CS_275 Automata and Formal Language Theory

Course Notes
Additional Material
(This material is no longer taught and not exam relevant)
Part III: Limits of Computation
Chapt. III.1: Introduction

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Examples for Limits of Computation

- Define a function is_sorting_fun : String $\to \{\mathrm{true}, \mathrm{false}\}$,

$$\text{is\_sorting\_fun}(p) := \begin{cases} \mathrm{true} & \text{if } p \text{ is a syntactically correct Java program, which has as input a list and returns a sorted list,} \\ \mathrm{false} & \text{otherwise.} \end{cases}$$

Is is_sorting_fun *computable?*

Explanation

- Assume is_sorting_fun were computable.
- Then we can construct (compute) a program which computes terminate as follows:
  - Assume as input a string p.
  - Check whether it is a syntactically correct Java program with no input and outputs.
  - If no, terminate(p) $= \mathrm{false}$, so return false.
  - Otherwise, create from p a program $q(p)$ which is a potential sorting function as follows:
    - $q(p)$ takes as input a list l.
    - Then it runs p.
    - If p has terminated, then it runs a known sorting function on l, and returns the result.

Explanation

  - If p terminates, then $q(p)$ will be a sorting function, so is_sorting_fun($q(p)$) $= \mathrm{true} =$ terminate(p).
  - If p does not terminate, then $q(p)$ does not terminate on any input, so is_sorting_fun($q(p)$) $= \mathrm{false} =$ terminate(p).
  - Our program returns now is_sorting_fun($q(p)$) which is the result of terminate(p).
- So we have obtained by using a program for is_sorting_fun a program which computes terminate.
- But terminate is non-computable, therefore is_sorting_fun cannot be computable.

Diophantine Equations

- Here is a short description of Diophantine Equations.
- This is the question, whether an indeterminate polynomial equation has solutions where the variables are instantiated as integers.
- Examples:
  - Solve for integers a, b the equation $ax + by = 1$ using integers x, y.
  - Solve for given n the equation $x^n + y^n = z^n$.
    - For $n \geq 3$ this is unsolvable by Fermat's Last Theorem.