

Object-oriented Programming in Dependent Type Theory

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Coalgebras in Dependent Type Theory

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Old Version of Coalgebras: Codata Types

- ▶ Idea of Codata Types:

`codata Stream : Set where`
`cons : $\mathbb{N} \rightarrow \text{Stream} \rightarrow \text{Stream}$`

- ▶ Same definition as inductive data type but we are allowed to have infinite chains of constructors

`cons n_0 (cons n_1 (cons n_2 ...))`

Objects as Elements of Coalgebras

- ▶ Coalgebras are used for modelling various phenomena related **infinite sequences of computations**.
 - ▶ Correspond to **non-well-founded trees**.
 - ▶ Arise when dealing with **interactive programs**.
 - ▶ Interactive programs often don't terminate unless terminated by the user.
- ▶ Coalgebras arise as representations of **real numbers**.
 - ▶ Examples: **streams of digits**, **Cauchy sequences**.
 - ▶ In general approximations by finite values
- ▶ Coalgebraic programming is heavily used in **object-oriented Programming**.
 - ▶ See section on objects below.

Solution: Coalgebras Defined by Observations

- ▶ **Problem of codata types:** Non-normalisation and undecidability of equality.
- ▶ Instead we define define coalgebras by their observations.
Tentative syntax

```

coalg Stream : Set where
  head  : Stream → ℕ
  tail  : Stream → Stream
  
```

- ▶ **Stream** is the largest set of terms which allow arbitrary many applications of **tail** followed by **head** to obtain a natural numbers.
- ▶ From this one can develop a general model for coalgebras (see our paper [Set16]).
- ▶ Therefore no infinite expansion of streams:
 - for each expansion of a stream one needs one application of **tail**.

Syntax in Agda

- ▶ In Agda the record type has been reused for defining coalgebras:

```
record Stream (A : Set) : Set where
  coinductive
  field
    head : A
    tail : Stream A
```

Principle of Guarded Recursion

- ▶ Define

$$\begin{aligned}
 f &: A \rightarrow \text{Stream} \\
 \text{head} \quad (f \ a) &= \dots : \mathbb{N} \\
 \text{tail} \quad (f \ a) &= \dots : \text{Stream}
 \end{aligned}$$

where

$$\begin{aligned}
 \text{tail} \ (f \ a) &= f \ a' \quad \text{for some } a' : A \\
 \text{or} \\
 \text{tail} \ (f \ a) &= s' \quad \text{for some } s' : \text{Stream} \text{ given before}
 \end{aligned}$$

- ▶ **No** function can be applied to the corecursion hypothesis.
- ▶ Using sized types one can apply size preserving or size increasing functions to co-IH (Abel).
- ▶ Above is example of **copattern matching**.

Example

- ▶ Constant stream of a, a, a, \dots

$$\begin{aligned} \text{const} &: \{A : \text{Set}\} \rightarrow A \rightarrow \text{Stream } A \\ \text{head } (\text{const } a) &= a \\ \text{tail } (\text{const } a) &= \text{const } a \end{aligned}$$

- ▶ The increasing stream $n, n + 1, n + 2, \dots$

$$\begin{aligned} \text{inc} &: \mathbb{N} \rightarrow \text{Stream } \mathbb{N} \\ \text{head } (\text{inc } n) &= n \\ \text{tail } (\text{inc } n) &= \text{inc } (n + 1) \end{aligned}$$

- ▶ Cons is **defined**:

$$\begin{aligned} \text{cons} &: X \rightarrow \text{Stream } X \rightarrow \text{Stream } X \\ \text{head } (\text{cons } x \ l) &= x \\ \text{tail } (\text{cons } x \ l) &= l \end{aligned}$$

Nested Pattern/Copattern Matching

- ▶ We can even define functions by a combination of pattern and copattern matching and nest those:
The following defines the stream

`stutterDown` $n\ n = n, n, n - 1, n - 1, \dots, 0, 0, n, n, n - 1, n - 1, \dots$

```

stutterDown : ℕ → ℕ → Stream ℕ
head (stutterDown n m)           = m
head (tail (stutterDown n m))    = m
tail (tail (stutterDown n (suc m))) = stutterDown n m
tail (tail (stutterDown n 0))    = stutterDown n n

