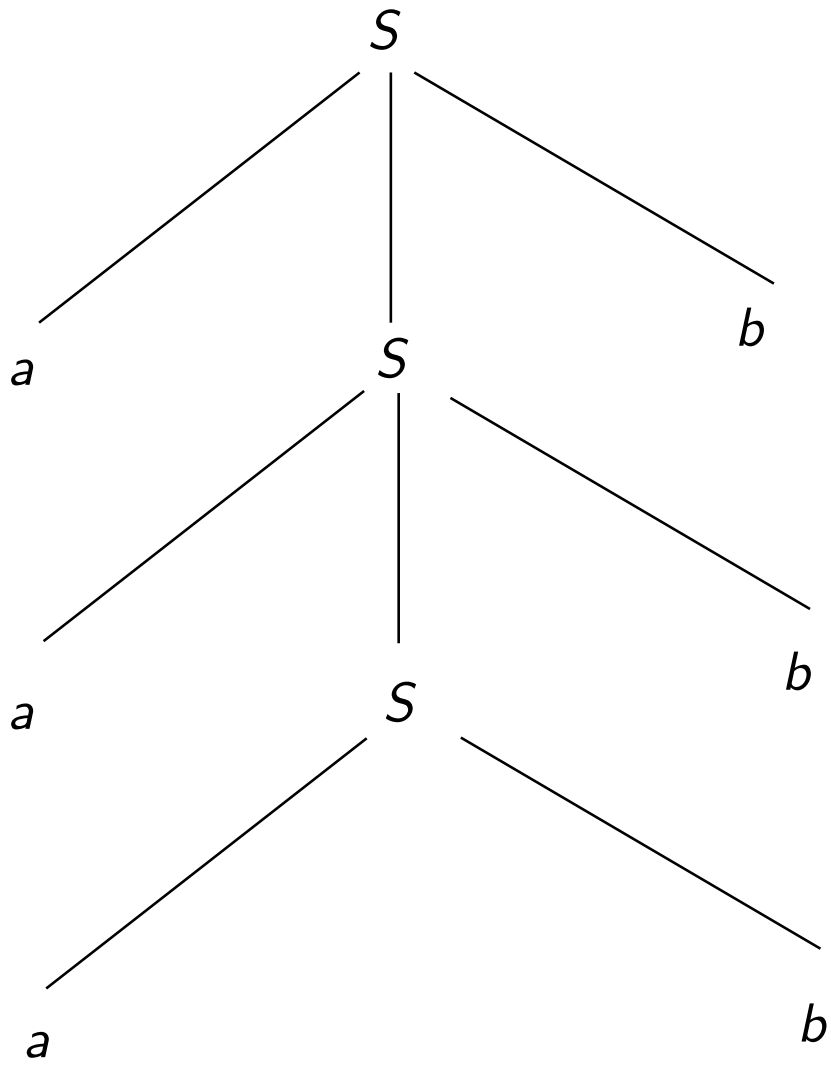
Example Derivation

We derive *aaabbbb* in it:

$$\begin{aligned} S &\Rightarrow aSb \\ &\Rightarrow aaSbb \\ &\Rightarrow aaabbb \end{aligned}$$

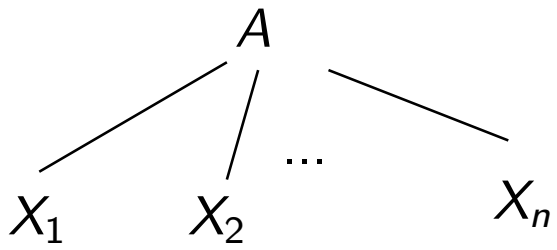
Derivation Tree

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaabbb$



Form of the Derivation Tree

- Nodes are labelled with elements of $N \cup T \cup \{\epsilon\}$.
- A node with label A has a subtree



only if A is a non-terminal and there is a production

$$A \longrightarrow X_1 X_2 \cdots X_n$$

where $X_i \in T \cup N$.

- All leaves of the tree together read from left to right form the string derived, namely *aaabbb*.
  This is called the **frontier** of the derivation tree.
- We will as well consider derivation trees not ending in a string of terminals, so the frontier is an element of $(T \cup N)^*$.

Definition Derivation Tree

Definition

Let $G = (T, N, S, P)$ be a CFG. A **derivation tree** or **parse tree** for G is a finite tree with

- nodes labelled by elements of $N \cup T \cup \{\epsilon\}$,
- s.t. a node A has children with labels $X_1, \ldots, X_n$ only if $A \in N$ and there is a production
$$A \longrightarrow X_1 X_2 \cdots X_n$$
- If the node of one of the children of A is ϵ, then this node is the only child of this tree.

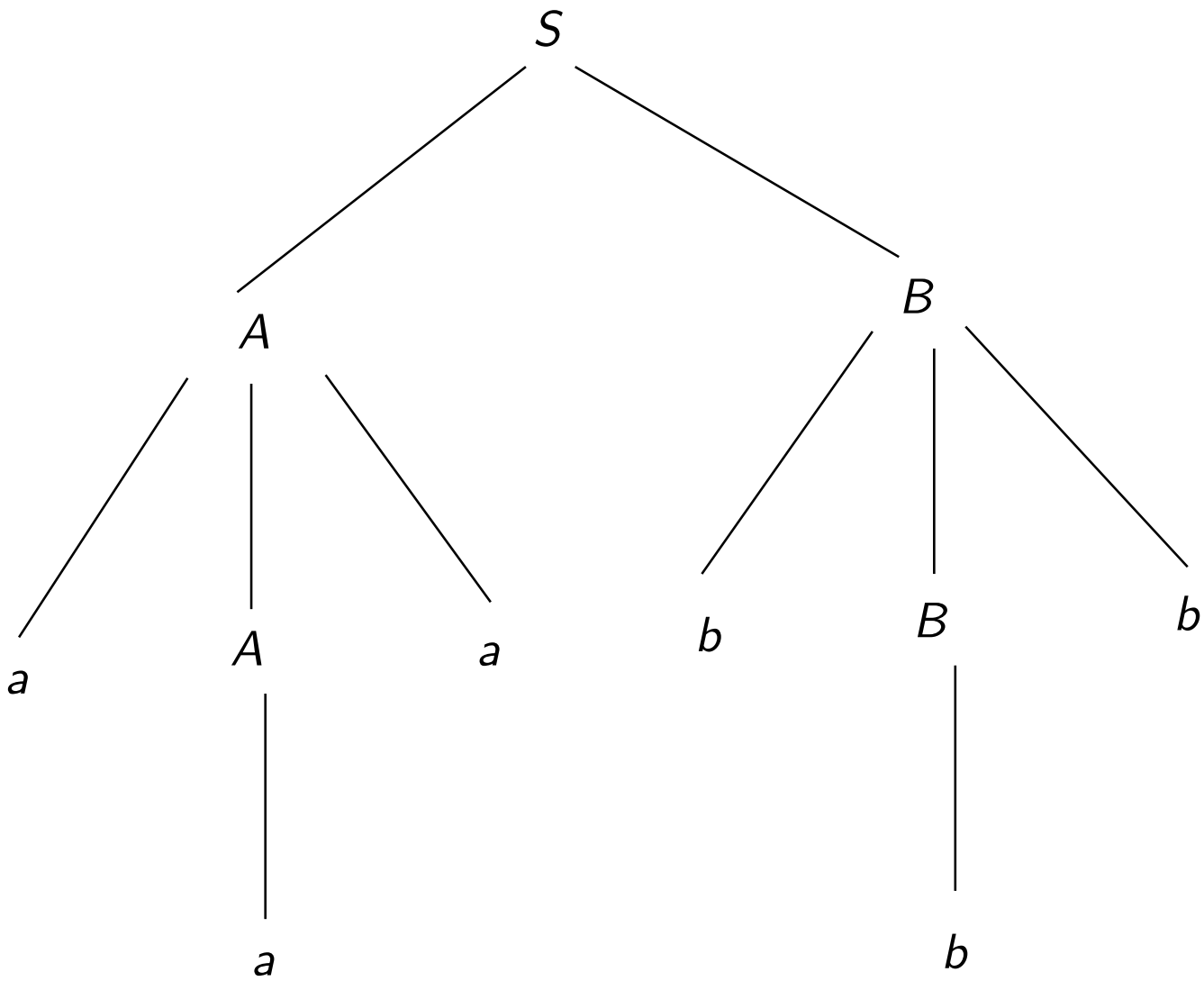
The **frontier** of the tree is the set of leaves red from left to right in sequence, which is an element $(T \cup N)^*$.
The **root** of the tree is the node at the to of the derivation tree.

Left-Most and Right-Most Derivations

From a derivation tree we can obtain a derivation in various orders.
Consider the grammar

grammar	G
terminals	a, b
nonterminals	S, A, B
start symbol	S
productions	$S \longrightarrow AB,$ $A \longrightarrow aAa,\ A \longrightarrow a$ $B \longrightarrow bBb,\ B \longrightarrow b$

Example Derivation Tree



Different Derivations of *aaabbb*

We can derive *aaabbb* in different ways:

$$S \Rightarrow AB \Rightarrow aAaB \Rightarrow aaaB \Rightarrow aaabBb \Rightarrow aaabbb$$

A **left most derivation**

$$S \Rightarrow AB \Rightarrow AbBb \Rightarrow Abbb \Rightarrow aAabbb \Rightarrow aaabbb$$

A **right most derivation**

$$S \Rightarrow AB \Rightarrow aAaB \Rightarrow aAabBb \Rightarrow aaabBb \Rightarrow aaabbb$$

$$S \Rightarrow AB \Rightarrow aAaB \Rightarrow aAabBb \Rightarrow aAabbb \Rightarrow aaabbb$$

$$S \Rightarrow AB \Rightarrow AbBb \Rightarrow aAabBb \Rightarrow aaabBb \Rightarrow aaabbb$$

$$S \Rightarrow AB \Rightarrow AbBb \Rightarrow aAabBb \Rightarrow aAabbb \Rightarrow aaabbb$$

Left-Most Derivation

Definition

Let $G = (T, N, S, P)$ be a CFG.
A single-step derivation $w \Rightarrow w'$ is **left-most** if a rule was applied to the left-most non-terminal in w, i.e.

- $w = sAt$ for some $A \in N$, $s \in T^*$ (consisting only of terminals), $t \in (S \cup T)^*$,
- and there exist a production $A \longrightarrow v$
- s.t. $w' = svt$.

Left-Most Derivation

Definition

Let $G = (T, N, S, P)$ be a CFG.
A single-step derivation $w \Rightarrow w'$ is **right-most** if a rule was applied to the right-most non-terminal in w, i.e.

- $w = sAt$ for some $A \in N$, $s \in (S \cup T)^*$, $t \in T^*$ (consisting only of terminals),
- there exist a production $A \longrightarrow v$
- s.t. $w' = svt$.

Left/Right-Most Derivation Sequence

Definition

Let $G = (T, N, S, P)$ be a CFG

1. A derivation sequence $w_0 \Rightarrow w_1 \Rightarrow w_2 \Rightarrow \cdots w_n$ is left-most, if each derivation step $w_i \Rightarrow w_{i+1}$ is left-most.
2. Right-most derivation sequences are defined analogously.

Theorem II.5.1.1. (Derivation Trees and Language Generation)

Theorem

Let $G = (T, N, S, P)$ be a CFG, $A \in T$, $w, w' \in (T \cup N)^*$, Then the following are equivalent

(1) *There exist a derivation tree with root labelled by A and frontier w'.*

(2) *$A \Rightarrow^* w'$.*

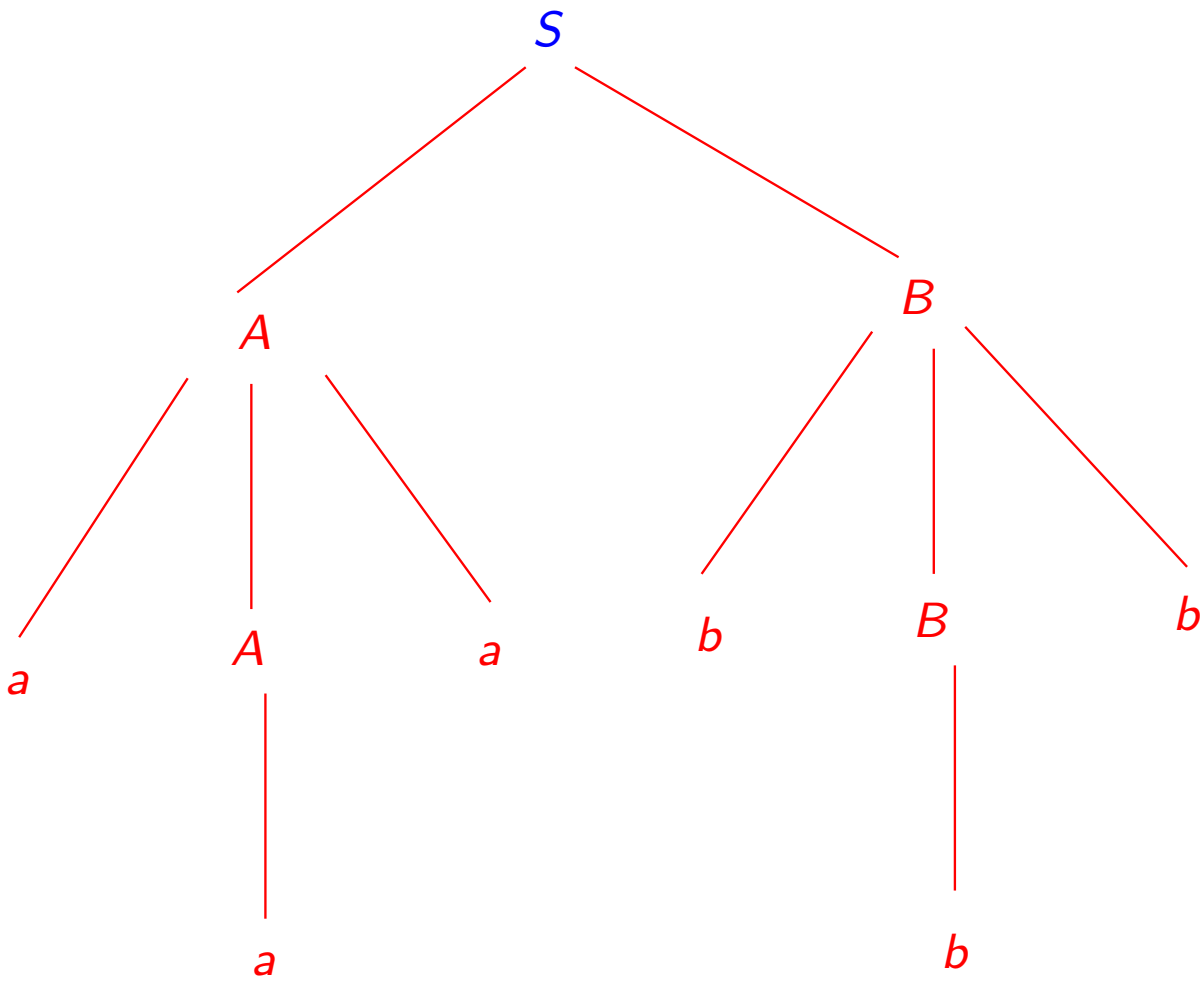
In case $w' \in T^*$, the derivation sequence $w \Rightarrow^* w'$ can both be chosen as a left-most and as a right-most derivation sequence

Proof of Theorem II.5.1.1.

- A proof of this theorem can be found in the additional material.
- We illustrate this theorem by an example.
  - We will first present a left-most derivation.
  - Then we will present a right most derivation.

