CS_336/CS_M36 (Part 2)/CS_M46 Interactive Theorem Proving

http://www.cs.swan.ac.uk/∼csetzer/lectures/intertheo/07/index.html

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Lent Term 2008

0. Introduction

(a) What is interactive theorem proving?

(b) Administrative Issues.

(c) Plan.

(d) Literature

(a) What is Interact. Theorem Provi

Need for Theorem Proving

- We need to prove theorems in order to establish mathematical theorems.
  - E.g. that certain problems are decidable, undecidable, polynomial computable etc.
- We need them as well in order to establish the correctness of software and hardware.
  - Is floating point division for the Intel processor correct?
  - Is a railway control system safe?
    - When verifying the Swedish railway system, lots of bugs were found.

4 Ways of Proving Theorems

1. **Theorem proving by hand.**
   - What mathematicians do all the time.
   - Will remain in the near future the main way for proving theorems.
   - Problem: Errors.
     - As in programs after a certain amount of lines there is a bug, after a certain amount of lines a proof has a bug.
     - The problem can only be reduced by careful proof checking, but not eliminated completely.
   - Unsuitable for verifying large software and hardware systems.
     - Data usually too large.
     - Likely that one makes the same mistakes as in the software.

4 Ways of Proving Theorems

2. **Theorem proving with some machine support.**
   - Machine checks the syntax of the statements, creates a good layout, translates it into different languages.
   - Theorem proving still to be done by hand.
   - **Example:** most systems for specification of software (e.g. CSP-CASL, as used by Dr. Roggenbach).
   - **Advantages:**
     - Less errors.
     - User is forced to obey a certain syntax.
     - Specifications can be exchanged more easily.
   - **Disadvantage:** Similar to 1.

4 Ways of Proving Theorems

3. **Interactive Theorem Proving.**
   - Proofs are fully checked by the system.
   - Proof steps have to be carried out by the user.
   - **Advantages:**
     - Correctness guaranteed (provided the theorem prover is correct).
     - Everything which can be proved by hand, should be possible to be proved in such systems.

4 Ways of Proving Theorems

- (Interactive theorem proving)
  - **Disadvantages:**
    - It takes much longer than proving by hand.
      - Similar to programming:
        To say in words what a program should do, doesn't take long.
        To write the actual program, can take a long time, since much more details are involved than expected.
    - Requires experts in theorem proving.

4 Ways of Proving Theorems

4. **Automated Theorem Proving.**
   - The theorem is shown by the machine.
   - It is the task of the user to
     - state the theorem,
     - bring it into a form so that it can be solved,
     - usually adapt certain parameters so that the theorem proving solves the problem within reasonable amount of time.
   - Espec. Dr. Kullmann is an expert in this area.

4 Ways of Proving Theorems

- (Automated theorem proving)
  - **Advantages**
    - Less complicated to "feed the theorem into the machine" rather than actually proving it.
      Might be done by non-specialists.
    - Sometimes faster than interactive theorem proving.

4 Ways of Proving Theorems

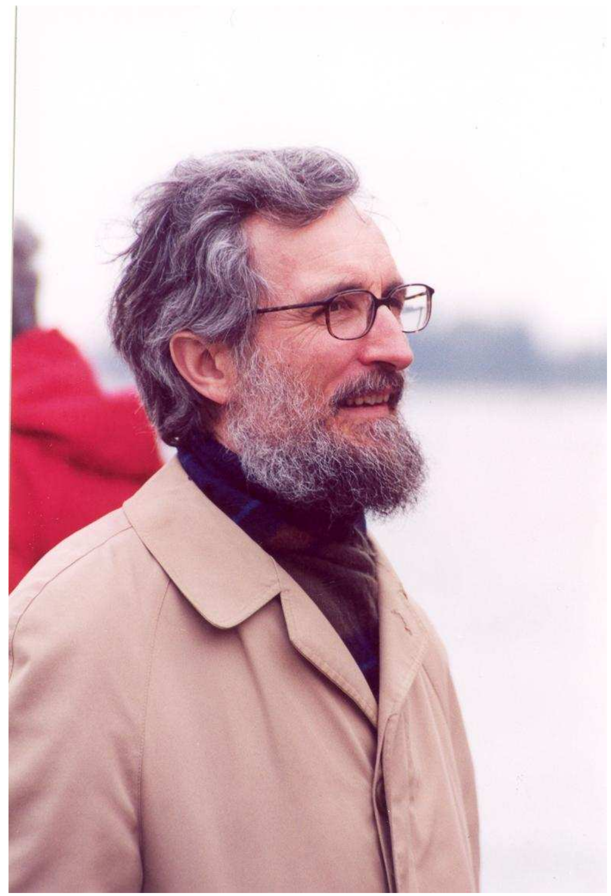
- (Automated theorem proving)
  - **Disadvantages**
    - Many problems cannot be proved automatically.
    - Can often deal only with finite problems.
      - We can show the correctness of one particular processor.
      - But we cannot show a theorem, stating the correctness of a parametric unit (like a generic n-bit adder for arbitrary n.
      - In some cases this can be overcome.
    - Limits on what can be done (some hardware problems can be verified as 32 bit versions, but not as 64 bit versions).