```

Hello World in Agda

We can develop IO programs based on coalgebras and get the following hello world program:

```
module helloWorld where

open import ConsoleLib

main : ConsoleProg
main = run (WriteString "Hello World")
```

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Object-Oriented/Based Programming

- ▶ Object-oriented (OO) programming is currently main programming paradigm.
 - ▶ Means that the main programming paradigm is essentially **coalgebraic programming**.
- ▶ Good for bundling operations into one objects, hiding implementations and reuse of code.
- ▶ Here restriction to **object-based programming**.
 - ▶ Only notion of an object covered.
 - ▶ Steps towards full OO programming work in progress.
- ▶ Ultimate goal: use objects in order to **organise proofs** in a better way.

Example: cell in Java

```
class cell <A> {  
  
    /* Instance Variable */  
    A content;  
  
    /* Constructor */  
    cell (A s) { content = s; }  
  
    /* Method put */  
    public void put (A s) { content = s; }  
  
    /* Method get */  
    public A get () { return content; }  
}
```

Modelling Methods as Objects

- ▶ The Type (interface) `cell` modelled as a coalgebra `Cell`.
- ▶ A method

$$B \text{ m } (A \times)$$

is modelled as observation

$$m : \text{Cell} \rightarrow A \rightarrow B \times \text{Cell}$$

- ▶ Return type `void` is modelled as `Unit` (one element type).
- ▶ A constructor with argument `A` modelled as a function defined by guarded recursion

$$\text{cell} : A \rightarrow \text{Cell}$$

Cell in Agda

```
record Cell (X : Set) : Set where
  coinductive
  field
```

```
  put : X → ( Unit × Cell X )
```

```
  get : Unit → ( X × Cell X )
```

```
cell : {X : Set} → X → Cell X
```

```
put (cell x) y = (unit , cell y)
```

```
get (cell x) _ = (x , cell x)
```


Generic Version

An interface for an object consist of methods and the result type:

```
record Interface : Set1 where
  field Method : Set
        Result  : Method → Set
```

An Object of an interface I has a method which for every method returns an element of the result type and the updated object:

```
record Object (I : Interface) : Set where
  coinductive
  field objectMethod : (m : Method I) → Result I m × Object I
```

Example: A Cell

A cell contains one element.

The methods allow to get its content and put a new value into the cell:

```
data CellMethod A : Set where
  get  : CellMethod A
  put  : A → CellMethod A
```

```
CellResult      : ∀{A} → CellMethod A → Set
CellResult {A} get  = A
CellResult (put _) = Unit
```

```
cell           : (A : Set) → Interface
Method (cell A) = CellMethod A
Result (cell A) m = CellResult m
```

Definition of Cell

The cell object is defined as follows:

$$\text{Cell} : \text{Set} \rightarrow \text{Set}$$

$$\text{Cell } A = \text{Object } (\text{cell } A)$$

$$\text{cell} : \{A : \text{Set}\} \rightarrow A \rightarrow \text{Cell } A$$

$$\text{objectMethod } (\text{cell } a) \text{ get} = (a, \text{cell } a)$$

$$\text{objectMethod } (\text{cell } a) (\text{put } b) = (\text{unit}, \text{cell } b)$$

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State Dependent Interface

record $\text{Interface}^s : \text{Set}_1$ where
 field

$\text{State}^s : \text{Set}$

$\text{Method}^s : \text{State}^s \rightarrow \text{Set}$

$\text{Result}^s : (s : \text{State}^s) \rightarrow (m : \text{Method}^s s) \rightarrow \text{Set}$

$\text{next}^s : (s : \text{State}^s) \rightarrow (m : \text{Method}^s s) \rightarrow \text{Result}^s s m$
 $\rightarrow \text{State}^s$

State Dependent Object

Assuming $I : \text{Interface}^s$ we define the set of state dependent objects:

```
record Objects (I : Interfaces) (s : States I) : Set where
  coinductive
  field
    objectMethod : (m : Methods I s)
                  →  $\Sigma [ r \in \text{Result}^s I s m ] \text{Object}^s I (\text{next}^s I s m r)$ 
```

