Closing the Validation Gap or Verifying Railway Interlockings in Agda

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Examples of Validation Problems

Closing the Validation Gap

Case Study: Formalisation of Railway Interlocking System

Proof of Safety
Proviso

- Background in mathematical logic, proof theory and type theory.
- Be prepared of misuse or naive use of terminology from software engineering.
Examples of Validation Problems

Closing the Validation Gap

Case Study: Formalisation of Railway Interlocking System

Proof of Safety
Exam Question

▶ Assume you have two planes:
  ▶ The code for the first one has been fully verified using automated and interactive theorem proving, but the plane has not been tested.
  ▶ The code for the second one has not been verified this way, but the plane has been thoroughly tested.

▶ Which one do you choose to use?
Examples of Validation Problems

Validation Gap

- **Verification** can be done in a machine checked way.
- Verification is only relative to a given specification.
- How do you know that the specification guarantees that the program fulfils the requirements?
- **Validation** checks that a program fulfils the requirements or a specification guarantees that the requirements are fulfilled.
  - Cannot be done formally.
Example Incomplete Specification

- We have written a program for controlling a railway interlocking system using SPARK Ada.
- Specification based on Hoare logic (pre and post conditions).
- Verification was carried out in a *machine checked* way.
- When running the program it was incorrect.
  - Trains disappeared.
  - Forgotten to add to the specification that trains should not get lost.
  - This happened in real world as well (disappearance of trains from a US control system of railways).
Examples of Validation Problems

Complexity of Specification

- Tobias Nipkow has verified the security of a hotel key system.
- **Specification was substantially longer than the program.**
- Maybe it is easier to see that the program is secure than that the specification guarantees security?
Examples of Validation Problems

Closing the Validation Gap

Case Study: Formalisation of Railway Interlocking System

Proof of Safety
Verification can be done provably correct or using systematic thorough testing.
  - We can guarantee (up to a certain degree).

Validation can only be done using semi-formal, systematic methods.
  - We cannot guarantee it.

We cannot avoid a gap between specification and requirements.

However, we can make the gap as small as possible.
Requirements - Specification - System

Real World

Requirements

Validation

Model

Specification

Verification

System

Validation
Suggestion to have two Specifications

- **Requirements specification** which is as close as possible to the requirements.
  - Corresponds as close as possible to a model of the real world situation.
  - Example: In railway interlocking systems model of railways.

- **Program specification** which is used to verify the program.
  - Should make it easy to verify that a program fulfils the specification.
  - Example: In railway interlocking systems signalling principles
  
  E.g.: If signal A is green, signal B is red.
Interactive vs Automated Theorem Proving

- That the **program fulfils the program specification** is typically provable by **automated theorem proving**.
  - In case of railway interlocking systems show that a railway interlocking system fulfils signalling principles.

- That the **program specification implies the requirements specification** is typically provable by **interactive theorem proving**.
Requirements and Program Specification

1. Requirements
   - Validation
2. Requirements Specification
   - Interactive Theorem Proving
3. Program Specification
   - Automated Theorem Proving or Testing
4. System
Examples of Validation Problems

Closing the Validation Gap

Case Study: Formalisation of Railway Interlocking System

Proof of Safety
The basic unit into which one divides a rail yard is that of a track segment.

A track segment is stretch of a track without any further smaller parts, which are significant for an analysis of an interlocking system.

- there are no sets of points in between (but a set of points might form one segment)
- there are no crossings in between,
- they are not divided by signals into parts.
In the following example we have track segments s1 - s6.
- The two branches of the set of points p1 form segment s2.
- The two branches of the set of points p2 form segment s4.
Signals

- Signals control the access from one train segment to the next one.
- They are drawn in the direction of use, e.g. Signal sig2 is visible from s1 and controls access to s2.
- In the example sig2, sig7, sig9, control access to the set of points p1, and sig3, sig6, sig10 control access to p2.
- sig1, sig5 control access to s1, s5 respectively, and sig8, sig4 control access to the neighbouring rail yards.
The control system for such a rail yard has several **train routes**.

