Here follows a list of errata in the script Computability, CS226. All mistakes have been changed in the lecture notes available on the Internet.

2: Encoding of Data Types into \( \mathbb{N} \)

- Slide 2a-12: Replace in the second displayed formula \( n \) by \( k \): Instead of
  \[
  A^k := \{ (x_0, \ldots, x_{n-1}) \mid x_0, \ldots, x_{k-1} \in A \}.
  \]
  write
  \[
  A^k := \{ (x_0, \ldots, x_{k-1}) \mid x_0, \ldots, x_{k-1} \in A \}.
  \]

- Slide 2a-14 In (a), replace \( A \) by \( A' \): instead of \( f[A'] := \{ f(a) \mid a \in A \} \)
  write \( f[A'] := \{ f(a) \mid a \in A' \} \)

- Slide 2a-31: Replace in the second, third and fourth bullet \( Q(a) \) by \( Q(q) \). So we have
  - in the third bullet: “both \( P(a) \) and \( Q(q) \) are true”.
  - in the fourth bullet: “\( P(a) \) or \( Q(q) \) is true”. a
  - and (Esp., if both \( P(a) \) and \( Q(q) \) are true, then \( P(a) \lor Q(q) \) is true as well).

- Slide 2-55: All \( A \) except for the one in line 1 \( \mathcal{P}(A) \), the one in line 2 in \( A \to \{0,1\} \), and the two \( A \)s in the last line have to be replaced by \( B \).

So this slide reads as follows:

- Let for \( B \in \mathcal{P}(A) \)

\[
\chi_B : A \to \{0,1\}
\]

\[
\chi_B(x) := \begin{cases} 
1 & \text{if } x \in B, \\
0 & \text{if } x \notin B.
\end{cases}
\]

\( \chi_B \) is called the \textbf{characteristic function of} \( B \).

* \( \chi \) is a function from \( \mathcal{P}(\mathbb{N}) \) to \( \mathbb{N} \to \{0,1\} \), where we write the application of \( \chi \) to an element \( B \) as \( \chi_B \) instead of \( \chi(B) \).

- We show that \( \chi \) is a bijection.

* Then it follows that \( \mathcal{P}(A) \approx (A \to \{0,1\}) \).
• Slide 2-56: In the first displayed formula (in orange), replace \( A \) by \( B \) (3 times): It should read
\[
\chi_B(x) := \begin{cases} 
1 & \text{if } x \in B, \\
0 & \text{if } x \notin B.
\end{cases}
\]

• Slide 2-57:
  – Same change as for slide 2-56, replace in the first displayed formula \( A \) by \( B \), i.e. we obtain
\[
\chi_B(x) := \begin{cases} 
1 & x \in B, \\
0 & \text{otherwise}.
\end{cases}
\]
  – Replace in the last bullet, \( A \) by \( B \) – except for \( \{x \in A \mid \} \) which remains as it stands – and \( \mathbb{N} \) by \( A \). One obtains:
  * If \( B \subseteq A \), then
\[
\chi^{-1}(\mathbb{N}) = \{x \in A \mid \chi_B(x) = 1\} = \{x \in A \mid x \in B\} = B
\]

• Slide 2-58, slide 2-59: Same change as for slide 2-56, replace in the first displayed formula \( A \) by \( B \), i.e. we obtain
\[
\chi_B(x) := \begin{cases} 
1 & x \in B, \\
0 & \text{otherwise}.
\end{cases}
\]

• Slide 2-98:
  – The second bullet should start with: Assume \( A \) is uncountable.
  – In the third bullet, replace \( A = (A \setminus B) \cup B \) by \( A = (A \setminus B) \cup (A \cap B) \). Add in front of it a clarifying item “Then \( A \cap B \) is countable (since \( A \cap B \subseteq B \)).”.

So the complete slide reads as follows:

  – To be shown:
    If \( A \) is uncountable and \( B \) is countable, then \( A \setminus B \) is uncountable.
  – Assume \( A \) is uncountable, \( B \) is countable and \( A \setminus B \) were countable.
  – Then \( A \cap B \) is countable (since \( A \cap B \subseteq B \)).
  – Therefore \( A = (A \setminus B) \cup (A \cap B) \) is countable as well, a contradiction.

• Slide 2-177: In the subitem, starting with “If \( t = x \) is a variable”, replace “\( t \ni x : \Leftrightarrow a = x \)” by “\( t \ni b : \Leftrightarrow x = b \)”.