Example Safe Stack

$$\text{StackState}^s = \mathbb{N}$$

data StackMethod^s (A : Set) : StackState^s → Set where
 push : {n : StackState^s} → A → StackMethod^s A n
 pop : {n : StackState^s} → StackMethod^s A (suc n)

$$\text{StackResult}^s : (A : \text{Set}) \rightarrow (s : \text{StackState}^s) \rightarrow \text{StackMethod}^s A s \rightarrow \text{Set}$$

$$\text{StackResult}^s A .n (\text{push } \{ n \} x_1) = \text{Unit}$$

$$\text{StackResult}^s A (\text{suc } .n) (\text{pop } \{ n \}) = A$$

$$n^s : (A : \text{Set}) \rightarrow (s : \text{StackState}^s) \rightarrow (m : \text{StackMethod}^s A s) \rightarrow (r : \text{StackResult}^s A s m) \rightarrow \text{StackState}^s$$

$$n^s A .n (\text{push } \{ n \} x) \quad r = \text{suc } n$$

$$n^s A (\text{suc } .n) (\text{pop } \{ n \}) \quad r = n$$

Safe Stack

$$\text{StackInterface}^s : (A : \text{Set}) \rightarrow \text{Interface}^s$$

$$\text{State}^s \quad (\text{StackInterface}^s A) = \text{StackState}^s$$

$$\text{Method}^s \quad (\text{StackInterface}^s A) = \text{StackMethod}^s A$$

$$\text{Result}^s \quad (\text{StackInterface}^s A) = \text{StackResult}^s A$$

$$\text{next}^s \quad (\text{StackInterface}^s A) = n^s A$$

$$\text{stackO} : \forall \{E : \text{Set}\} \{n : \mathbb{N}\} (v : \text{Vec } E \ n)$$

$$\rightarrow \text{Object}^s (\text{StackInterface}^s E) \ n$$

$$\text{objectMethod} \quad (\text{stackO } es) \quad (\text{push } e) = (_ , \text{stackO } (e :: es))$$

$$\text{objectMethod} \quad (\text{stackO } (e :: es)) \quad \text{pop} = (e , \text{stackO } es)$$

Example Fibonacci Stack

data FibState : Set where

fib : $\mathbb{N} \rightarrow \text{FibState}$

val : $\mathbb{N} \rightarrow \text{FibState}$

data FibStackEl : Set where

$_+ \cdot$: $\mathbb{N} \rightarrow \text{FibStackEl}$

$\cdot + \text{fib}__$: $\mathbb{N} \rightarrow \text{FibStackEl}$

FibStack : $\mathbb{N} \rightarrow \text{Set}$

FibStack = Object^s (StackInterface^s FibStackEl)

emptyFibStack : FibStack 0

emptyFibStack = stackO []

Reduce

$$\begin{aligned}
 \text{reduce} &: \text{Stackmachine} \rightarrow \text{Stackmachine} \uplus \mathbb{N} \\
 \text{reduce} (n, \text{fib } 0, \text{stack}) &= \text{inj}_1 (n, \text{val } 1, \text{stack}) \\
 \text{reduce} (n, \text{fib } 1, \text{stack}) &= \text{inj}_1 (n, \text{val } 1, \text{stack}) \\
 \text{reduce} (n, \text{fib} (\text{suc} (\text{suc } m)), \text{stack}) &= \\
 &\quad \text{objectMethod } \text{stack} (\text{push } (\cdot + \text{fib } m)) \triangleright \lambda \{ (-, \text{stack}_1) \rightarrow \\
 &\quad \text{inj}_1 (\text{suc } n, \text{fib} (\text{suc } m), \text{stack}_1) \} \\
 \text{reduce} (0, \text{val } m, _) &= \text{inj}_2 m \\
 \text{reduce} (\text{suc } n, \text{val } m, \text{stack}) &= \\
 &\quad \text{objectMethod } \text{stack} \text{pop} \triangleright \lambda \{ (k + \cdot, \text{stack}_1) \rightarrow \\
 &\quad \text{inj}_1 (n, \text{val } (k + m), \text{stack}_1) ; \\
 &\quad \quad \quad (\cdot + \text{fib } k, \text{stack}_1) \rightarrow \\
 &\quad \text{objectMethod } \text{stack}_1 (\text{push } (m + \cdot)) \triangleright \lambda \{ (-, \text{stack}_2) \rightarrow \\
 &\quad \text{inj}_1 (\text{suc } n, \text{fib } k, \text{stack}_2) \} \}
 \end{aligned}$$

