Lab Session 3 (Critical Systems, CS_411)

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1 Preliminary Remarks

- Don't forget to check for termination.
- You might keep your results for the next coursework.
- Hints on how to print the results out nicely (using latex) can be found at the end of the instructions for labsession 2.

2 Basic Examples on Natural Numbers

The goal of this part is to obtain basic skills in dealing with natural numbers in Agda.

- Introduce the set `N` of natural numbers.
- Introduce `zero`, the successor operation and elements `one`, `two`, `three` of `N`.
- Introduce addition and multiplication.
  - Note: If you introduce them as `(+)`, `(*)`, you can write later `n + m`, `n * m`.
- Check your definition by evaluating expressions like `two * two`.
  - Use menu-item agda-compute-WHNF or agda-compute-WHNF-strict in a goal.
- Introduce, depending on `A:: Set` and `n::N` the set `Vec A n` of vectors of length `n` with elements in `A`.
  - Note that `Vec A zero` is the one element set,
  - and `Vec A (S n)` is the product of `A` and `Vec A n`.
  - It is convenient to use
    - constructor `nil` for the one element set `Vec A zero` and
    - constructor `cons` for the product of `A` and `A n` in `Vec A (S n)`.

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3 Proving Basic Properties of N and +

- Introduce the equality on natural numbers.
- Prove that this equality is reflexive, ie.
  define `refl :: (n::N) -> Eqnat n n`.
  Hint: Use induction (essentially case distinction).
- Prove that it is symmetric, ie. define

```
sym (n::N)
    (m::N)
    (p::Eqnat n m)
    ::  Eqnat m n
```

- Prove that this equality is transitive, ie.
  define

```
trans (n::N)
      (m::N)
      (k::N)
      (p::Eqnat n m)
      (q::Eqnat m k)
      ::Eqnat n k
```

  - Hint: Use induction (essentially case distinction).
  - If $n \neq m$ or $m \neq k$ you can use the empty case distinction on p.
  - You might need to prove auxiliary properties like $0 + n = n$.
- Prove that + is commutative, ie. $n + m = m + n$.
  Hints:
  - You might need some auxiliary lemma like $(\mathrm{S}\ n) + m = \mathrm{S}(n + m)$.
  - You might need to use reflexivity or transitivity.

4 For those of you who are very fast

- Introduce the $n \times m$ Matrices with elements in N.
  - A matrix is a vector of columns, each column being a vector of elements of N.
- Define the functions which you need later: the projection of a matrix with S n rows to its first row and to the other n rows.
- Now define the product of two matrices:

- First define the result of multiplying one row with one column, ie.

$$(a_1, \ldots, a_n) \cdot \begin{pmatrix} b_1 \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{pmatrix} = a_1 \cdot b_1 + \cdots + a_n \cdot b_n$$

- Now define the result of multiplying an $n \times m$ matrix with one column:

$$\begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ \cdot \\ \cdot \\ \cdot \\ b_m \end{pmatrix}$$

  * It's probably easiest to work by recursion on n.
  * You will probably need to make use of the projections defined above.

- Now define the product of matrices.