

CS_275 Automata and Formal Language Theory

Course Notes

Additional Material

(This material is no longer taught and not exam relevant)

Part III: Limits of Computation

Chapt. III.1: Introduction

Anton Setzer

<http://www.cs.swan.ac.uk/~csetzer/lectures/automataFormalLanguage/current/index.html>

March 30, 2017

CS_275

Chapt. III.1 (Additional Material)

1 / 8

Examples of Difficulty Defining Computability

Examples of Difficulty Defining Computability

Diophantine Equations

CS_275

Chapt. III.1 (Additional Material)

2 / 8

Examples of Difficulty Defining Computability

Examples for Limits of Computation

Examples of Difficulty Defining Computability

Diophantine Equations

- Define a function is_sorting_fun : String → {true, false},

$$\text{is_sorting_fun}(p) := \begin{cases} \text{true} & \text{if } p \text{ is a syntactically correct} \\ & \text{Java program, which has as input} \\ & \text{a list and returns a sorted list,} \\ \text{false} & \text{otherwise.} \end{cases}$$

Is is_sorting_fun *computable*?

CS_275

Chapt. III.1 (Additional Material)

3 / 8

CS_275

Chapt. III.1 (Additional Material)

4 / 8

Explanation

- ▶ Assume `is_sorting_fun` were computable.
- ▶ Then we can construct (compute) a program which computes `terminate` as follows:
 - ▶ Assume as input a string p .
 - ▶ Check whether it is a syntactically correct Java program with no input and outputs.
 - ▶ If no, `terminate(p) = false`, so return `false`.
 - ▶ Otherwise, create from p a program $q(p)$ which is a potential sorting function as follows:
 - ▶ $q(p)$ takes as input a list l .
 - ▶ Then it runs p .
 - ▶ If p has terminated, then it runs a known sorting function on l , and returns the result.

Explanation

- ▶ If p terminates, then $q(p)$ will be a sorting function, so `is_sorting_fun($q(p)$) = true = terminate(p)`.
- ▶ If p does not terminate, then $q(p)$ does not terminate on any input, so `is_sorting_fun($q(p)$) = false = terminate(p)`.
- ▶ Our program returns now `is_sorting_fun($q(p)$)` which is the result of `terminate(p)`.
- ▶ So we have obtained by using a program for `is_sorting_fun` a program which computes `terminate`.
- ▶ But `terminate` is non-computable, therefore `is_sorting_fun` cannot be computable.

Examples of Difficulty Defining Computability

Diophantine Equations

- ▶ Here is a short description of Diophantine Equations.
- ▶ This is the question, whether an indeterminate polynomial equation has solutions where the variables are instantiated as integers.
- ▶ Examples:
 - ▶ Solve for integers a, b the equation $ax + by = 1$ using integers x, y .
 - ▶ Solve for given n the equation $x^n + y^n = z^n$.
 - ▶ For $n \geq 3$ this is unsolvable by Fermat's Last Theorem.