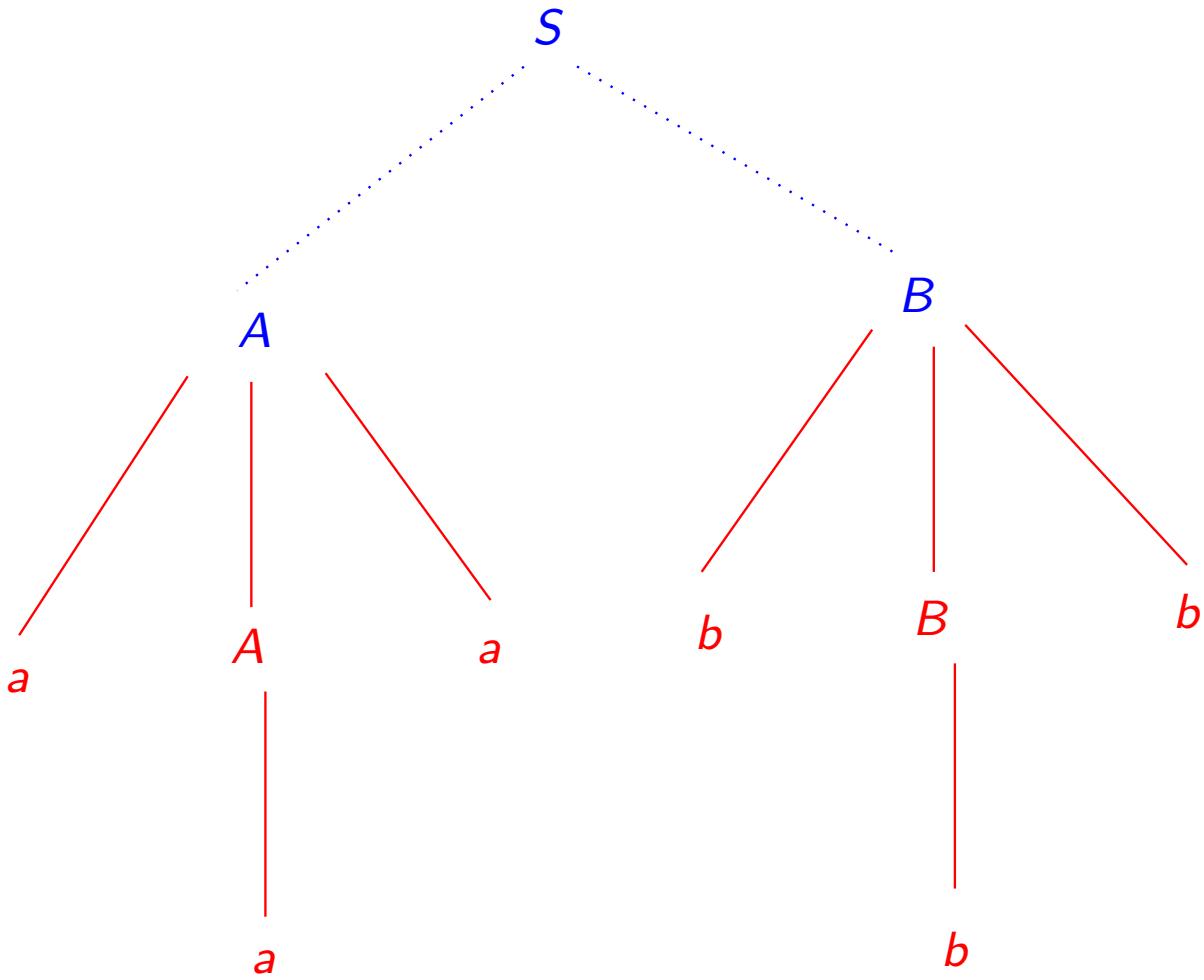
Example Left-Most Derivation (Step 1)

S



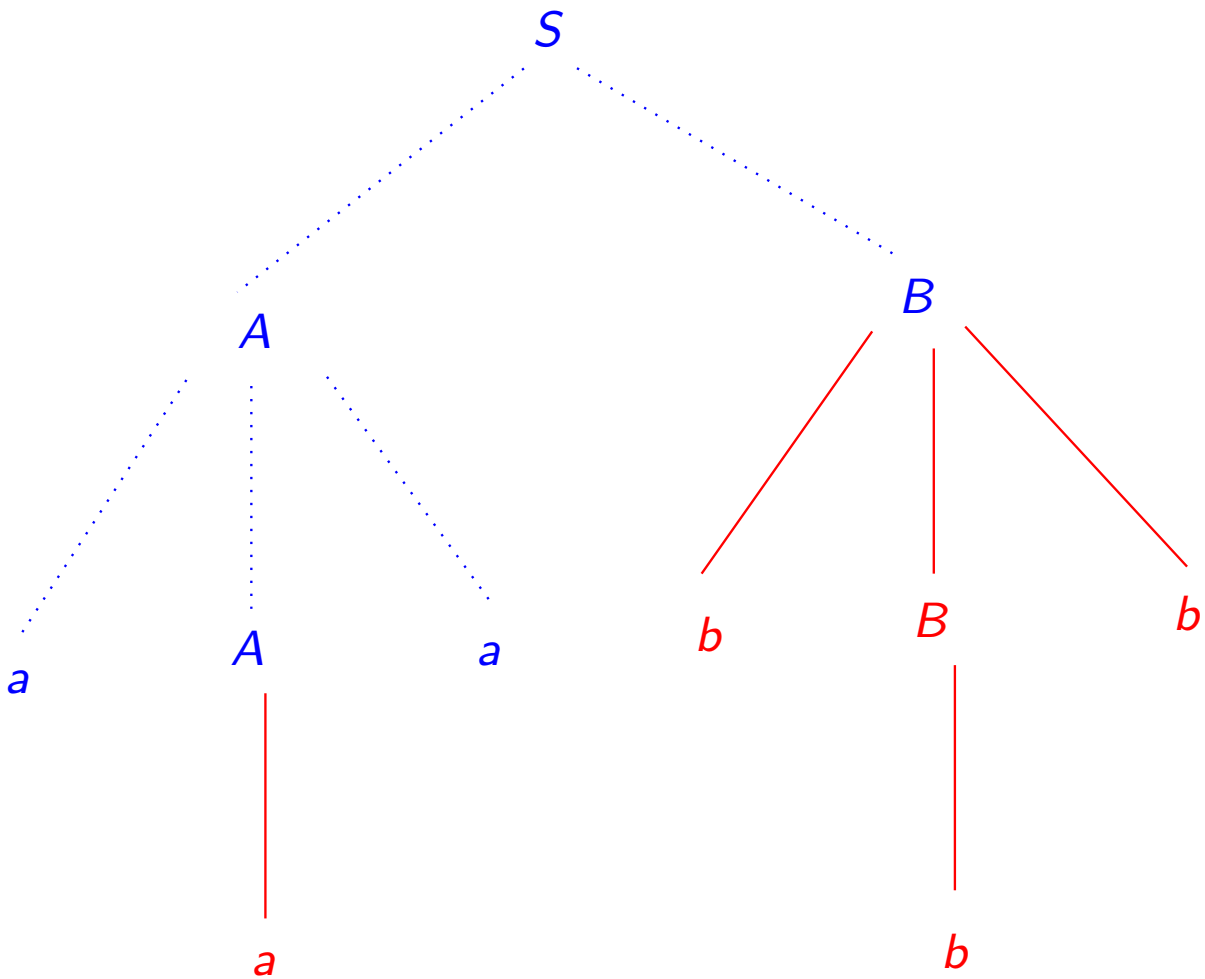
Example Left-Most Derivation (Step 2)

$S \Rightarrow AB$



Example Left-Most Derivation (Step 3)

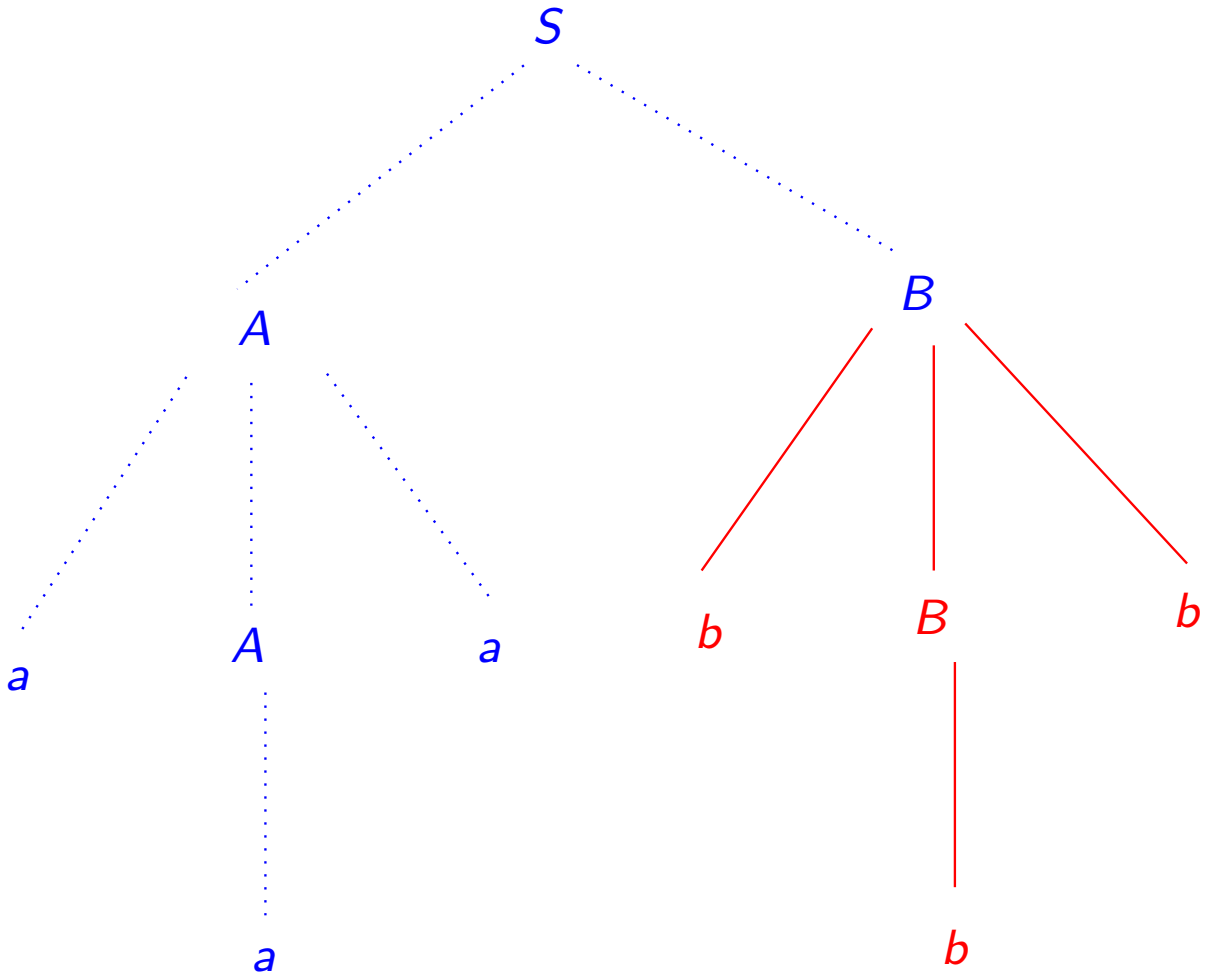
$S \Rightarrow AB \Rightarrow aAaB$



Derivation tree for a is trivial.

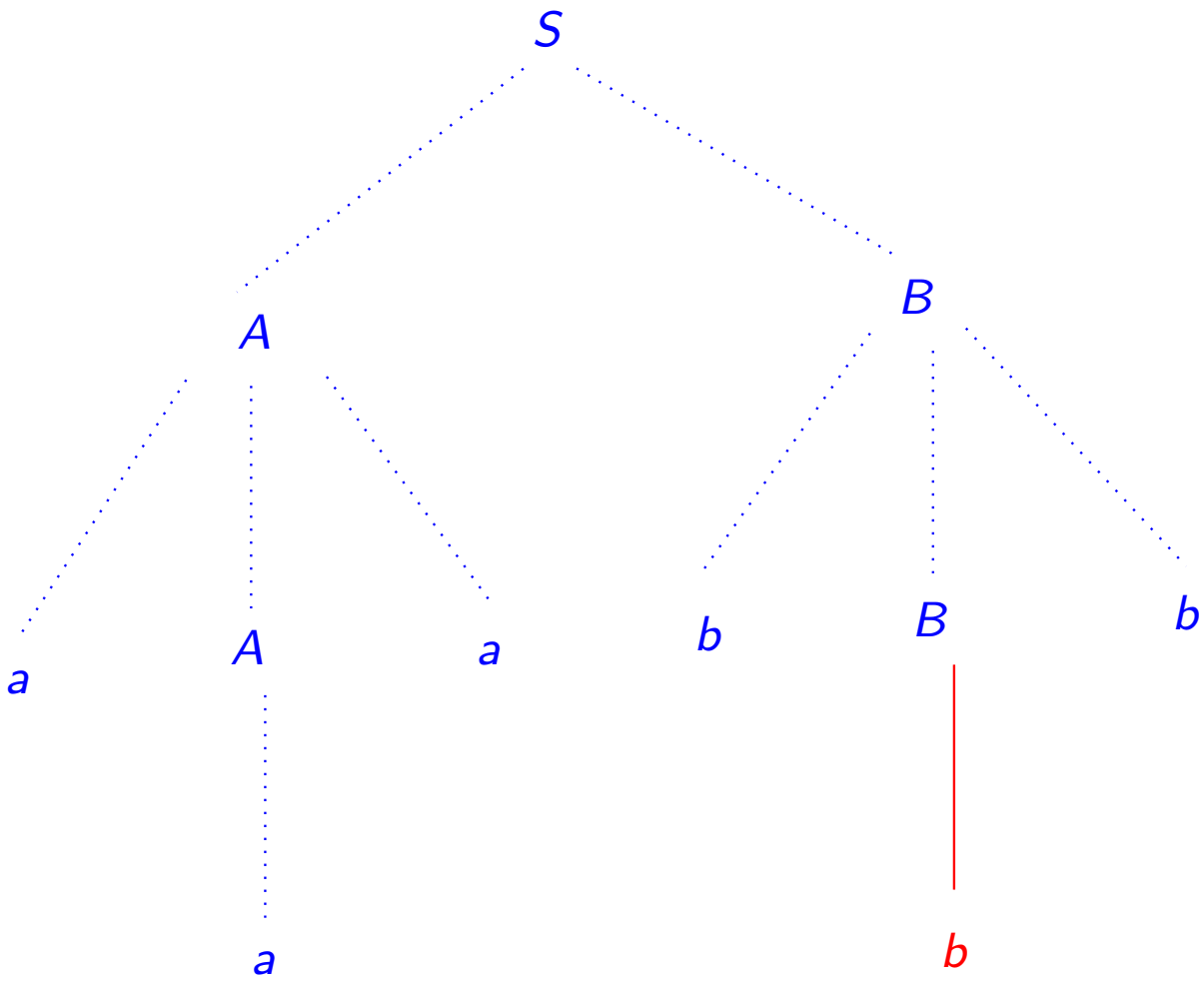
Example Left-Most Derivation (Step 4)