This Lecture

- In this lecture we will consider approach (c).
- We will make use of the theorem prover **Agda**, based on **Martin-Löf type theory**.
  - The theory was developed by **Per Martin-Löf**.
    - Per Martin-Löf is professor at Stockholm University for philosophy and mathematical logic.
    - The lecturer is and was collaborating with him, especially while working as a research associate at Uppsala University (Sweden).

Per Martin-Löf

The Father of Martin-Löf Type Theory, the variant of dependent type theory used in this module.



This Lecture

- The students will learn (in practical lab sessions) how to actually carry out proofs in Agda.
  - **Proving theorems** will not be much different from **programming**.
  - In Martin-Löf type theory “proving” and “programming” is the same.
- With a slight shift of emphasis, this module could as well be called “**programming with dependent types**”.

(b) Administrative Issues

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Assessment

- **CS_336:**
  - 80% exam and 20% coursework
  - 3 **small** assignments. Each counts 6% or 7%.
- **CS_M36/CS_M46:**
  - 70% exam and 30% coursework
  - Same assignments as before.
  - One extra assignment to demonstrate level M learning outcome.

Course home page

- Located at
  http://www.cs.swan.ac.uk/∼csetzer/lectures/
  intertheo/07/index.html
- There is an open version,
- and a password protected version.
- The password is ______________.
- Errors in the notes will be corrected on the slides and noted on the list of errata.
- In order to reduce plagarism, coursework and solutions to coursework will **not** be made available in electronic form (e.g. on this web site).

Timetable, Course Material

- Two lectures per week.
  - Tuesday, 12:00, Faraday-B
  - Thursday, 12:00, Faraday-C
- Web page contains overhead slides from the lectures. Course material will be continually updated.

(c) Plan

0. Introduction.
1. From simple to dependent types.
2. Reduction systems and term rewriting.
3. The λ-calculus and implication.
4. The λ-calculus with products and conjunction.
5. The logical framework.
6. Data types.

(d) Literature

- In general, the module is self-contained.

Main Course Literature

- B. Nordström, K. Peterson, J. M. Smith:
  Programming in Martin-Löf's type theory. Available via
  http://www.cs.chalmers.se/Cs/Research/Logic/book/.
  Course book, although a little bit too high level.
- B. Nordström, K. Peterson, J. M. Smith:
  Martin-Löf's type theory. Handbook of Computer Science, Vol 5, 1-37. Oxford Univ. Press, 2000.
  Available via
  http://www.cs.chalmers.se/ bengt/papers/hlcs.pdf
  Similar as the previous book, but shorter.

Other Introductory Books

- Martin-Löf, Per: *Intuitionistic Type theory*. Bibliopolis, Naples, 1984.
  Relatively easy short book, from the father of the type theory we are using. Intended for philosophers.
- Aarne Ranta: *Type-theoretic grammar.* Clarendon Press, 1995.
  Use of type theory in linguistics and for translation between languages. Has a good and simple introduction into type theory.

More advanced Books

- Troelstra, A. S. and van Dalen, D.:
  Constructivism in mathematics. Vol. I. North Holland, 1988.
  Contains some material of interest (e.g. BHK interpretation of logical connective). Postgraduate level.
- Chapter 11 of Troelstra, A. S. and van Dalen, D.:
  Constructivism in mathematics. Vol. II. North Holland, 1988.
  Book on postgraduate level. Deviates from "official Martin Löf type theory".

Books on Term Rewriting

- Terese: *Term Rewriting Systems*. Cambridge Tracts in Theoretical Computer Science 55, 2003.
  Very thick and detailed study of term rewriting written by some of the most important people in term rewriting.
- Baader, Franz; Nipkow, Tobias: *Term rewriting and all that*. Cambridge University Press, 1999.
  Shorter then the book by Terese.

Books on the λ-Calculus

- J. Roger Hindley and Jonathan P. Seldin:
  Introduction to Combinators and λ-Calculus. Cambridge University Press, 1986.
  Best book on the λ-calculus.
  Mainly chapter 1 relevant for this module.
- J. R. Hindley, B. Lercher and J.P. Seldin:
  Introduction to combinatory logic. Cambridge University Press, 1972.
  Mainly chapter 1 relevant for this module.
- J. Roger Hindley: *Basic simple type theory.* Cambridge University Press, 1997.
  Introduces a slightly more powerful type theory than used in this module.

Books on the λ-Calculus

- H.P. Barendregt:
  The Lambda Calculus. It's syntax and semantics.
  Revised Edition. Elsevier, 1984.
  Thorough monograph, the bible of the λ-calculus. Level too high for this module.