

Searching

Markus Roggenbach

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Examples: where do we search?

- Google
- Telephone book
- Train connection

Making things precise: Searching as a Computational Problem

Searching Problem:

Input: array $A[1..n]$ of n numbers;
number d

Output: if d appears in $A[1..n]$:
 smallest index $i \in \{1, \dots, n\}$ with $A[i] = d$
otherwise: 0

Definition 1 (computational problem)

A *computational problem* specifies in general terms the desired input/output relationship.

We formulate such problems using the scheme:

<ProblemName>:

Input: <*list of inputs*>

Output: <*list of desired outputs*>

Algorithms which shall solve the Sorting Problem

SEQSEARCH($A, p, q; d$) (iterative version)

```
1  $l \leftarrow 0$ 
2 WHILE  $p \leq q$  DO
3     IF  $A[p] = d$  THEN  $l, q \leftarrow p$ 
4      $p \leftarrow p + 1$ 
5     OD
6 output  $l$ 
```

$\text{SEQSEARCH}(A, p, q; d)$ (recursive version)

```
1 IF  $p > q$  THEN output 0
2     ELSE DO
3         IF  $A[p] = d$ 
4             THEN output  $p$ 
5             ELSE  $\text{SEQSEARCH}(A, p + 1, q; d)$ 
6         OD
```

Correctness

Question: Are the two variants of SEQSEARCH doing what we expect them to do?

Definition 2 (Correctness)

*A **algorithm** is said to be correct if, for every input instance, it halts with the correct output. We say that a correct algorithm **solves** the given computational problem.*

Remark 1 (Computational Problems and correct Algorithms)

*For **one** computational problem*

*there are **many** different algorithms solving it.*

Running time

Question: How many steps do the variants of SEQSEARCH need to produce an output?

Definition 3 (Running time)

*The **running time** of an algorithms on a particular input is the number of primitive operations or “steps” executed.*

*Usually we concentrate on the **worst case running time**, that is, the longest running time for any input of size n .*

General approach:

1. Formulate a **computational problem** P .
2. Write an **algorithm** A
3. Prove: A **solves** P , i.e.
 - A halts for every input
 - A produces the correct output
4. Analyse the **running time** of A .

Links to the Course Text:

- Algorithm, computational problem, correctness: section 1.1 of [CFRS03]
- Running time: section 2.2. of [CFRS03]
- Pseudo code conventions: pp.19-20 in [CFRS03]