$S \Rightarrow AB \Rightarrow aAaB \Rightarrow aaaB$



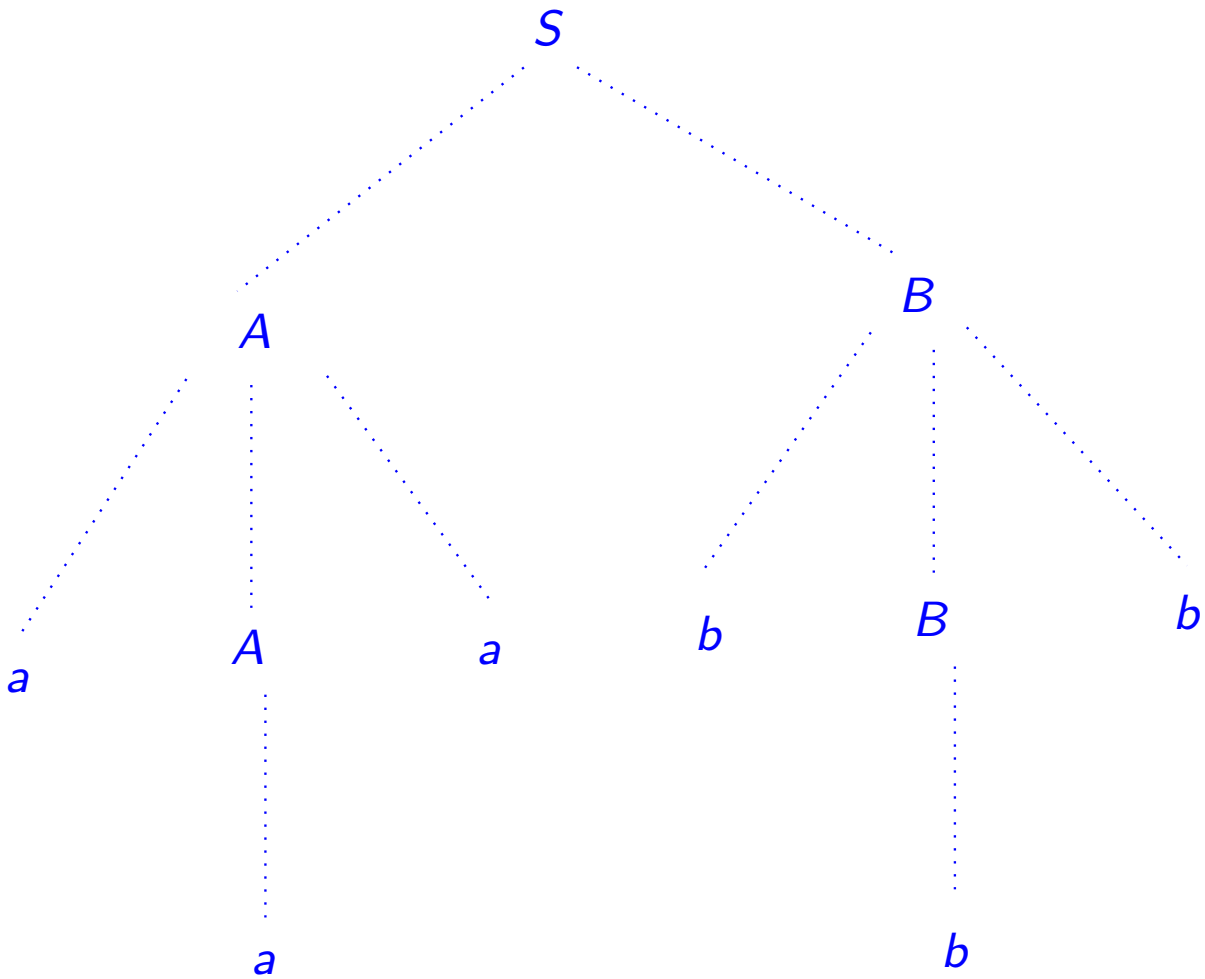
Example Left-Most Derivation (Step 5)

$S \Rightarrow AB \Rightarrow aAaB \Rightarrow aaaB \Rightarrow aaabBb$



Example Left-Most Derivation (Step 6)

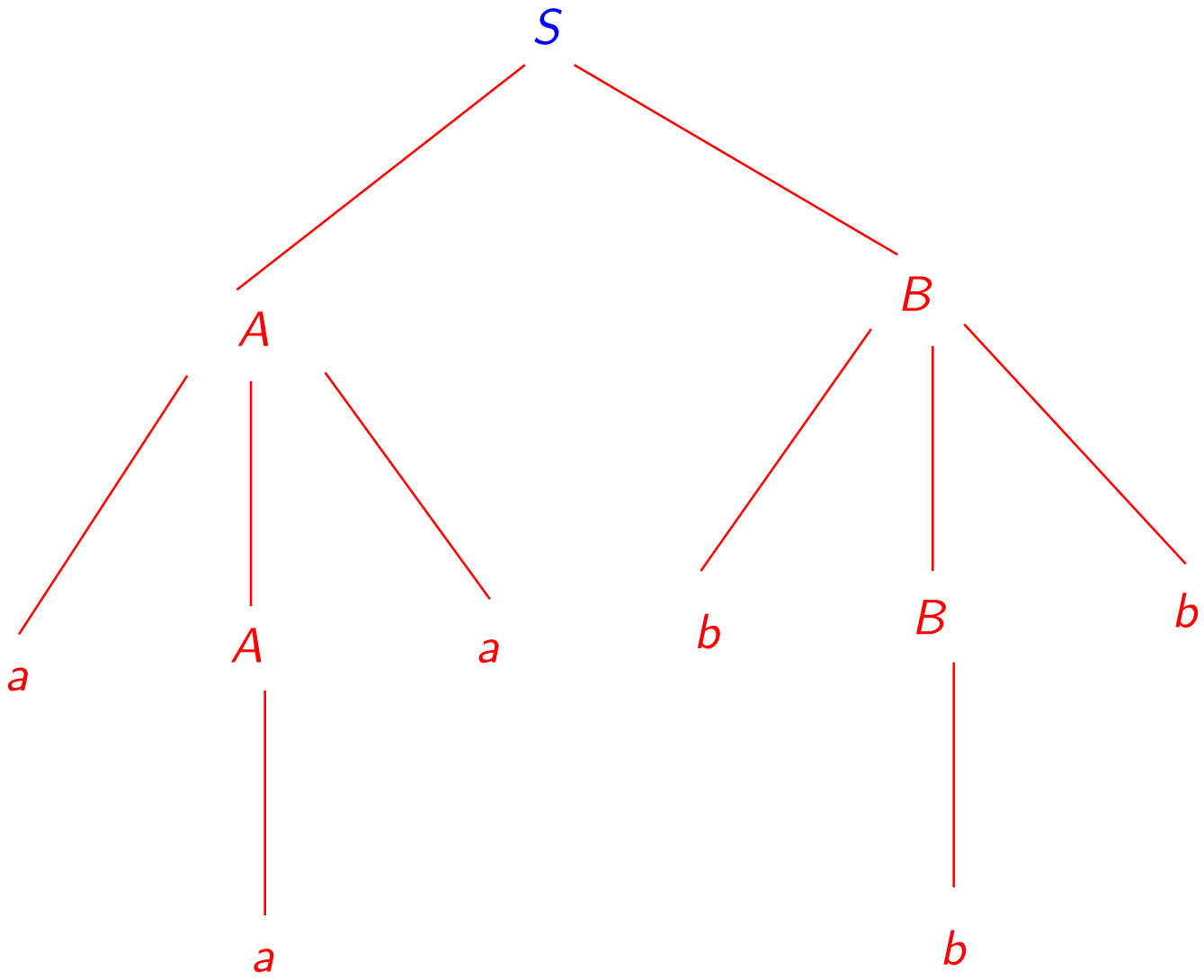
$S \Rightarrow AB \Rightarrow aAaB \Rightarrow aaaB \Rightarrow aaabBb \Rightarrow aaabbb$



Final derivation.

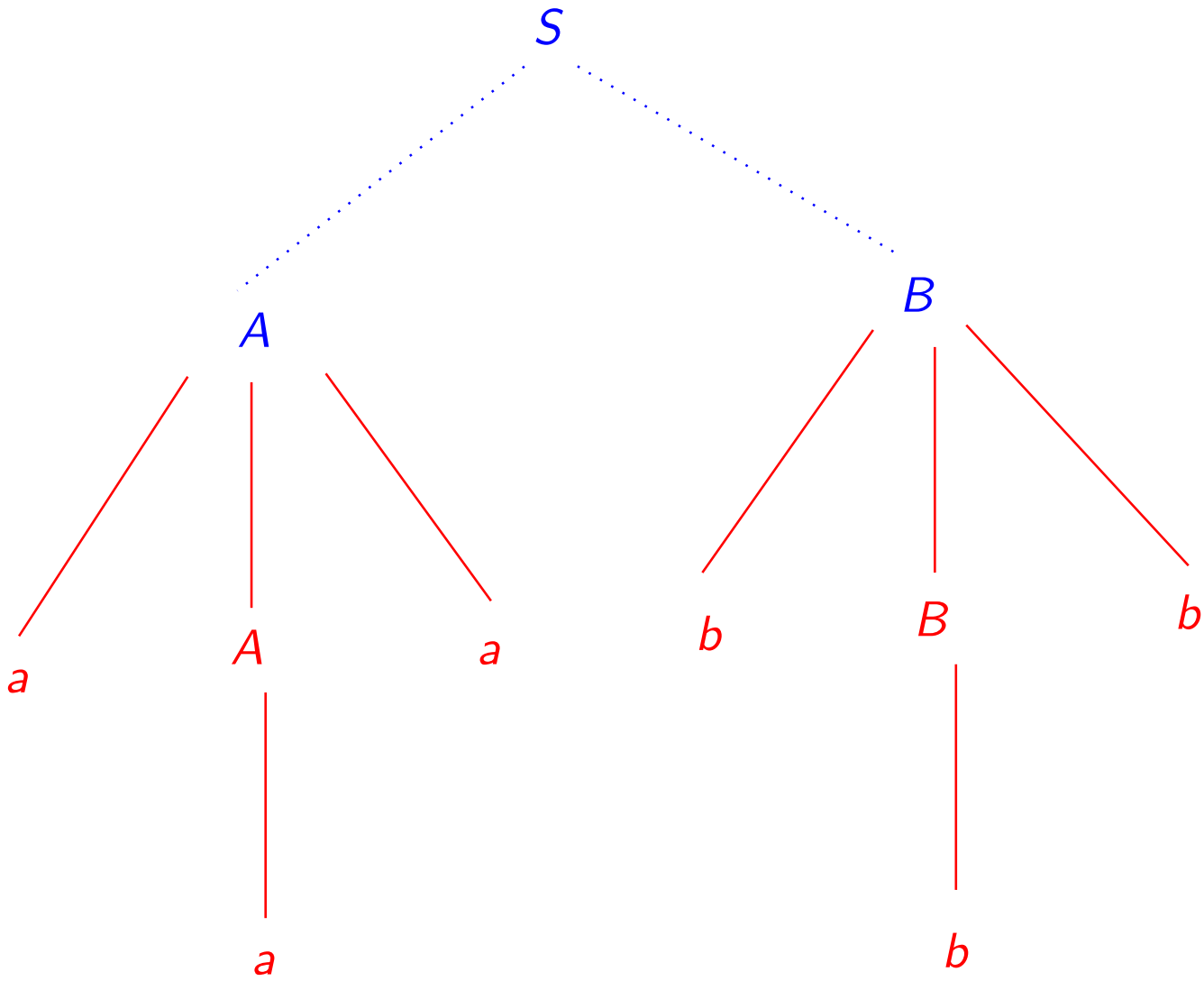
Example Right-Most Derivation (Step 1)

S



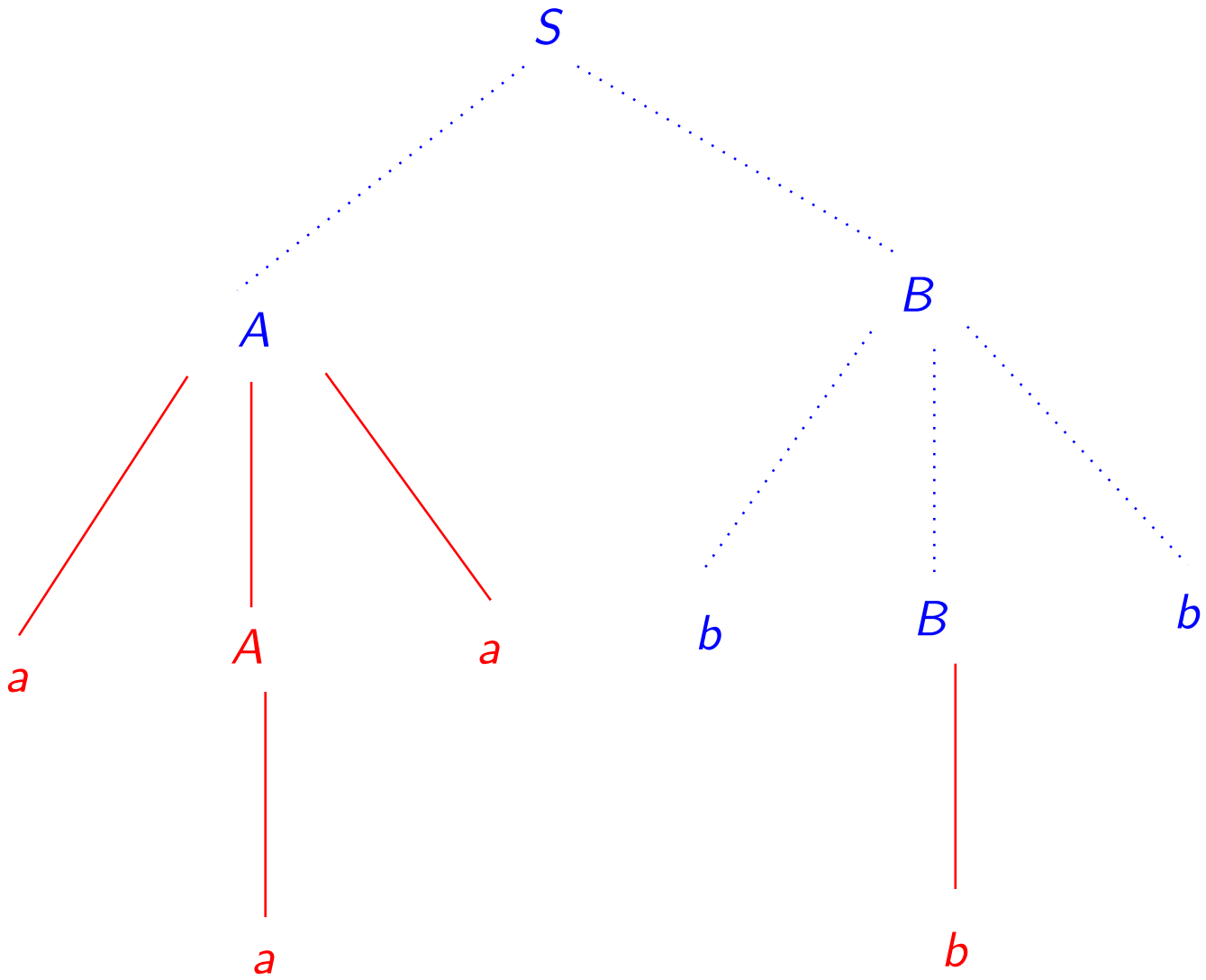
Example Right-Most Derivation (Step 2)

