Antrag
auf Förderung einer wissenschaftlichen Zusammenarbeit zwischen Deutschland und Großbritannien

1 Allgemeine Angaben

Dies ist ein Neuantrag.

Es wird die Förderung einer Kooperation zwischen deutschen und britischen Wissenschaflern im Rahmen der Vereinbarung zwischen der DFG und dem EPSRC beantragt. Die neben dem Antragsteller Prof. D. Spreen auf der deutschen und Dr. U. Berger auf der britischen Seite beteiligten weiteren Wissenschaftler sind unter Punkt 6 aufgeführt. Dr. Berger wird gleichzeitig einen inhaltlich gleichlautenden Antrag beim EPSRC einreichen.

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1.2 Topic (Thema)

Logic and algorithms for continuous data spaces (Logik und Algorithmen für kontinuumsähnliche Datenräume)

1.3 Key word (Kennwort)

Logic and algorithms (Logik und Algorithmen)

1.4 Scientific discipline and field of work (Fachgebiet und Arbeitsrichtung)

Mathematik und Theoretische Informatik: Logik, Berechenbarkeit, Komplexität, topologische Strukturen

1.5 Scheduled duration in total (Voraussichtliche Gesamtdauer)

3 Jahre

1.6 Application period (Antragszeitraum)

3 Jahre

1.7 Start of support (Gewünschter Beginn der Förderung)

1. Juli 2009

1.8 Summary (Zusammenfassung)

In safety-critical applications it is not sufficient to produce software that is only tested for correctness: its correctness has to be formally proven. This is true as well in the area of scientific computation. An important example are autopilot systems for aircrafts. The problem here is that the current mainstream approach to numerical computing uses programming languages that do not possess a sound mathematical semantics. Hence, there is no way to provide formal correctness proofs.

The reason is that on the theoretical side one deals with well-developed analytical theories based on the non-constructive concept of a real number. Implementations, on the other hand, use floating-point realisations of real numbers which do not have a well-studied mathematical structure. Ways to get out of these problems are currently promoted under the slogan "Computing with Exact Real Numbers". They are based on the following ideas:

In traditional analytical mathematics, points of spaces represent the "ideal (total)" result of approximations. In order to give a sound mathematical semantics to programs that have data structures representing such points, the "partial (finite)" objects used to approximate them have to be given the same status of points of a space as the total ones. From a computational view point they are even the first-class citizens, as the other—the total—ones are derived via a limiting process. This has led to new structures (e.g. "domains" in the sense of D.S. Scott and Yu.L. Ershov).

For proving the existence of certain objects, for example the roots of polynomials or fixed points of operators, one can proceed in a purely abstract way using e.g. proofs by contradiction or compactness arguments. In applications, however, we need not only know that such objects exist, we need a concrete way to get hold of them. This can be achieved in a logic-oriented way by either using a constructive framework in the existence proof or by disclosing the constructive content of classical proofs, or by deriving an effective version of the classical existence result, i.e. by studying whether and how the postulated object can be computed. The latter approach—to a large extent developed by K. Weihrauch and his students and known as Type-Two Theory of Effectivity (TTE)— has led to extensions of traditional models used for computations on discrete objects and the study of their complexity. The full relationship of these approaches is still under investigation.

To compute with continuous data like the real numbers, these are represented by streams of finite objects. Computations proceed by transforming the streams. It is therefore not surprising that coalgebraic methods figure extensively in the research we are proposing.

The research in this project will concentrate on the following interrelated problem areas:

1. Computability and complexity over continuous data structures.
2. Domains and computation.
3. Exact real number computation.
4. Continuous data and coalgebras.

5. Program extraction.

6. Proof mining.

7. Constructive theories and their strength.

There are many connections between these problem areas which, so far, have only partly, or not at all, been explored. Apart from strengthening existing contacts between researchers working in the respective areas, the project also aims at creating new collaborations in order to fully explore these connections.

The main connections of the project we are applying for are depicted by the graph in Figure 1.