- A **train route** is a sequence of track segments, the train can follow without ever having to stop in between (except in emergency cases).
- The beginning of a train route and its end should be delimited by signals.
  - The first one prevents entering the train route, the second one, delimits access from this train route to the following train routes.
- The segment before the guarding signal belongs to the route.
Train Routes

- So we have a train route (s1, s2, s6)
  - with segments s1, s2, s6
  - guarded by signal sig2
- Routes r1, r2 are connected if after having traversed route r1 one can proceed to route r2
  - route (s1, s2, s6) and route (s6, s4, s5) are connected.
We have sets and relations

- Segment : Set
- Train : Set
- Route : Set
- Connected : Route → Route → Set
- SegInRoute : Segment → Route → Set
Model

- Time is given as

  \[ \text{Time} = \mathbb{N} : \text{Set} \]

- Depending on \( t : \text{Time} \) we assume

  \[
  \begin{align*}
  \text{trainRoute}_t & : \text{Train} \rightarrow \text{Route} \\
  \text{signalAspect}_t & : \text{Route} \rightarrow \{\text{proceed, danger}\}
  \end{align*}
  \]
Abstract Assumptions about Routes and Trains

- Single-Entry-Point: If two routes $route_1$ and $route_2$ are connected to route $route_3$, there is a segment (the one before the signal of $route_3$ which is in $route_1$ and $route_2$):

$$\forall route_1, route_2, route_3. \text{Connected } route_1 \text{ } route_3 \rightarrow \text{Connected } route_2 \text{ } route_3 \rightarrow \exists \text{segment} . (\text{SegInRoute } segment \text{ } route_1 \land \text{SegInRoute } segment \text{ } route_2)$$

- Trains follow connected routes and obey signals:

$$\forall t, train. (\text{trainRoute}_t \text{ } train \equiv \text{trainRoute}_{t+1} \text{ } train) \lor (\text{Connected } (\text{trainRoute}_t \text{ } train) (\text{trainRoute}_{t+1} \text{ } train) \land \text{signalAspect}_t (\text{trainRoute}_{t+1} \text{ } train) \equiv \text{proceed})$$
Abstract Signal Principle 1: Opposing Signals are not both Green

- If a segment is in two different routes, the signal of one of the routes must have aspect danger:

\[
\forall t, \text{route}_1, \text{route}_2, \text{segment}.
\]
\[
\text{route}_1 \not\equiv \text{route}_2
\]
\[
\rightarrow \text{SegInRoute segment route}_1
\]
\[
\rightarrow \text{SegInRoute segment route}_2
\]
\[
\rightarrow (\text{signalAspect}_t \text{ route}_1 \equiv \text{danger} \\
\forall \text{signalAspect}_t \text{ route}_2 \equiv \text{danger})
\]
Abstract Signal Principle 2: Routes of Trains are Guarded

- If a train is using a route, all routes with access to the segments of this route are guarded by red signal:

\[ \forall t, \text{train}, \text{segment}, \text{route}. \]

\[ \text{SegInRoute} \text{segment} \left( \text{trainRoute}_t \text{train} \right) \]

\[ \rightarrow \text{SegInRoute} \text{segment} \text{route} \]

\[ \rightarrow \text{signalAspect}_t \text{route} \equiv \text{danger} \]
Initial Condition

\( \forall \text{train}_1, \text{train}_2, \text{segment} \)
\( \text{train}_1 \not\equiv \text{train}_2 \)
\( \rightarrow \neg (\text{SegInRoute } \text{segment} (\text{trainRoute}_0 \text{train}_1) \land \text{SegInRoute } \text{segment} (\text{trainRoute}_0 \text{train}_2)) \)
Examples of Validation Problems

Closing the Validation Gap

Case Study: Formalisation of Railway Interlocking System

Proof of Safety
Collision Free

Theorem

Assume the above abstract conditions. Then trains don’t collide, i.e.

$$\forall t, train_1, train_2, segment$$

$$\quad train_1 \not\equiv train_2$$

$$\quad \rightarrow \neg (\text{SegInRoute} \ segment \ (\text{trainRoute}_t \ train_1))$$

$$\quad \land \text{SegInRoute} \ segment \ (\text{trainRoute}_t \ train_2))$$
Proof of Theorem

- Induction on $t$: Time.
- $t = 0$ follows by the initial condition.
- For $t \to t + 1$ assume $\text{train}_1$, $\text{train}_2$, segment s.t.