$S \Rightarrow AB$



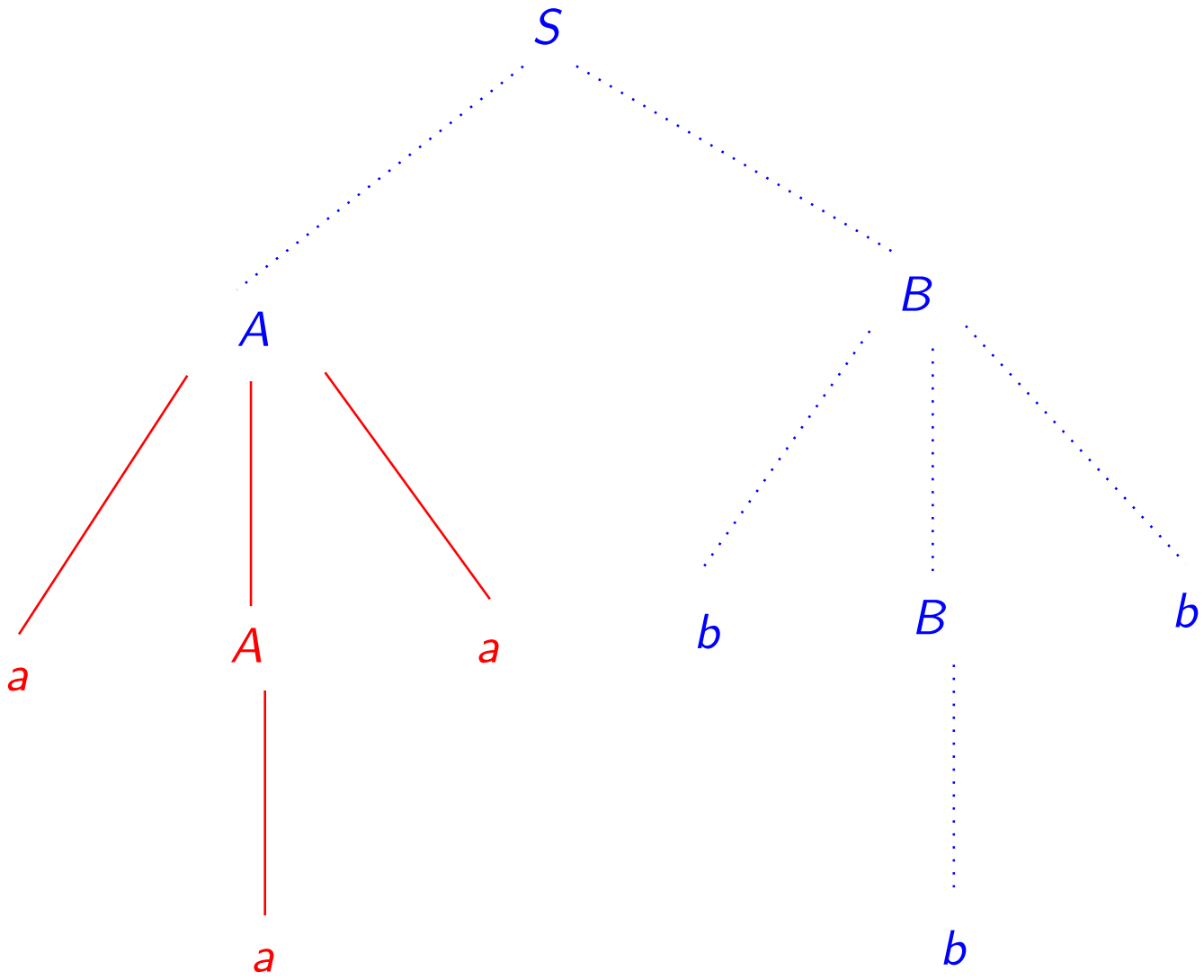
Example Right-Most Derivation (Step 3)

$S \Rightarrow AB \Rightarrow AbBb$



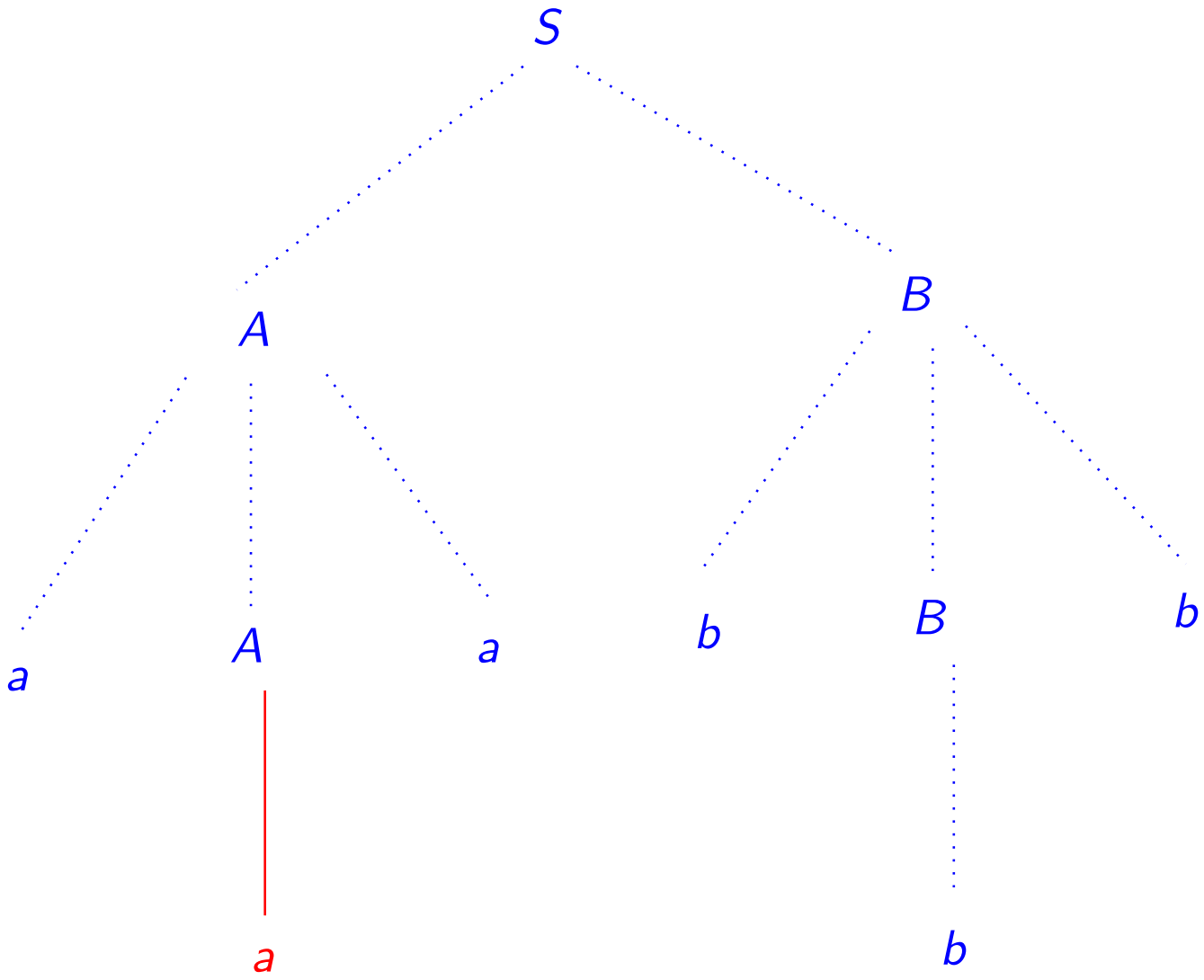
Example Right-Most Derivation (Step 4)

$S \Rightarrow AB \Rightarrow AbBb \Rightarrow Abbb$



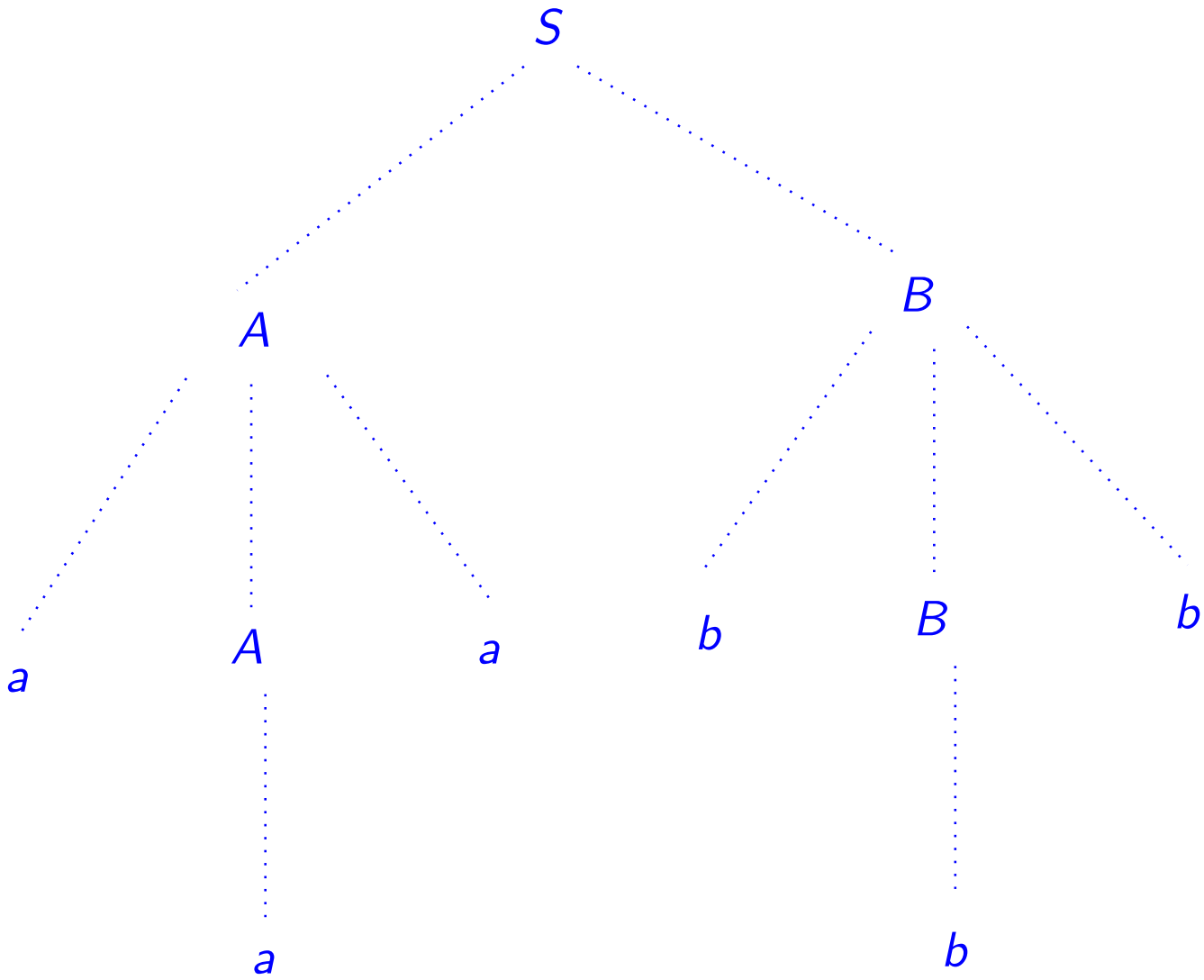
Example Right-Most Derivation (Step 5)

$S \Rightarrow AB \Rightarrow AbBb \Rightarrow Abbb \Rightarrow aAabbb$



Example Right-Most Derivation (Step 6)

$S \Rightarrow AB \Rightarrow AbBb \Rightarrow Abbb \Rightarrow aAabbb \Rightarrow aaabbb$



Theorem II.5.2.4 Uniqueness of Derivation (Trees)

Theorem (II.5.2.4)

Let $G = (T, N, S, P)$ be a CFG, $w \in T^*$. The following are equivalent:

(1) *There exist exactly one derivation tree with label S and frontier w.*

(2) *There exist exactly one left-most derivation sequence $S \Rightarrow^* w$.*

(3) *There exist exactly one right-most derivation sequence $S \Rightarrow^* w$.*

Proof: See Additional Material.

Ambiguous Grammars

Definition

A CFG $G = (T, N, S, P)$ is **ambiguous**, if there is a string $w \in L(G)$ having more than one derivation tree (or, equivalently, having more than one left-most or more than one right-most derivation).

Example 1

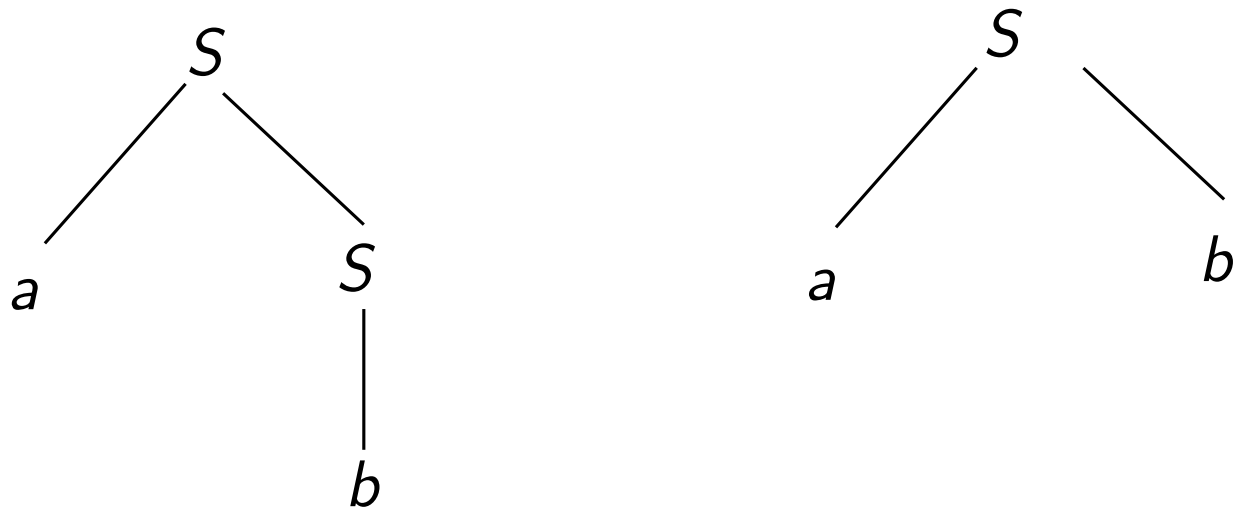
grammar	G
terminals	S
nonterminals	a, b
start symbol	S
productions	$S \longrightarrow aS$
	$S \longrightarrow b$
	$S \longrightarrow ab$

Example 1

There are two left-most derivations of ab:

$$S \Rightarrow aS \Rightarrow ab \text{ and } S \Rightarrow ab$$

And two derivation trees:



Example 2: Dangling Else

Assume the following grammar which is a cut down version of the grammar G^{while} introduced in I.2.4 with **if_then_else_fi** replaced by **if_then** and a **if_then_else_**):

grammar	$G^{Dangling\_else}$
import	$G^{Identifier}$, $G^{Arithmetic\_Expression}$, $G^{Boolean\_Expression}$
terminals	**if**, **then**, **else**, :=
nonterminals	*Program*
start symbol	*Program*
productions	$Program \longrightarrow Id \ := \ AExp$
	$Program \longrightarrow$ **if** $BExp$ **then** $Program$ **else** $Program$
	$Program \longrightarrow$ **if** $BExp$ **then** $Program$

"import" means that we add all the ingredients of those grammars, including the terminals. The grammars $G^{Identifier}$, $G^{Arithmetic\_Expression}$, $G^{Boolean\_Expression}$ have start symbols *Id*, *AExp*, *BExp*, respectively.

