From winning Strategies to Nash equilibria

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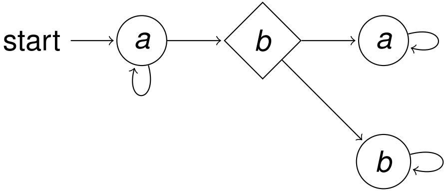
Introduction

- Games played on finite graphs are of great interest (recall Patrick Totzke's talk)
- Usually, people care about winning strategies realizable by finite automata.
- We want to generalize to multiplayer multioutcome games.
- We then care about finite memory Nash equilibria.

Question

Under what conditions can we obtain the existence finite memory Nash equilibria in the multiplayer multioutcome version of a game from the existence of finite memory winning strategies in the two-player win/lose version?

Games played on graphs



Preferences

- Vertices are labelled by colours $c \in C$.
- Players now have preferences $\prec_a$ over C^ω.
- A Nash equilibrium is an assignment of strategies to players where no player can do better by changing their own strategy.

The notions for our result

- A strategy is optimal, if it guarantees the *best worst-case*.
- A preference has the *optimality is regular* property, if for any game and any finite memory strategy there is a finite automaton deciding whether the strategy is optimal from some history onwards.
- A preference $\prec$ is prefix-linear, if $p \prec q \Leftrightarrow hp \prec hq$ for any finite history $h \in C^*$ and infinite histories $p, q \in C^*$.
- Being (automatic-piecewise) prefix-linear implies optimality-is-regular.

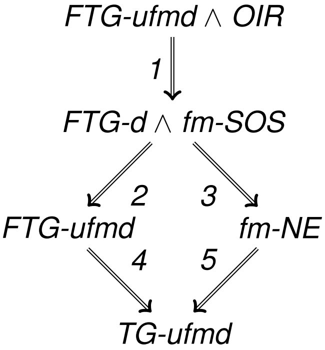
The notions for our result II

- A threshold game is derived win/lose game based on some outcome: One player wins, if he can do better than that, otherwise everyone else wins.
- Future threshold games start with some history.
- *Uniformly* finite memory means that the required memory depends only on the size of the game graph, not on the history.

The result

Theorem

Let $(\prec_a)_{a \in A}$ be closed under antagonism. The statements below refer to all the games built with C, A, and $(\prec_a)_{a \in A}$.



- **OIR:** *Optimality is regular.*
- **fm-SOS:** *There are finite-memory subgame-optimal strategies.*
- **fm-NE:** *There are finite-memory Nash equilibrium.*
- **FTG-d:** *The future threshold games are determined.*
- **TG-ufmd** *In every game, the threshold games are determined using uniformly finite memory.*
- **FTG-ufmd:** *In every game, the future threshold games are determined using uniformly finite memory.*

Strictness

We know that Implications 1 and 2 do not reverse, but no more.

Question

Is there a preference $\prec$ *such that all threshold games for* $\prec$ *are uniformly finite memory determined, but not all future threshold games?*

Why talking about all graphs is needed

Example (Based on ideas by Axel Haddad and Thomas Brihaye)

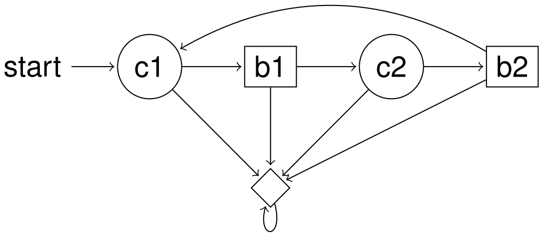


Figure: The graph for the game in Example 2

The game *g* in Figure 1 involves Player 1 (2) who owns the circle (box) vertices. Who owns the diamond is irrelevant. The payoff for Player 1 (2) is the number of visits to a box (circle) vertex, if this number is finite, and is -1 otherwise.

The paper

Stéphane Le Roux & Arno Pauly:
Extending finite-memory determinacy to multi-player games.
Information & Computation. Vol 261 (2018)

Preceding work

T. Brihaye, J. De Pril, S. Schewe:
Multiplayer cost games with simple Nash equilibria
Logical Foundations of Computer Science. (2013)

Further reading

Stéphane Le Roux, Arno Pauly & Mickael Randour:
Extending Finite-Memory Determinacy by Boolean Combination of Winning Conditions.
FCT&TCS 2018