3: The URM

• Slide 3-46:
  – In “Assume \( m \in \mathbb{N}, h : \mathbb{N}^2 \rightarrow \mathbb{N} \)” , replace \( m \) by \( n \).
    So this sentence should read:
    “Assume \( n \in \mathbb{N}, h : \mathbb{N}^2 \rightarrow \mathbb{N} \)”.
  – In “by primitive recursion from \( m \) and \( h \) as follows:” replace \( m \) by \( n \), so this piece of a sentence should read:
    “by primitive recursion from \( n \) and \( h \) as follows:”
- In “We write $\text{primrec}(m, h)$ for $f$, so $\text{primrec}(m, h) : \mathbb{N} \hookrightarrow \mathbb{N}$,” replace twice $m$ by $n$, so this sentence should read: “We write $\text{primrec}(n, h)$ for $f$, so $\text{primrec}(n, h) : \mathbb{N} \hookrightarrow \mathbb{N}$.”

- Slide 3-52: Replace “Let $g : \mathbb{N}^n + 1 \hookrightarrow \mathbb{N}$.” by “Let $g : \mathbb{N}^{n+1} \hookrightarrow \mathbb{N}$.”

- Slide 3-57: Replace “$g(\bar{x}, y') |$” by “$g(\bar{x}, y') |$”.

- Slide 3-66: Replace “First we compute $g_i(\bar{z})$ for $i = 0, \ldots, k - 1$, store result in registers $y_i$.” by “First we compute $g_i(\bar{z})$ for $i = 0, \ldots, n - 1$, store result in registers $y_i$.”

- Slide 3-68: Replace “We show $U^{(n)}(\bar{z}) \simeq (f \circ (g_0(\bar{z}), \ldots, g_{n-1}(\bar{z})))$” by “We show $U^{(k)}(\bar{z}) \simeq (f \circ (g_0(\bar{z}), \ldots, g_{n-1}(\bar{z})))$.”

- Slide 3-69: Replace “$U^{(n)}$” by “$U^{(k)}$”.

- Slide 3-70: Replace twice “$U^{(n)}$” by “$U^{(k)}$”.

- Slide 3-71: Replace “$U^{(n)}$” by “$U^{(k)}$”.

- Slide 3-84: Replace twice $R_{N-1}$ by $R_N$.

So the item affected reads:

- the content of $R_0, \ldots, R_N$, if $R_N$ is the maximum register referenced to in the URM program.

- Slide 3-85: Replace in the second item the variable $k$ by $l$. So that item reads:

  - Let $N$ be the maximum number $l$ s.t. an instruction $\text{succ}(l)$, $\text{pred}(l)$, $\text{ifzero}(l, q)$ is amongst $I_0, \ldots, I_n$.

- Slide 3-89: In the second item, replace $\chi_M(\bar{x})$ by $\chi_{\text{Multiple}}(x, y)$ and $M(\bar{z})$ by $\text{Multiple}(x, y)$. This item should read:

  - $\chi_{\text{Multiple}}(x, y)$ decides, whether $\text{Multiple}(x, y)$ holds (then it returns 1 for yes), or not.

- Slide 3-97: In the last line, replace three times $U'$ by $V$. So that line should read as follows: “$f(\text{encode}(V)) \neq V^{(1)}(\text{encode}(V))$.”

4: Turing Machines

- Slide 4-15: In the label of the graph, the upper part of $D$ was cut of. The label should read “$a/b, D$”.

- Slide 4-24: In the graph, the right state should be $s2$ instead of $s1$. 
5: Algebraic View of Computation

- Slide 5-60: Replace

\[ f(x) := \left| \{ y < x \mid P(x) \} \right| \]

by

\[ f(x) := \left| \{ y < x \mid P(y) \} \right| \]

- Slide 5-86: Replace

\[
\begin{align*}
  f(\vec{x}, y) &= (\langle f(\vec{x}, 0), \ldots, f(\vec{x}, y) \rangle) + 1 \\
  &= (h(\vec{x}, y + 1)) + 1
\end{align*}
\]

by

\[
\begin{align*}
  f(\vec{x}, y) &= (\langle f(\vec{x}, 0), \ldots, f(\vec{x}, y) \rangle) \\
  &= (h(\vec{x}, y + 1))
\end{align*}
\]

7: The Recursion Theorem

- Slide 7-17: Replace

\[
\{ (\vec{x}, y, z) \in \mathbb{N}^n \mid \varphi(\vec{x}, y, z) \}
\]

by

\[
\{ (\vec{x}, y, z) \in \mathbb{N}^n \mid \varphi(\vec{x}, y, z) \}
\]

(i.e. replace “(” by “(“).

Revision Lecture

- Slide Rev-33: Replace

\[ \{ e \}^n(\vec{x}) \simeq U(\mu u. T^n(e, \vec{x})) \]

by

\[ \{ e \}^n(\vec{x}) \simeq U(\mu u. T^n(e, \vec{x}, u)) \]