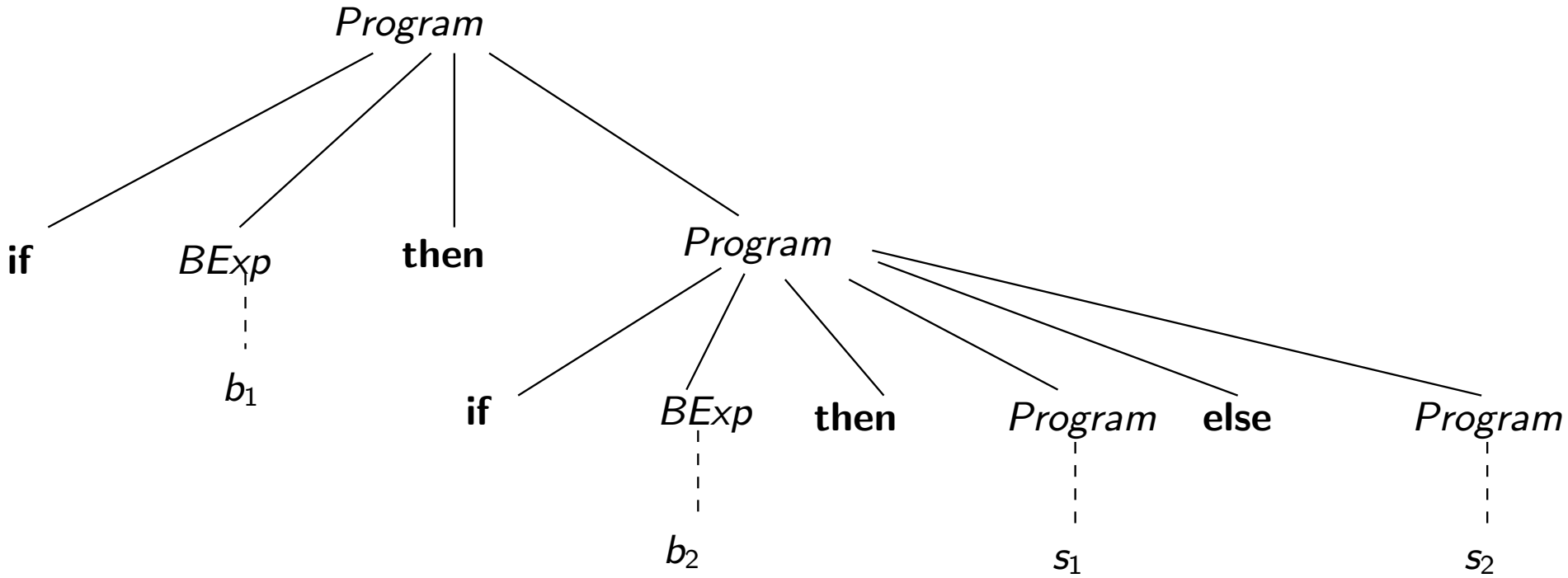
Example 2: Dangling Else

Assume strings b_1, b_2 deriving from *BExp* and string s_1, s_2 deriving from *Program*.
The string

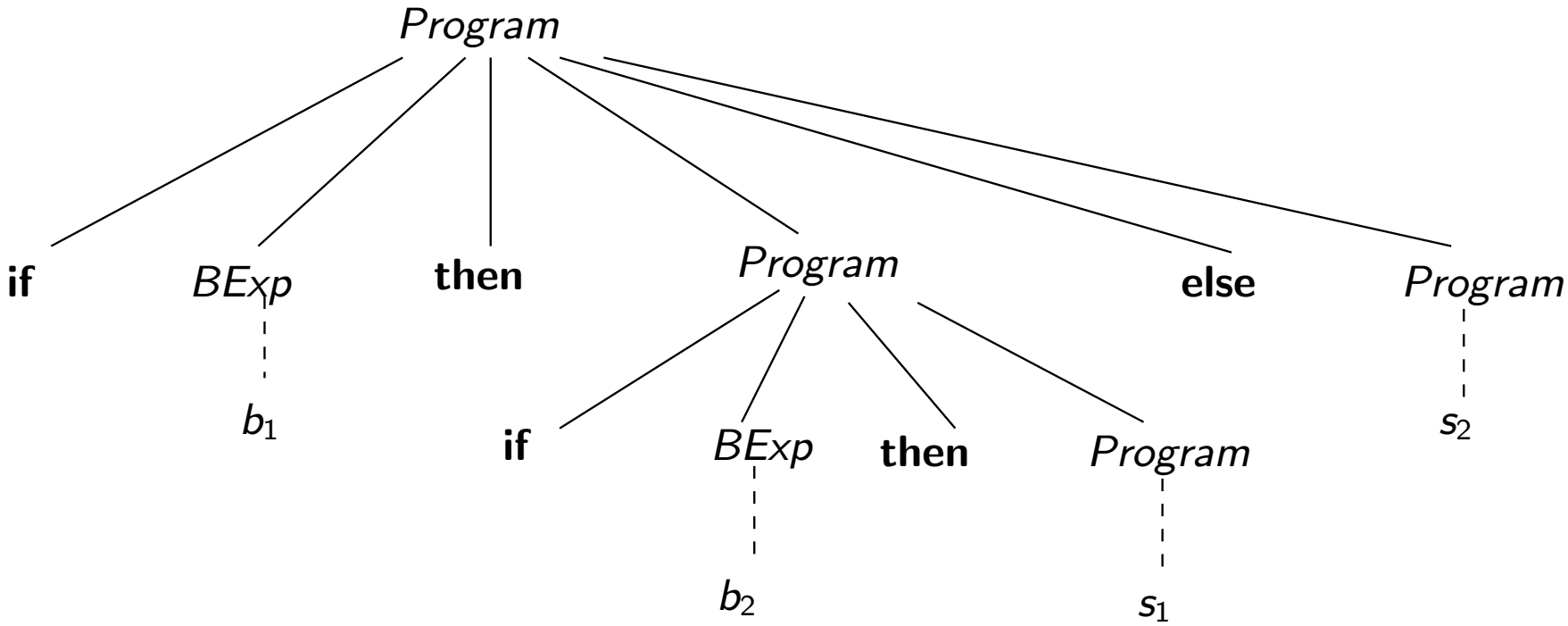
$$\textbf{if } b_1 \textbf{ then if } b_2 \textbf{ then } s_1 \textbf{ else } s_2$$

has two derivation trees (we omit the derivation trees for b_i, s_i.)

First Derivation Tree



Second Derivation Tree



Different Interpretations of the Program

The two different derivation trees of the program

$$\textbf{if } b_1 \textbf{ then if } b_2 \textbf{ then } s_1 \textbf{ else } s_2$$

correspond to two different ways of executing the program:

Execution following the Derivation Tree 1

- In the first the else case belongs to the second if. It is executed if b_1 is true and b_2 is false.
  The program can be using suggestive indentation be written as follows:

if b_1 **then**
  if b_2 **then**
    s_1
  else
    s_2

Execution following the Derivation Tree 2

- In the second derivation tree, the else case belongs to the first if. It is executed if b_1 is false.
  The program can be using suggestive indentation be written as follows:

if b_1 **then**
  if b_2 **then**
    s_1
  else
    s_2

2 Solutions for Solving the Problem

There are 2 solutions for solving this problem.
The first solution is to add to **if_then** and **if_then_else** a symbol **fi** (or some other keyword such as **endif**) labelling the end of the statement.

grammar	$G^{Unambiguous\_if}$
import	$G^{Identifier}$, $G^{Arithmetic\_Expression}$, $G^{Boolean\_Expression}$
terminals	**if**, **then**, **else**, :=
nonterminals	*Program*
start symbol	*Program*
productions	*Program* $\longrightarrow$ *Id* := *AExp* *Program* $\longrightarrow$ **if** *BExp* **then** *Program* **else** *Program* **fi** *Program* $\longrightarrow$ **if** *BExp* **then** *Program* **fi**

Solution 1

Now the two interpretations of the original string would be written in as two different strings:

- "Else" belonging to the second "if" is written as

$$\textit{if}\ \ b_1\ \textbf{then}\ \textit{if}\ \ b_2\ \textbf{then}\ s_1\textbf{else}\ s_2\ \ \textit{fi}\ \ \textit{fi}$$

- "Else" belong to the first "if" is written as

$$\text{if}\ b_1\ \textbf{then}\ \textit{if}\ \ b_2\ \textbf{then}\ s_1\ \ \textit{fi}\ \textbf{else}\ s_2\ \ \textit{fi}$$

- This solution has been taken for instance in Algol, in the bash shell (Linux), and in Ada (where fi is replaced by "end if").
- A similar solution was taken in Java and some other languages: they require that the subprograms of an **if_then_** or **if_then_else_** are enclosed by brackets $\{\cdots\}$.

Solution 2

- The 2nd solution is to modify the grammar so that the derivation tree will be possible only for one of the two choices.
- For this we modify the grammar so that the statement s_1 in

$$\textbf{if } b_1 \textbf{ then } s_1 \textbf{ else } s_2$$

is not matched by

$$\textbf{if } b_1' \textbf{ then } s_1'$$

but only by

$$\textbf{if } b_1' \textbf{ then } s_1' \textbf{ else } s_2'$$

- This solution has been taken in most other programming languages.

Solution 2

- For this we split Programs into two categories:
  - Those derived from MatchedIf. In a program deriving from MatchedIf, each **if** is matched by an **else** clause.
  - Those derived from UnmatchedIf. These have at least one **if** with no matching **else** clause.

Solution 2

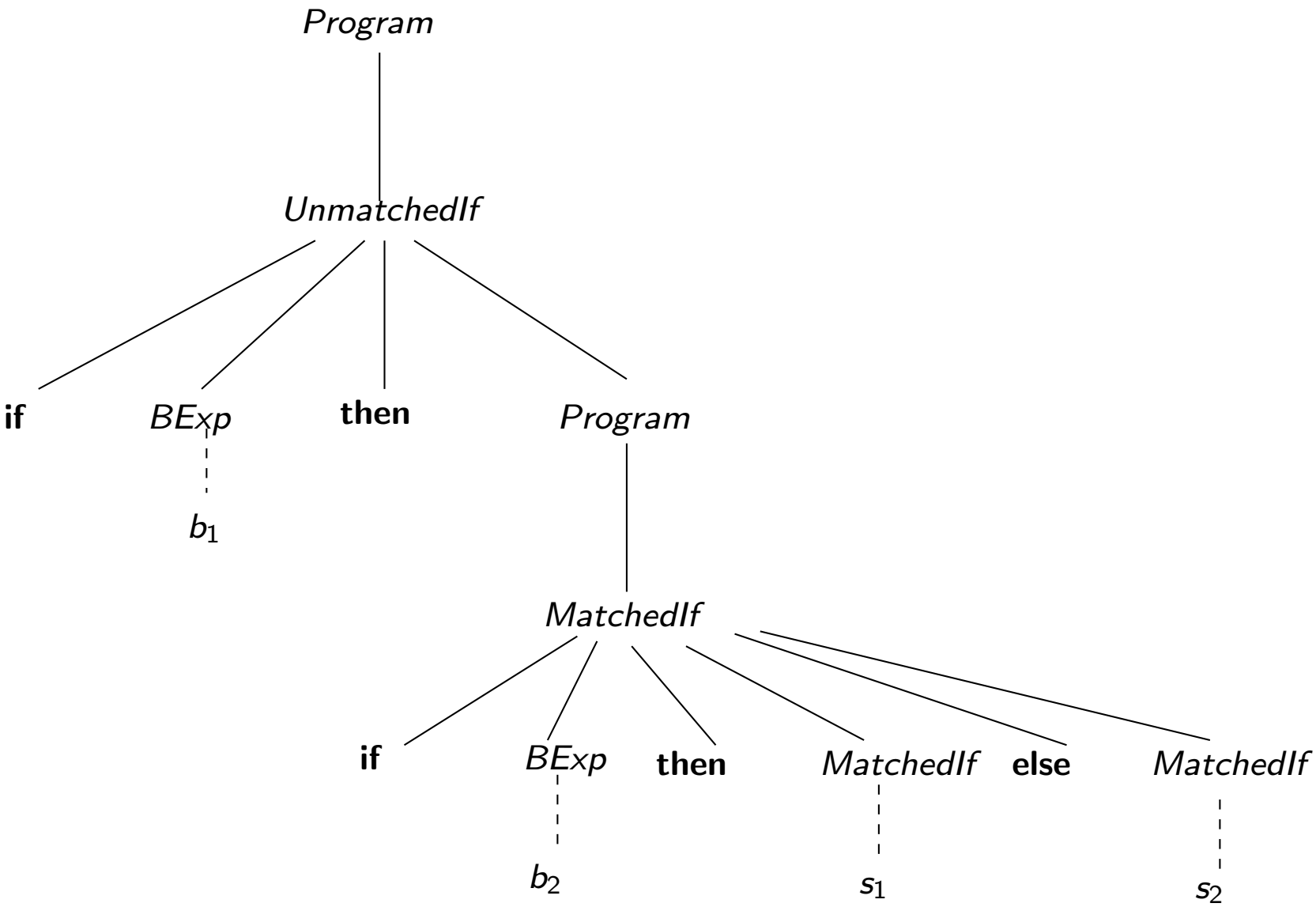
- The grammar will make sure that a **else** will always be associated with the first **if** to the left, which has no unmatched **else** yet.
- So **if**_**then**_*mathbf else* expression will be parsed as in the first derivation tree.

Solution 2

Here is the grammar:

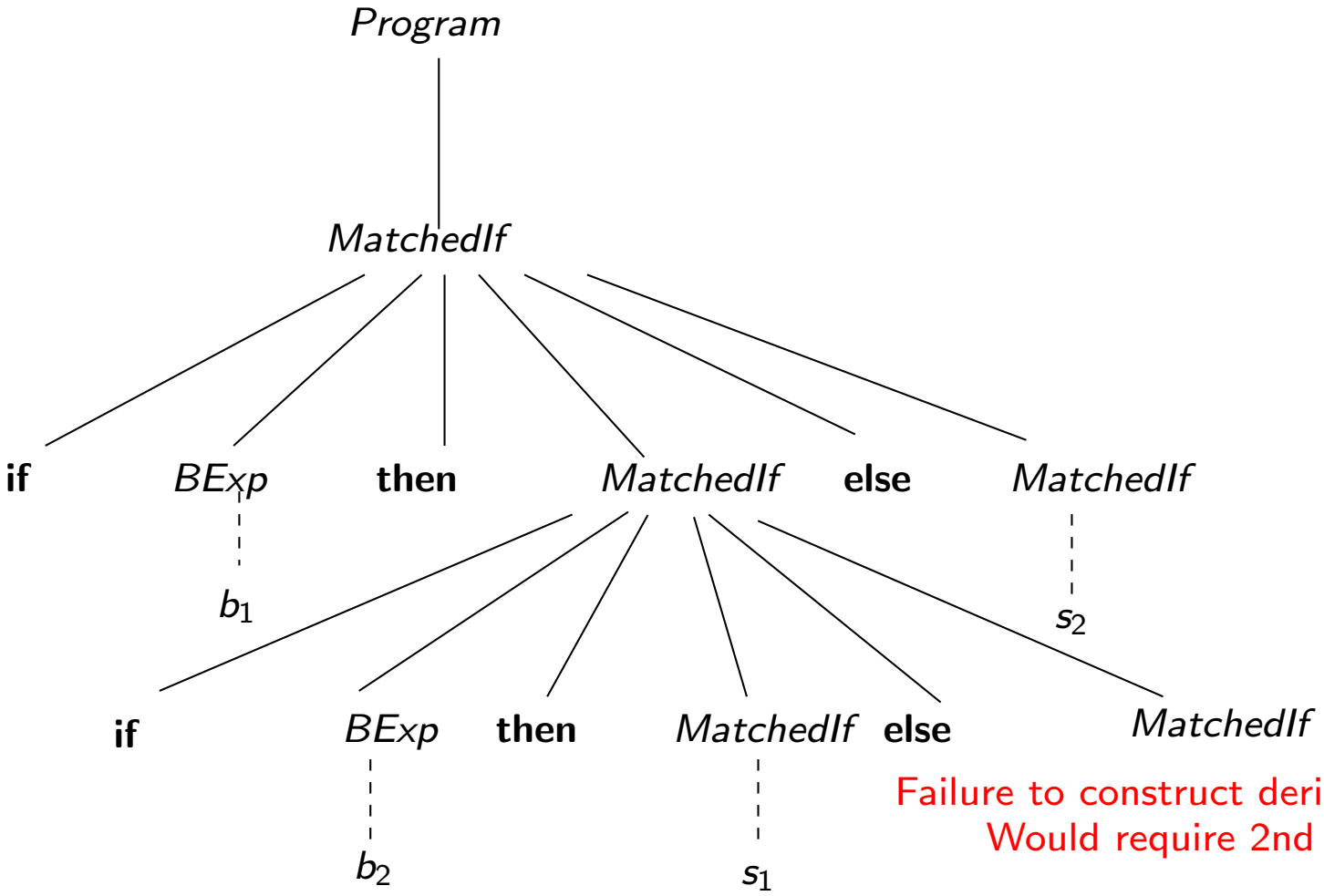
grammar	$G^{Dangling\_Else}$		
import	$G^{Identifier}$, $G^{Arithmetic\_Expression}$, $G^{Boolean\_Expression}$		
terminals	**if**, **then**, **else**, $:=$		
nonterminals	*Program*		
start symbol	*Program*		
productions	*Program*	$\longrightarrow$	*UnmatchedIf*
	Program	$\longrightarrow$	*MatchedIf*
	MatchedIf	$\longrightarrow$	*Id* $:=$ *AExp*
	MatchedIf	$\longrightarrow$	**if** *BExp* **then** *MatchedIf* **else** *MatchedIf*
	UnmatchedIf	$\longrightarrow$	**if** *BExp* **then** *Program*
	UnmatchedIf	$\longrightarrow$	**if** *BExp* **then** *MatchedIf* **else** *UnmatchedIf*