\[
\text{train}_1 \neq \text{train}_2
\]
\[
\text{SegInRoute segment (trainRoute}_{t+1} \text{train}_1)
\]
\[
\text{SegInRoute segment (trainRoute}_{t+1} \text{train}_2)
\]

and show a contradiction.
- If none of the trains have moved (so their routes are as before) this follows by IH.
Proof of Safety

Proof of Theorem

- If only $train_1$ has moved we have:

  $signalAspect_t(trainRoute_{t+1} \ train_1) \equiv proceed$
  (since $train_1$ obeys signals)
  $SegInRoute\ segment\ (trainRoute_{t+1} \ train_2)$
  $SegInRoute\ segment\ (trainRoute_t \ train_2)$
  (by $trainRoute_{t+1} \ train_2 = trainRoute_t \ train_2$)
  $SegInRoute\ segment\ (trainRoute_{t+1} \ train_1)$
  $signalAspect_t(trainRoute_{t+1} \ train_1) \equiv danger$
  (by Abstract Signal Principle 2)
  Contradiction

- The case where only $train_2$ has moved follows similarly.
Proof of Theorem

- If both \( train_1 \), \( train_2 \) have moved to the same route we have

\[
\text{trainRoute}_{t+1} \; train_1 \equiv \text{trainRoute}_{t+1} \; train_2
\]

Connected \( (\text{trainRoute}_t \; train_1) \) \( (\text{trainRoute}_{t+1} \; train_1) \)

Connected \( (\text{trainRoute}_t \; train_2) \) \( (\text{trainRoute}_{t+1} \; train_2) \)

Since train routes of trains are connected

\[ \exists segment \]

\[ \text{SegInRoute} \; segment \; (\text{trainRoute}_t \; train_1) \]

\[ \land \text{SegInRoute} \; segment \; (\text{trainRoute}_t \; train_2) \]

(by single entry to routes)

Contradiction to IH
Proof of Theorem

- If both $train_1$, $train_2$ have moved to different routes we have

$$signalAspect_t (\text{trainRoute}_{t+1} \ train_1) \equiv \text{proceed}$$
$$signalAspect_t (\text{trainRoute}_{t+1} \ train_2) \equiv \text{proceed}$$

(Since trains obey signals)

$$\text{trainRoute}_{t+1} \ train_1 \neq \text{trainRoute}_{t+1} \ train_2$$

SegInRoute $segment$ ($\text{trainRoute}_{t+1} \ train_1$)

SegInRoute $segment$ ($\text{trainRoute}_{t+1} \ train_2$)

($signalAspect_t (\text{trainRoute}_{t+1} \ train_1) \equiv \text{danger}$
$\lor signalAspect_t (\text{trainRoute}_{t+1} \ train_2) \equiv \text{danger}$)

Contradiction
Points in Routes of Trains are Locked

- Similarly we were able to show that under additional conditions on points we have
  If a set of points is in facing direction of a route of a train, then the set of points is locked.
The conditions on $\text{trainRoute}_t$ and $\text{signalAspect}_t$ are still abstract.

In order to reduce it to concrete interlockings we take the following steps:

- Formalise state (consisting of interlocking state, location circuits, trains).
- Formalise desired inputs to state.
- Define initial state.
- Define functions computing next state depending on state and desired input.
- Define concrete signalling principles and conditions on locations/trains for initial state and next state.
- Show that the functions above fulfil these concrete conditions.
- Compute $\text{trainRoute}_t$, $\text{signalAspect}_t$.
- Show that concrete conditions above imply the abstract conditions on $\text{trainRoute}_t$, $\text{signalAspect}_t$.
- Therefore the interlocking system is safe.
Evaluation

- Even in this simplified situation it is rather complicated to see that the signalling principles imply safety.
- In usual validation this is done by hand.
- In the above approach we have formalised it mathematically in Agda and shown that the signalling principles imply safety.
- Therefore the validation gap has been narrowed.
Conclusion

- **Validation gap** between Specification and Requirements.
- By having a **requirements specification** which is as close as possible to the requirements this gap can be narrowed.
- Replaces arguments which are carried out informally in the head of the validator by robust mathematical arguments.
- Two step verification:
  - **Step 1**: Program fulfils program specification.
  - **Step 2**: Program specification implies requirements specification.
- Full verification of real world interlocking system in Agda has been carried out (PhD thesis Karim Kanso). Interlocking system could be executed in Agda as an interactive program.
References