Fibonacci Function

$$\{-\# \text{NON_TERMINATING} \#\}$$

$$\text{iter} : \text{Stackmachine} \rightarrow \mathbb{N}$$

$$\text{iter } \textit{stack} \text{ with } \textit{reduce } \textit{stack}$$

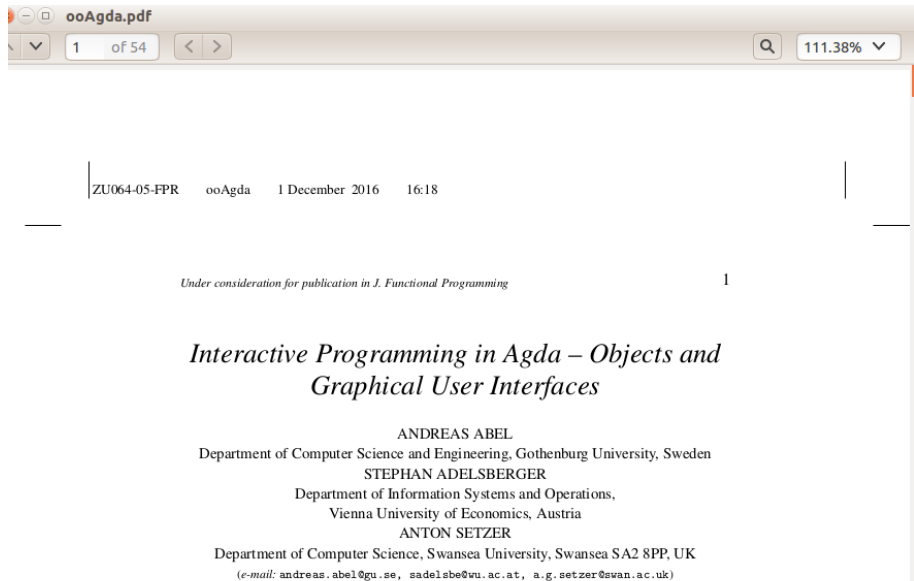
$$\dots \mid \text{inj}_1 s' = \text{iter } s'$$

$$\dots \mid \text{inj}_2 m = m$$

$$\text{fibUsingStack} : \mathbb{N} \rightarrow \mathbb{N}$$

$$\text{fibUsingStack } n = \text{iter } (0, \text{fib } n, \text{emptyFibStack})$$

Paper to appear in JFP [AAS16a]



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Under consideration for publication in J. Functional Programming 1

Interactive Programming in Agda – Objects and Graphical User Interfaces

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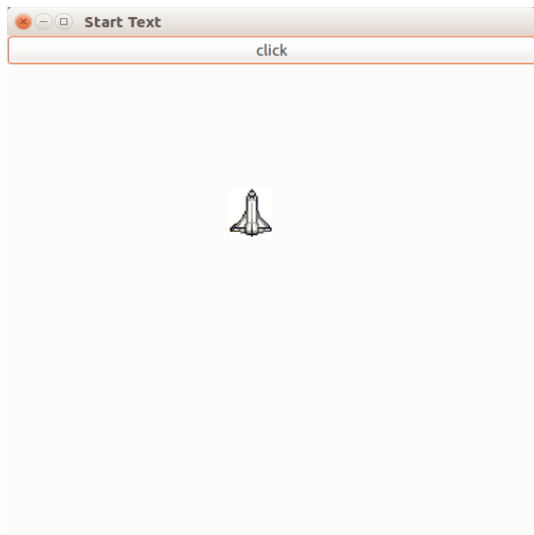
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Results in [AAS16a]

- ▶ Development of GUIs in Agda.
 - ▶ Based on **server-side** programmings.
 - ▶ Use of **action listeners** which are part of an **object**
- ▶ Verification of laws of a safe and equivalences of implementations of stacks using bisimilarity.
- ▶ Library ooAgda on github [AAS16b].
- ▶ **Remark:** Library CSP-Agda for the process algebra CSP in Agda is now on github, see [IS17], article: [IS16].