Unique Derivation Tree 2nd Solution



Failure of Starting Derivation Tree with MatchedIf

Trying to construct derivation tree for expression which matches else to first if fails:



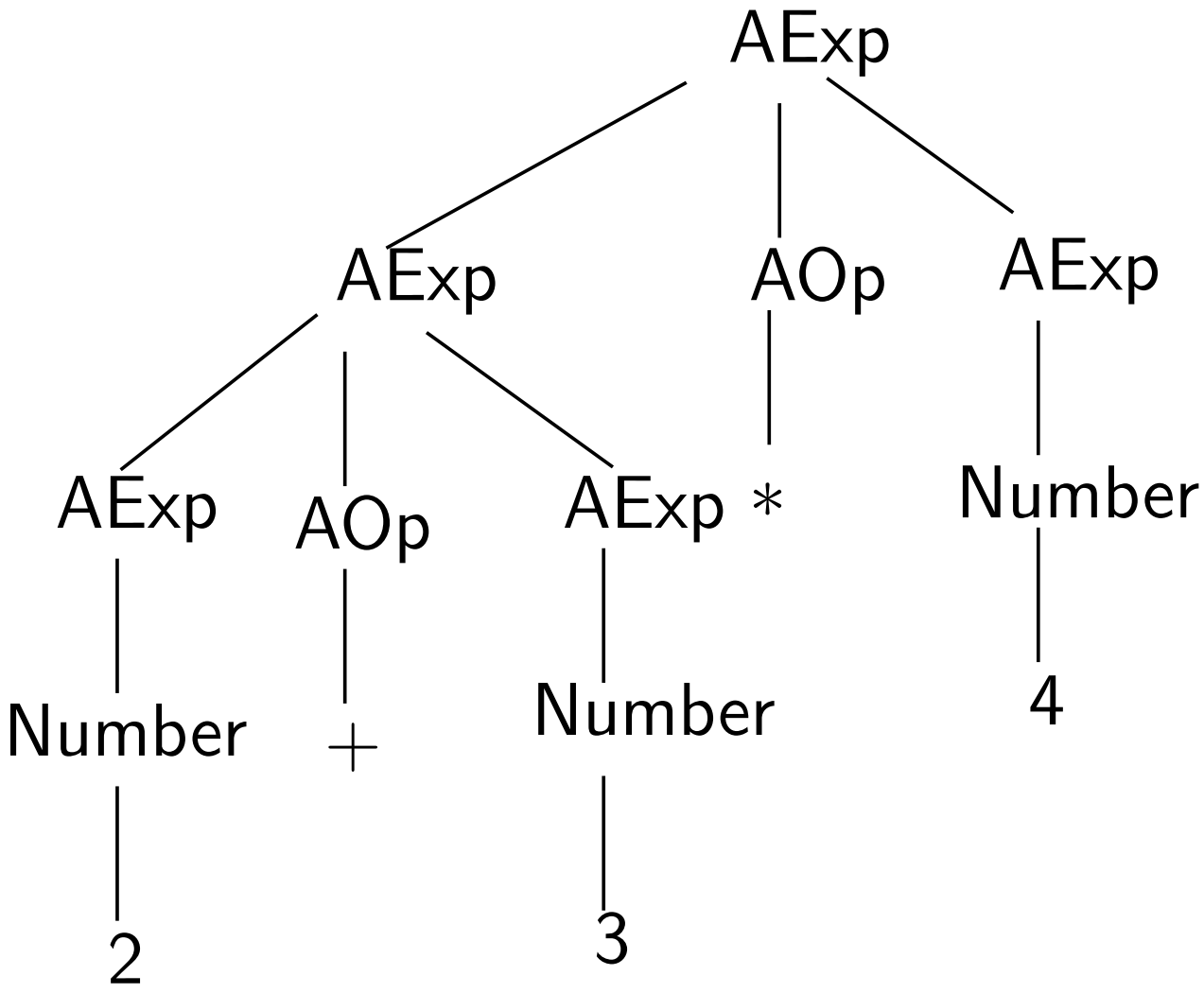
Failure to construct derivation tree
Would require 2nd else

Example: Grammar for Arithmetic Expressions

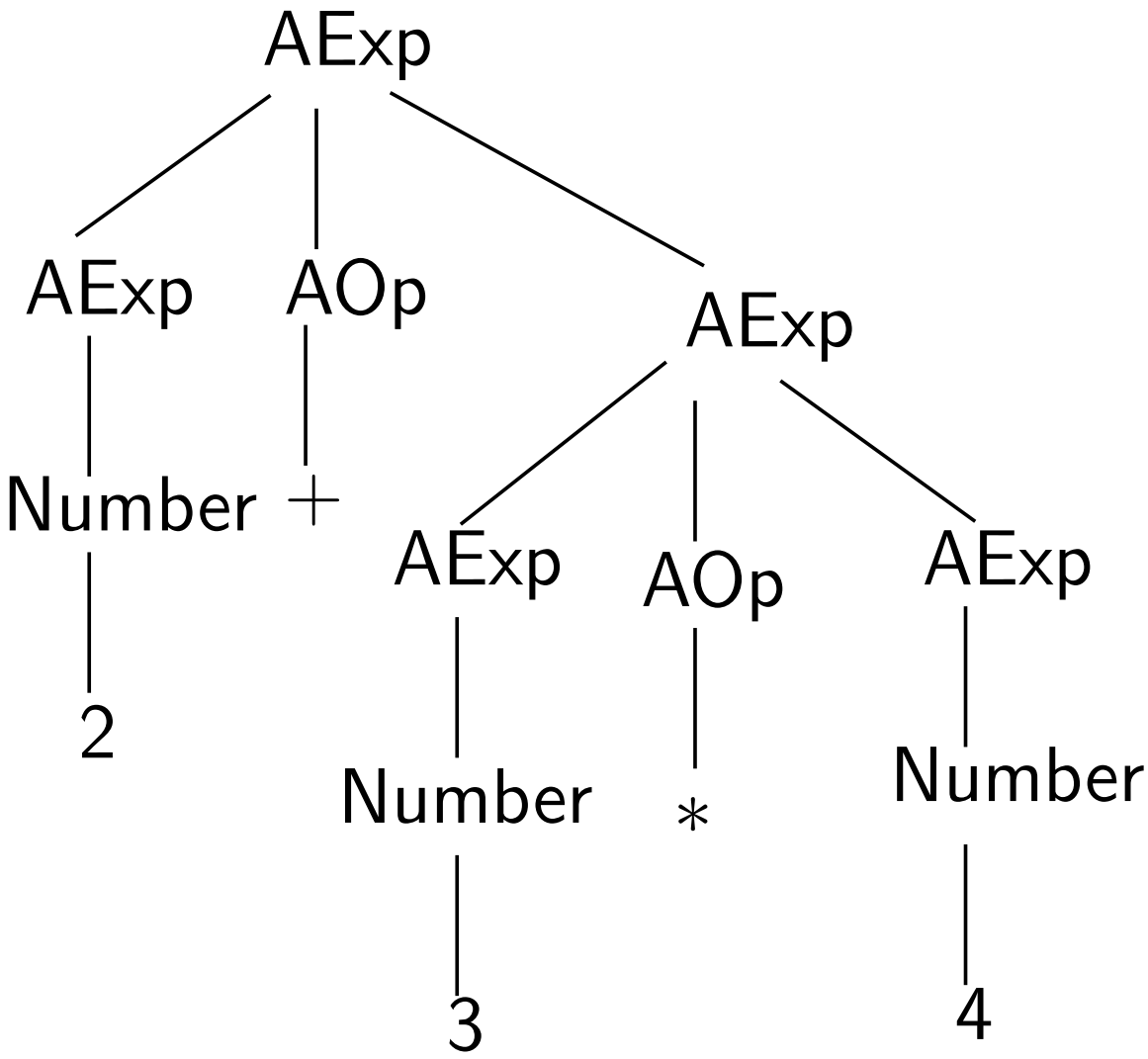
Remember the grammar for arithmetic expressions
(using elements of BNF notation)

grammar	$G^{Arithmetic\_Expression}$
import	$G^{Identifier}$, G^{Number}
terminals	$+, -, *, /, (,)$
nonterminals	$AExp, AOp$
start symbol	$AExp$
productions	$AExp \longrightarrow Id \mid Number$
	$AExp \longrightarrow (AExp)$
	$AExp \longrightarrow AExp\ AOp\ AExp$
	$AOp \longrightarrow + \mid - \mid * \mid /$

First Parse tree for $2 + 3 * 4$



Second Parse tree for $2 + 3 * 4$



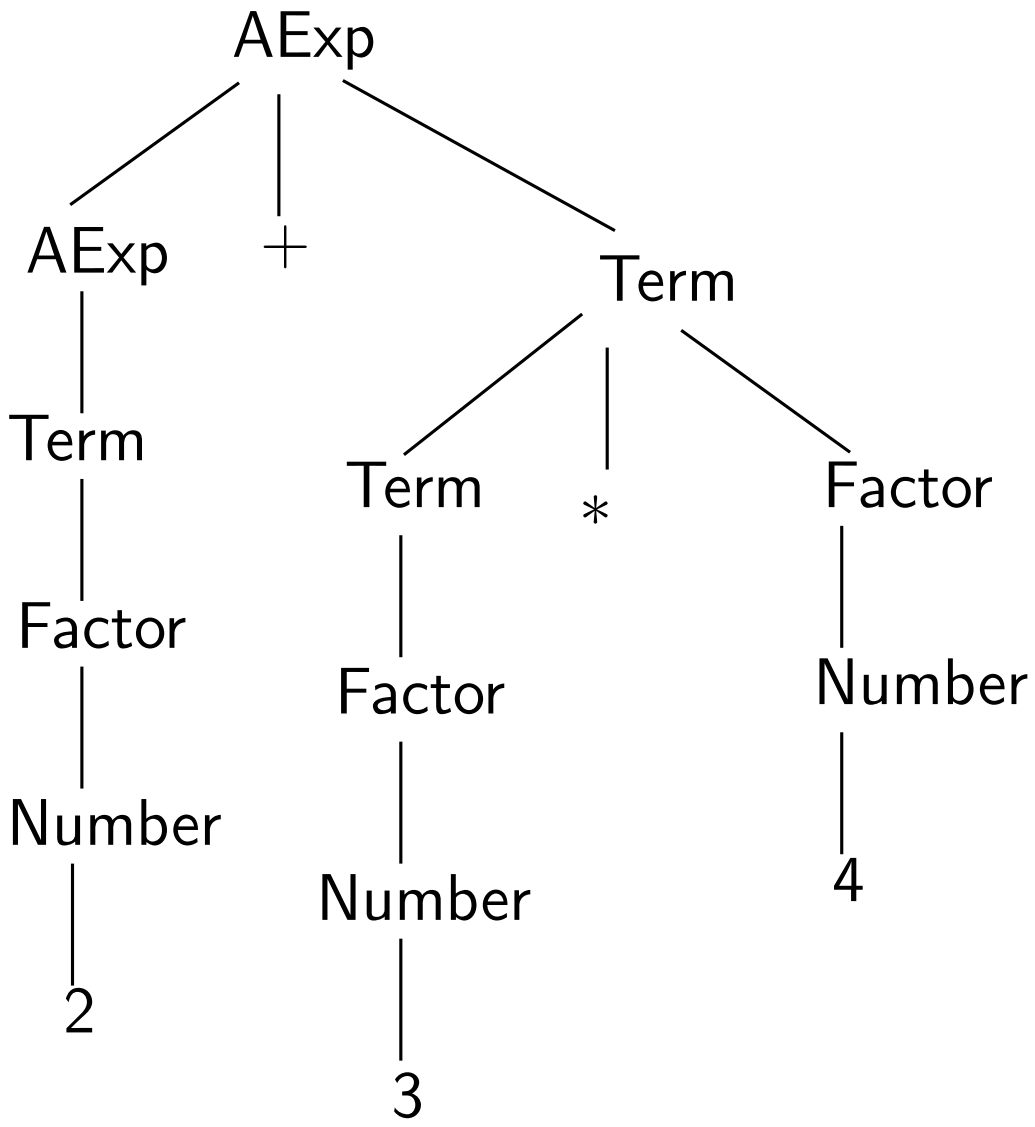
Difference in Evaluation

- The first parse tree corresponds to parsing it as if it were $(2 + 3) * 4$ Evaluation will return 20.
- The second parse tree corresponds to parsing it as if it were $2 + (3 * 4)$ Evaluation will return 14.

Unambiguous Version

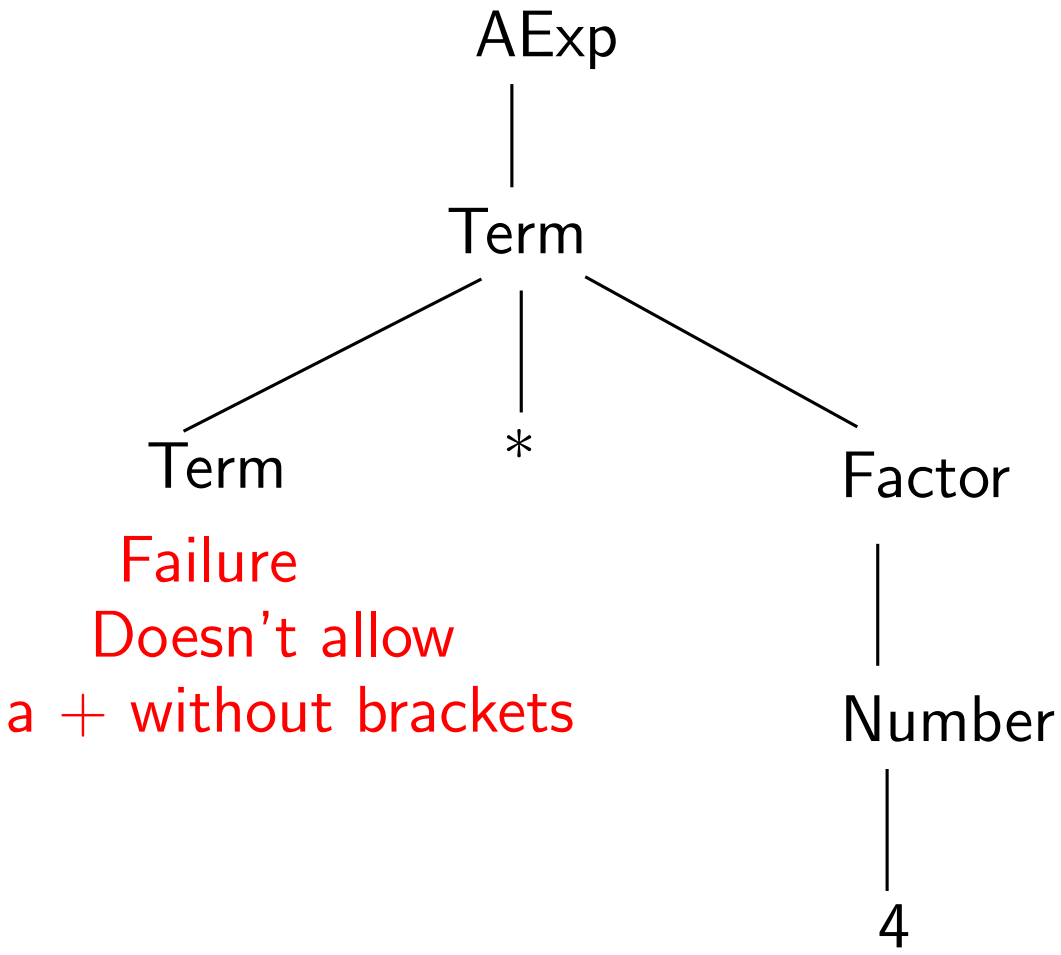
grammar	$G^{Arithmetic\_Expression}_{unambiguous}$
import	$G^{Identifier}$, G^{Number}
terminals	$+, -, *, /, (,)$
nonterminals	$AExp$, $Term$, $Factor$
start symbol	$AExp$
productions	$AExp \longrightarrow AExp + Term \mid AExp - Term \mid Term$
	$Term \longrightarrow Term * Factor \mid Term / Factor \mid Factor$
	$Factor \longrightarrow Id \mid Number \mid (AExp)$

Unique Parse Tree for $2 + 3 * 4$ in $G_{unambiguous}^{Arithmetic\_Expression}$



Failure to Parse $2 + 3 * 4$ with $*$ Binding Stronger Than $+$

Trying to construct derivation tree for expression which parses $2 + 3 * 4$ as $2 + 3 * 4$ fails:



Making Context Free Grammars Unambiguous

The following is known about Context Free Grammars:

- There are languages defined by context free grammars which **cannot be defined by an unambiguous grammar**.
  - Context free grammars, for which there exist no equivalent unambiguous grammar, are called **inherently ambiguous grammars**.
    See Hopcroft/Motwani/Ullman [HMU07], 5.4.4, p. 213.
- It is **undecidable** whether a grammar is **ambiguous**. (Same book, 7.4.5, p. 307.)
- It is **undecidable** whether a grammar is **inherently ambiguous** grammars. (Same book, 7.4.5 and 9.5.2, p. 413).

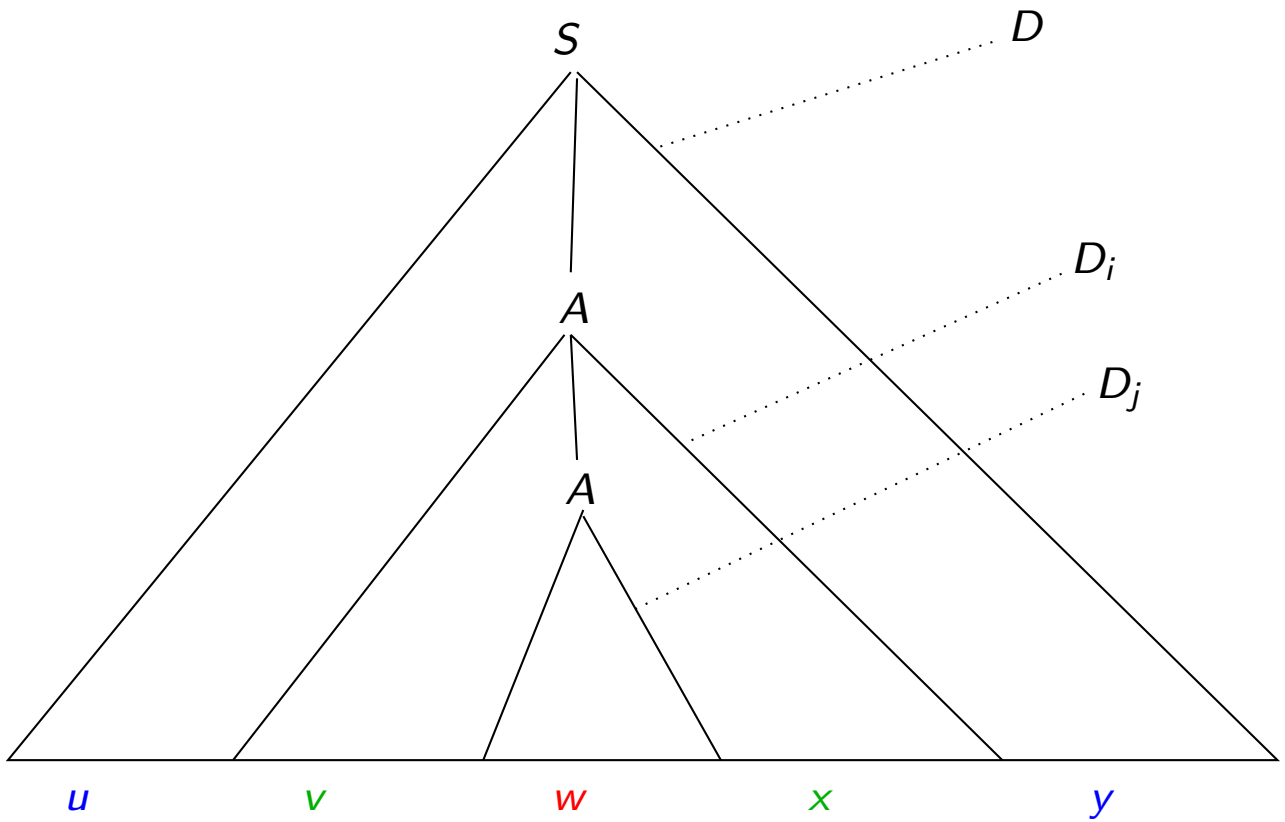
II.5.2. Normal Forms for Context-Free Grammars (14.2)

This section has been moved to "Additional Material".

Idea of the Pumping Lemma for CFG

- The **Pumping Lemma for Regular Languages** is based on the fact that if a string derived by a finite state automaton is sufficiently big, we pass **through at least one state twice**.
- We could equivalently have used the fact that in a **left-** or **right-linear grammar**, if a string derived is sufficiently big, **we pass through one non-terminal at least twice**.
- For **CFG**, we need that if a string derived is sufficiently big, we **pass through one non-terminal** at least **twice**.
- However, if this **occurrence** is **in two disjoint subtrees** of the derivation trees, there is **no relation between them**.
- What we need that there is a **path in the subtree** which **passes through one non-terminal at least twice**.

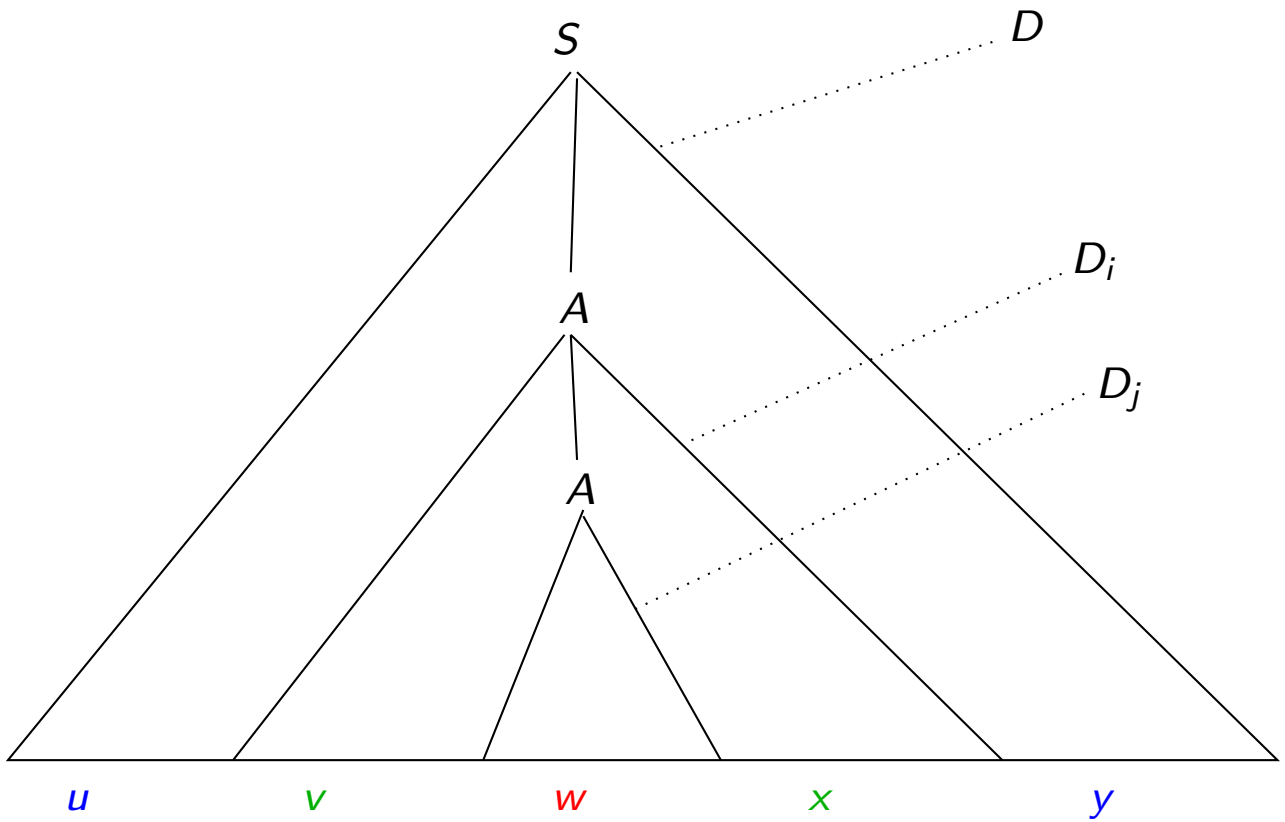
Picture



Idea of the Pumping Lemma for CFG

- **If** in a derivation tree of a CFG with l non-terminals there is **no repetition of a non-terminal in any path**, **then** the **tree can have height at most** l.
  - Here the height of a tree consisting of the root only is defined as 0.
- Since at any node of a derivation tree there are only finitely many rules to apply to, one can easily see that there are only **finitely many derivation trees in a CFG with height at most** $l + 1$ and arbitrary non-terminal as root.
- Let k' be the **maximum length of any string derived from any non-terminal with height at most** $l + 1$. Let $k := k' + 1$.
- Assume a derivation of a string z with $|z| \geq k$.
- We can omit in the derivation any subderivations where $A \Rightarrow^* A$ and this derivation takes more than one step.
- The derivation must have **height** $\geq l + 1$, and therefore contain a **subderivation of height exactly** $l + 1$ **of a string from some non-terminal**.

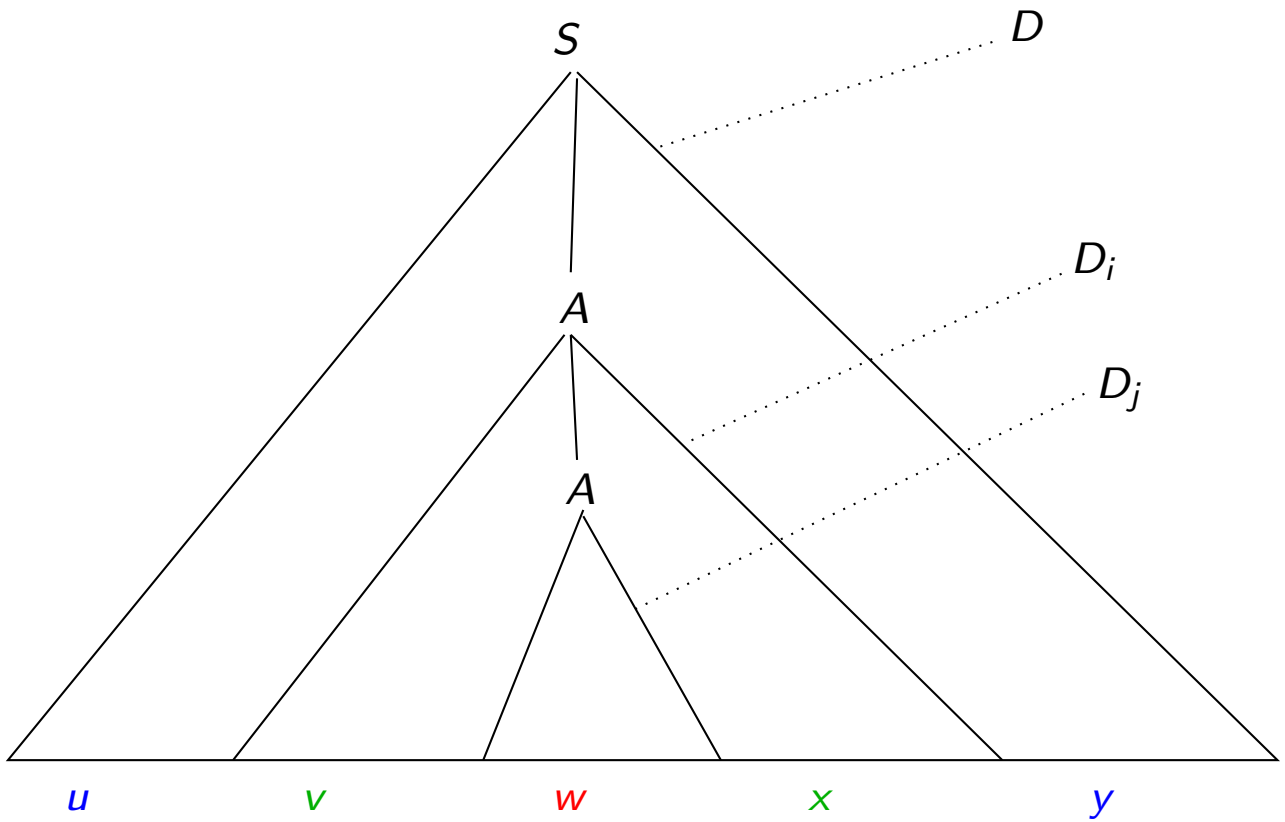
Picture



Idea of the Pumping Lemma for CFG

- In this subderivation there is a path from the root to a terminal, in which at least one non-terminal A occurs twice.
- And we can have that subtree starting with upper occurrence of A has height $\leq l + 1$.
- Therefore the string derived from that A has length at most $k' < k$.
- Let
  - w be the string deriving from the lower A,
  - vwx be the string deriving from the upper A, with v and x deriving to the left and right of the lower A,
  - $z = uvwxy$, with u, y, deriving to the left and right of the upper A,
- Then $|vwx| \leq k' < k$.