SpaceShip Example



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Dynamic Creation of Objects

- ▶ Idea is to create a set **Heap**, and pointers on the heap which dereference as objects.
- ▶ We have a state dependent object heap
 - ▶ depending on the size of the heap,
 - ▶ and methods for
 - ▶ dereferencing pointers,
 - ▶ updating pointers,
 - ▶ creating new pointers (which increases the size)
- ▶ Currently working on linked and double linked lists built on the heap.
 - ▶ Goal is to create a proper queue.
 - ▶ Idea: Develop and verify the networking protocols such as the Chord protocol.

Current Challenges

- ▶ What is the right language?
 - ▶ At the moment in OO programs in Agda we need to introduce for every new instance of an object a new variable.
 - ▶ Can we write a library which hides these new variables?
 - ▶ Do we need new language constructs in Agda or (preferred) can we achieve this using the library?
- ▶ How to execute programs involving the heap efficiently.
 - ▶ At the moment heap is implemented as a list of heap elements.
 - ▶ Can we in a compiled version override this by calls to the “real” heap?
 - ▶ Do we obtain good performance of a queue?
 - ▶ Is it possible to write a true heap directly in Agda without overriding?

Conclusion

- ▶ Definition of **coinductive data types** (coalgebras) by their **observations**.
 - ▶ Use of **copattern** matching
- ▶ **Objects** as examples of coalgebras.
- ▶ **State dependent objects**.
- ▶ **Current work:** Developing of heap and dynamic creation of objects on the heap.

Coalgebras in Dependent Type Theory

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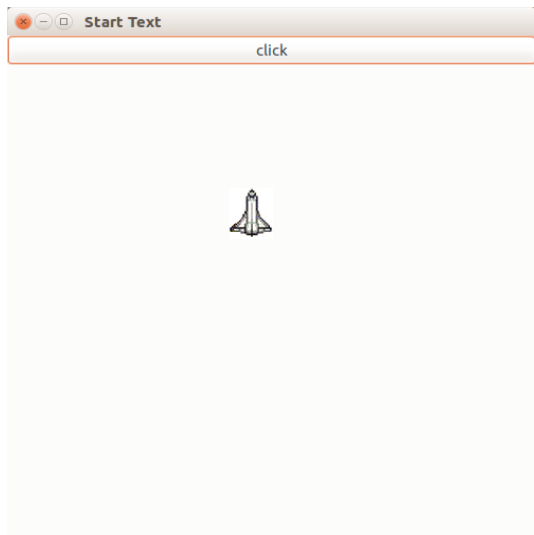
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SpaceShip Example



Graphics Interface Level1

```

data GuiLev1Command : Set where
  makeFrame   : GuiLev1Command
  makeButton  : Frame → GuiLev1Command
  addButton   : Frame → Button → GuiLev1Command
  drawBitmap  : DC    → Bitmap → Point → Bool
                → GuiLev1Command
  repaint     : Frame → GuiLev1Command

```

```

GuiLev1Response : GuiLev1Command → Set
GuiLev1Response makeFrame      = Frame
GuiLev1Response (makeButton _) = Button
GuiLev1Response _              = Unit

```

```

GuiLev1Interface : IOInterface

```

```

Command GuiLev1Interface = GuiLev1Command
Response GuiLev1Interface = GuiLev1Response

```

Graphics Level2 Commands

GuiLev2State : Set₁

GuiLev2State = VarList

```

data GuiLev2Command (s : GuiLev2State) : Set1 where
  level1C          : GuiLev1Command → GuiLev2Command s
  createVar        : {A : Set} → A → GuiLev2Command s
  setButtonHandler : Button
                  → List (prod s
                        → IO GuiLev1Interface ∞ (prod s))
                  → GuiLev2Command s
  setOnPaint       : Frame
                  → List (prod s → DC → Rect
                        → IO GuiLev1Interface ∞ (prod s))
                  → GuiLev2Command s