Picture

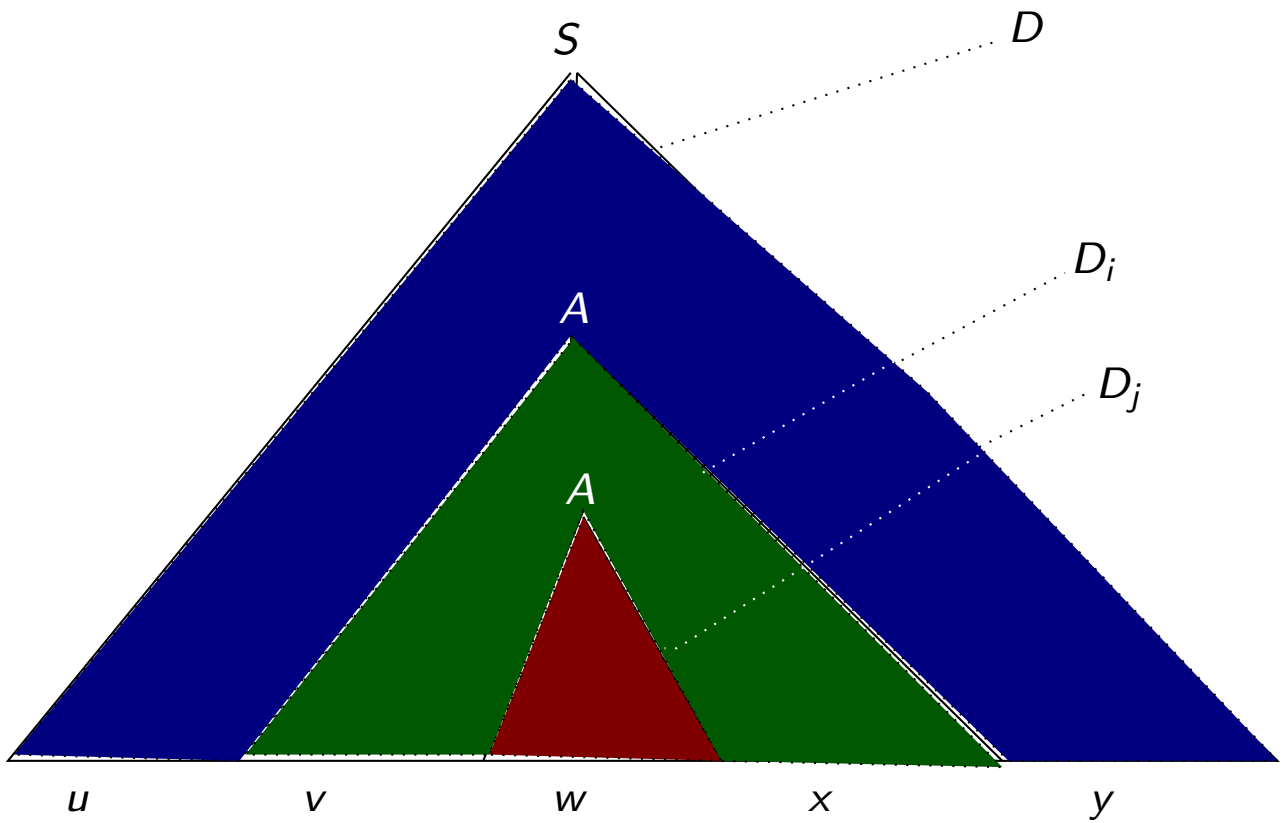


Idea of the Pumping Lemma for CFG

So for any CFG G we can find a constant k s.t.

- in any derivation tree of a word $z \in L(G)$ s.t. $|z| \geq k$,
- we can
  - decompose $z = uvwxy$
  - find a nonterminal A,
  - and derivations
    - $S \Rightarrow^* uAy$ (written blue on the next slide),
    - $A \Rightarrow^* vAx$ (written green on the next slide),
    - $A \Rightarrow^* w$ (written red on the next slide)
  - The subderivation $A \Rightarrow^* vAx$ plays the role of the loop we had in the pumping lemma for regular languages.
  - Furthermore the middle part vwx can be chosen to be of length $\leq k$.
  - $vx \neq \epsilon$ since we omitted subderivations $A \Rightarrow^* A$ taking more than one step.
- The following pictures don't **come out well on the black and white** handouts. Please look at them on the **online version**.

Picture



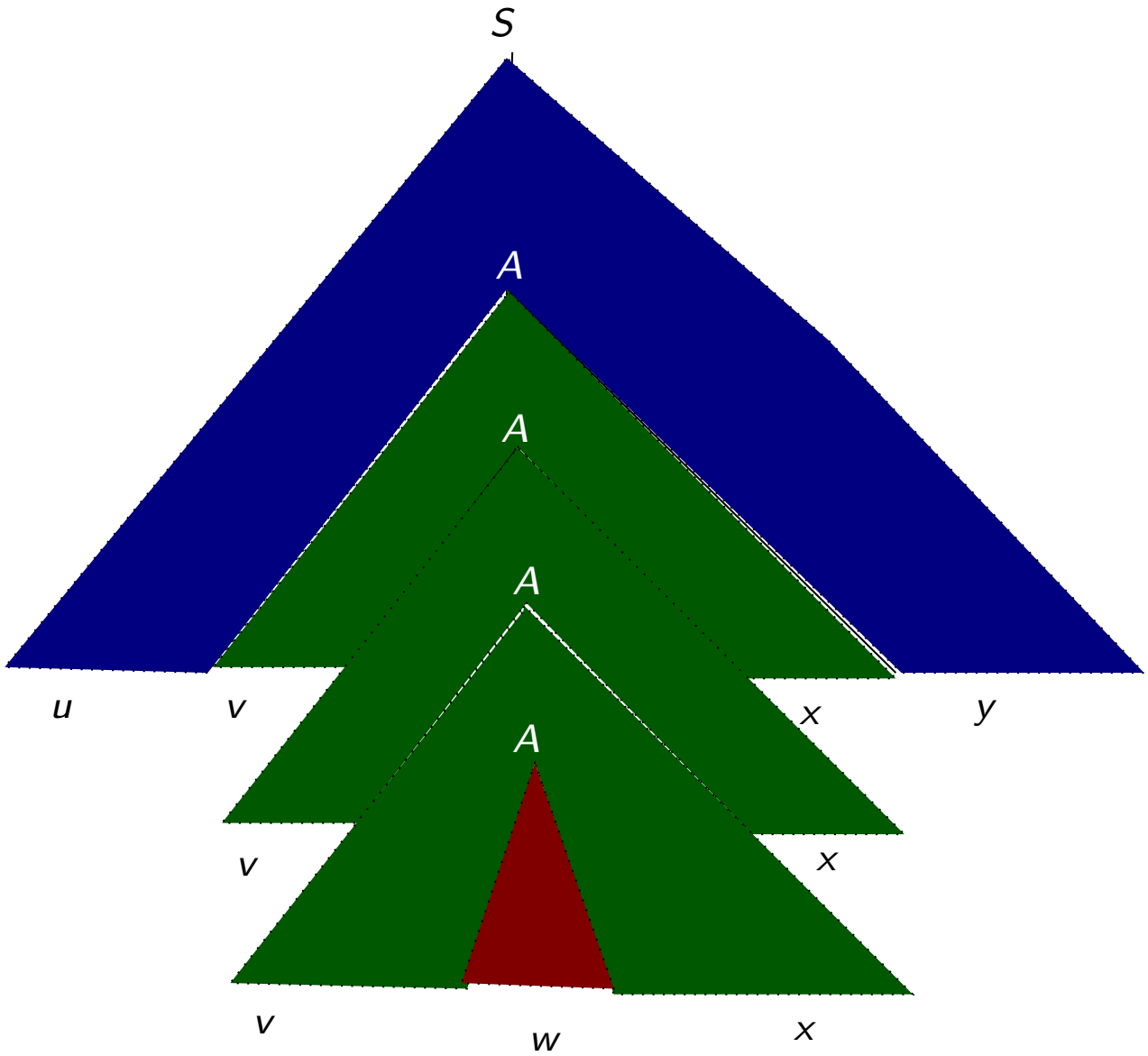
Idea of the Pumping Lemma for CFG

- Now we can repeat the subderivation $A \Rightarrow^* vAx$ several times and obtain

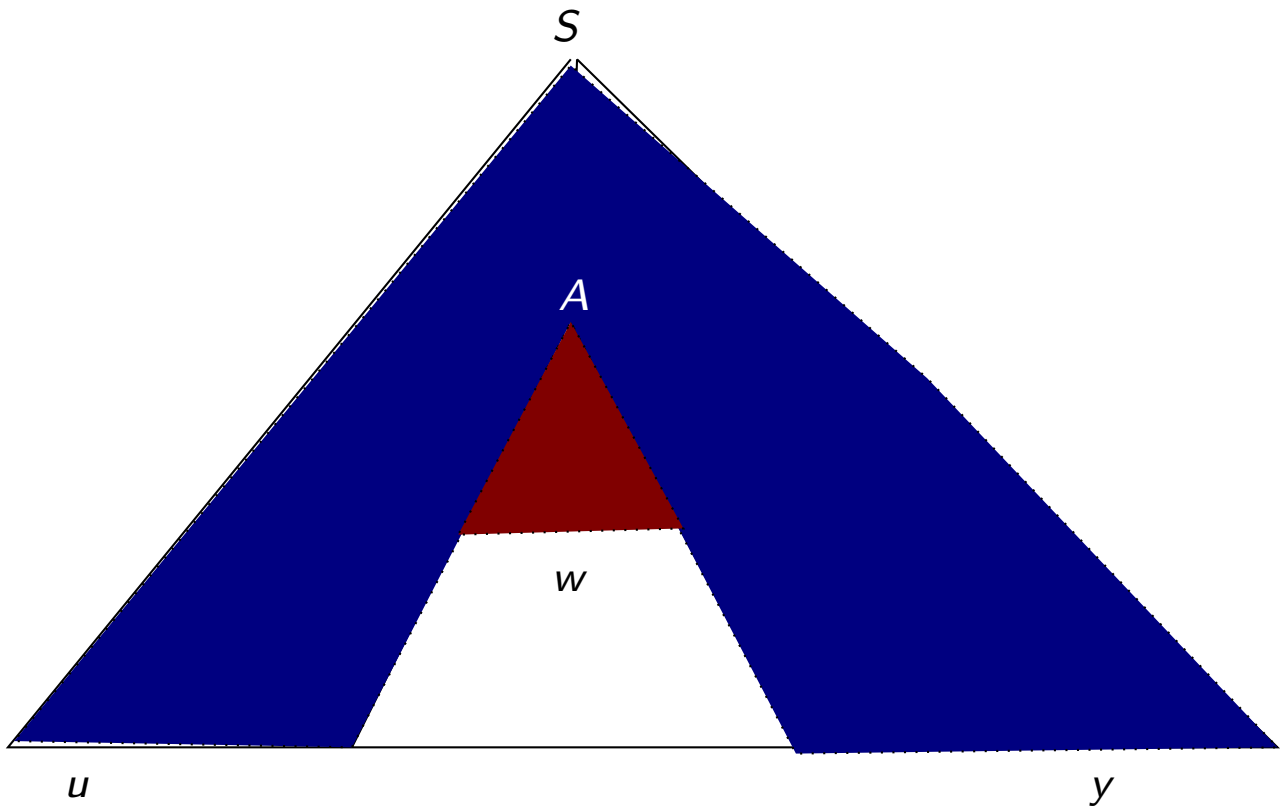
  $$S \Rightarrow^* uAy \Rightarrow^* uvAxy \Rightarrow^* uvvAxxy \Rightarrow^* \cdots \Rightarrow^* uv^iAx^iy \Rightarrow^* uv^ivx^iy$$

- And therefore we obtain that for all $i \geq 0$ we have $uv^ivx^iy \in L(G)$

Pumping it up to uv^3vx^3y



Pumping it down to uv^0vx^0yy



Pumping Lemma for CFG

Theorem

Let L be a context free language. Then there exists a constant k s.t. for all strings z of L s.t. $|z| \geq k$ there exist u, v, w, x, y s.t.

- $z = uvwxy$,
- $|vwx| \leq k$, *i.e. the middle portion is not too long,*
- $|vx| \geq 1$, *i.e. v or x are not ϵ,*
- $\forall i \geq 0. uv^i wx^i y \in L$.

Proof of the Pumping Lemma for CFG

A formal proof can be found in the Additional Material

Example 1

Lemma

The language $L = \{a^i b^i c^i \mid i \geq 0\}$ *is not context free.*

Proof (Example 1)

- Assume L is context free.
- Let k be the constant from the pumping lemma.
- Let $z := a^k b^k c^k \in L$.
- By the pumping lemma, $z = uvwxy$ s.t. $|vwx| \leq k$, $|vx| \geq 1$ and $\forall i \geq 0. uv^i wx^i y \in L$.
- If v contains a's and b's or b's and c's, uv^2wx^2y is not an element of $a^*b^*a^*b^*$ (i.e. the language defined by this regular expression), since there is an a after a b or a b after a c.
- Therefore v is part of a^k, b^k or c^k, similarly for x.
- But now $uv^2wx^2y = a^{k+i}b^{k+j}c^{k+l}$ where at most 2 of (i, j, l) can be $\neq 0$, and at least one is $\neq 0$.
  But then $a^{k+i}b^{k+j}c^{k+l} \notin L$, a **contradiction**.

Example 2

Lemma

The language $L = \{a^n b^m a^n b^m \mid n, m \geq 0\}$ *is not context free.*

Proof (Example 2)

- Assume L is context free.
- Let k be the constant from the pumping lemma.
- Let $z := a^k b^k a^k b^k \in L$.
- By the pumping lemma, $z = uvwxy$ s.t. $|vwx| \leq k$, $|vx| \geq 1$ and $\forall i \geq 0. uv^i wx^i y \in L$.
- If v contains both a's and b's uv^2wx^2y is not an element of $a^*b^*a^*b^*$, since there are more than 3 switches between as and bs.
- Therefore v is part of one of the subwords a^k, b^k, similarly for x.
- But now $uv^2wx^2y = a^{k+i}b^{k+j}a^{k+l}b^{k+m}$ where
  - at most 2 of (i, j, l, m) can be $\neq 0$,
  - if there are two they are consecutive,
  - at least one is $\neq 0$.

  But then $a^{k+i}b^{k+j}a^{k+l}b^{k+m} \notin L$, a **contradiction**.

Example 3

Lemma

The language $L = \{ww \mid w \in \{a, b\}^*\}$ *is not context free.*

Proof (Example 3)

- We use the fact that the intersection of a context free and a regular grammar is context free.
  - This fact is not shown in this module.
  - It can be shown using the equivalence of context free languages and languages definable by Push Down Automata.
- If L were context free, so were $L' := L \cap (a^*b^*a^*b^*)$.
- But L' is just the language of Example 2, which is not context free, a **contradiction**.

Intersection of CFG

- In Example 3 we used the fact that the intersection of a context free and a regular language is context free.
- The intersection of two context free languages is in general not context free:
  Consider

$$\begin{aligned} L_1 &:= \{a^n b^n c^m \mid n, m \in \mathbb{N}\} \\ L_2 &:= \{a^n b^m c^m \mid n, m \in \mathbb{N}\} \end{aligned}$$

  Both L_1, L_2 are context free.
  However

$$L_1 \cap L_2 = \{a^n b^n c^n \mid n \in \mathbb{N}\}$$

  which is tha language of Example 1 which is **not context free**.

Repetition of words is not context free

- We have seen in Example 3 of II.5.4 (using Pumping Lemma for Context Free Grammars), that

$$L := \{ww \mid w \in \{a, b\}^*\}$$

  is **not** context free.
  - Note that

$$\{ww^{\mathrm{R}} \mid w \in \{a, b\}^*\}$$

    is context free.
- A program language which expresses that a variable needs to be declared before contains as a sublanguage L.
  - More precisely, if we had a context free grammar for such a language, we could derive from it a context free grammar for L.
- This can be generalised to Floyd's theorem.

II.5.5. Floyd's Theorem

Theorem

Under weak assumptions a programming language, which requires that variables need to be declared before used, cannot be defined by a context free grammar.

A precise formulation and proof of Floyd's theorem can be found in "Additional Material".

II.5.5. Floyd's Theorem

- Therefore most programming languages cannot be defined by a context free grammar.
- However, one can define in most cases a context free grammar defining the basic syntax of a language.
  - Grammar allows to define a parse tree.
  - Languages which are defined by this grammar are those which can be parsed in such a way.
- Then one adds a program, which afterwards checks semantic properties of the program,
  - E.g. that a variable is declared before being used.
  - Or even more complicated features such as correctness of type checking.
- Full details can be found in the "Additional Material".

Chapter II.6.: Push Down Automata

This Chapter will not be taught this year.