```

Graphics Level2 Response + Next

$$\text{GuiLev2Response} : (s : \text{GuiLev2State}) \rightarrow \text{GuiLev2Command } s$$

$$\rightarrow \text{Set}$$

$$\text{GuiLev2Response } _ (\text{level1C } c) = \text{GuiLev1Response } c$$

$$\text{GuiLev2Response } _ (\text{createVar } \{A\} a) = \text{Var } A$$

$$\text{GuiLev2Response } _ _ = \text{Unit}$$

$$\text{GuiLev2Next} : (s : \text{GuiLev2State}) \rightarrow (c : \text{GuiLev2Command } s)$$

$$\rightarrow \text{GuiLev2Response } s \ c$$

$$\rightarrow \text{GuiLev2State}$$

$$\text{GuiLev2Next } s (\text{createVar } \{A\} a) \ \text{var} = \text{addVar } A \ \text{var } s$$

$$\text{GuiLev2Next } s _ _ = s$$

Graphics Level2 Interface

GuiLev2Interface : IOInterface^s

State^s GuiLev2Interface = GuiLev2State

Command^s GuiLev2Interface = GuiLev2Command

Response^s GuiLev2Interface = GuiLev2Response

next^s GuiLev2Interface = GuiLev2Next

Action Handling Object

```

data ActionHandlerMethod : Set where
  onPaintM      : DC    → Rect → ActionHandlerMethod
  moveSpaceShipM : Frame → ActionHandlerMethod
  callRepaintM  : Frame → ActionHandlerMethod
  
```

```

ActionHandlerResult : ActionHandlerMethod → Set
ActionHandlerResult _ = Unit
  
```

```

ActionHandlerInterface : Interface
Method ActionHandlerInterface = ActionHandlerMethod
Result ActionHandlerInterface = ActionHandlerResult
  
```

```

ActionHandler : Set
ActionHandler = IOObject GuiLev1Interface ActionHandlerInterface
  
```

Action Handling Object

```

actionHandler :  $\mathbb{Z} \rightarrow \text{ActionHandler}$ 
method (actionHandler z) (onPaintM dc rect) =
  do $\infty$  (drawBitmap dc ship (z , (+ 150)) true)  $\lambda$  _  $\rightarrow$ 
  return $\infty$  (unit , actionHandler z)
method (actionHandler z) (moveSpaceShipM fra) =
  return $\infty$  (unit , actionHandler (z + (+ 20)))
method (actionHandler z) (callRepaintM fra) =
  do $\infty$  (repaint fra)  $\lambda$  _  $\rightarrow$ 
  return $\infty$  (unit , actionHandler z)

```

```

actionHandlerInit : ActionHandler
actionHandlerInit = actionHandler (+ 150)

```

Action Handlers

```

onPaint : ActionHandler → DC → Rect
         → IO GuiLev1Interface ActionHandler
onPaint obj dc rect = mapIO proj₂ (method obj (onPaintM dc rect))

moveSpaceShip : Frame → ActionHandler
              → IO GuiLev1Interface ActionHandler
moveSpaceShip fra obj = mapIO proj₂
  (method obj (moveSpaceShipM fra))

```

Action Handlers

callRepaint : Frame \rightarrow ActionHandler
 \rightarrow IO GuiLev1Interface ActionHandler

callRepaint *fra obj* = mapIO proj₂ (method *obj* (callRepaintM *fra*))

buttonHandler : Frame \rightarrow List (ActionHandler
 \rightarrow IO GuiLev1Interface ActionHandler)

buttonHandler *fra* = moveSpaceShip *fra* :: [callRepaint *fra*]

Spaceship Program

```

program : IOS GuiLev2Interface (λ _ → Unit) []
program = doS (level1C makeFrame)          λ fra →
doS (level1C (makeButton fra))          λ bt →
doS (level1C (addButton fra bt))        λ _ →
doS (createVar actionHandlerInit)      λ _ →
doS (setButtonHandler bt (moveSpaceShip fra
                                     :: [ callRepaint fra ])) λ _ →
doS (setOnPaint fra [ onPaint ])
returnS

```

```

main : NativeIO Unit
main = start (translateLev2 program)

```

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