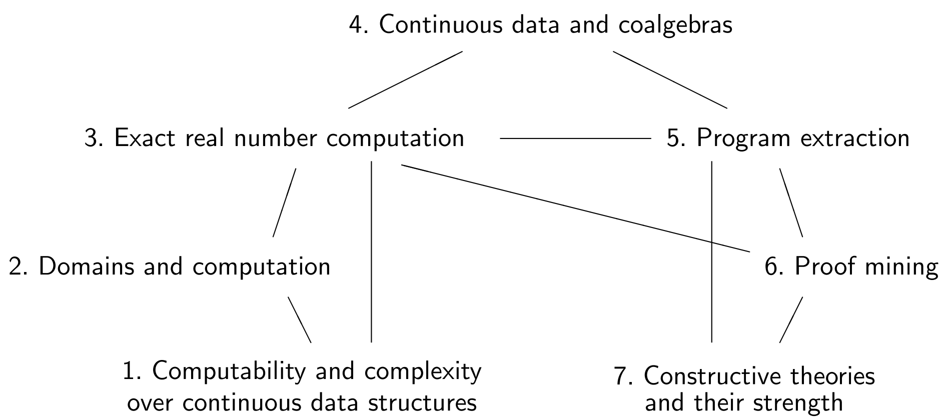

Figure 1: Main connections between the problem areas

Each of the four maximal cliques in this graph is united by a common theme:

1,2,3. Computability for continuous and higher-order data.

3,4,5. Algorithms for continuous and higher-order data.

3,5,6. Efficient computation with continuous and higher-order data.

5,6,7. Computational content of proofs.

These themes cover many aspects of the research topics of the project, but not all of them. In the following the goals sought for in the problem areas above are summarised.

1.8.1 Computability and complexity over continuous data structures

Data structures, computability and complexity for basic differential operators. There exist a well developed mathematical theory and established algorithmic recipes for many kinds of differential equations. In sharp contrast we currently lack a rigorous treatment of most of these problems concerning a natural notion of computational complexity or even computability.

Our first objective will therefore be a classification of several differential operators with respect to their computability and—where possible—with respect to their computational complexity. Such classifications are strongly connected to and depend on natural representations of domains for the Euclidean space and function spaces, both from a theoretical and practical point of view. Thus our second objective is to develop and compare such representations in the sense of Type-Two Theory of computability and complexity.

Computability and complexity on function spaces, and higher types in general. We plan to work on the one hand on problems concerning the way elements of spaces without countable base can be represented in a computer. Certain function spaces are of this type. On the other hand, in numerical mathematics many concrete problems over function spaces are analysed. In Information-Based Complexity the complexity of such problems is investigated with respect to an algebraic computation model, for well-posed problems and for ill-posed problems. We are working on the solvability and the complexity of such problems in the Turing machine model.

A particular problem in higher-type computability that is of interests to us is to try to extend the results of [187], to show that a wide class of realisability models has a larger subcategory in common than just the finite types over $\mathbb{N}$ and retracts thereof (e.g. that one can close up under the formation of various subspaces and quotients as well as exponentials, thus embracing additional spaces of interest in topology and analysis). A full treatment of this issue would probably be very hard, and would e.g. settle the question of whether the intensional and extensional type structures over the reals coincide. But we are hopeful of some progress in this direction, which would illuminate the extent to which results e.g. in computable analysis are (or are not sensitive) to the underlying model of computation.

Kolmogorov complexity and entropy of dynamical systems. We plan to analyse dynamical systems, especially cellular automata and shift spaces, with the tools provided by the theory of algorithmic randomness, in particular Kolmogorov complexity. Furthermore we are working on the question under which circumstances and in what sense the topological entropy of such dynamical systems can be computed.

Computability in real algebraic geometry. Real Algebraic Geometry is concerned with the solutions to systems of polynomial in-/equalities over the reals. Based on famous theorems like Hilbert's Nullstellensatz, it has emerged into an algorithmic discipline using e.g. Gröbner Bases as a means to quantifier elimination and with deep complexity-theoretic investigations. However these apply traditionally to the algebraic (also called BSS) model of real number computation—simply because in the complementary model of real number computation, namely Recursive Analysis, decision problems are trivially discontinuous in the given polynomial coefficient vectors and hence uncomputable. We are interested in conditions and restrictions to the possible inputs which render the solution computable: either as a set or as a (multi-valued) solution vector.

1.8.2 Domains and computation

Topological methods in semantics Domain theory is a powerful tool for providing rigorous semantics of programming languages. It models basic concepts, such as partial information, approximation, convergence, etc., with order-theoretical and topological tools. Originally, domain theory was based on ordered structures: directed-complete partially ordered sets (dcpos, for short) and continuous lattices [111]. However, to better meet the needs of semantics, its compass has recently been extended to include more general topological structures [135, 13, 14].

Modelling probabilistic features and other programming constructs requires the transfer of appropriate tools from mainstream mathematics to the context of domain theory. The case of probability has required the development of a kind of topological measure theory for domains [134], which involves adapting functional-analytic tools to a non-Hausdorff setting. The consideration of more general computational effects, such as nondeterminism, side effects, input/output, exceptions, etc., similarly requires the development of a universal algebra for non-Hausdorff spaces.

This strand of the project will push forward the development of the required functional-analytic and algebraic tools, both in the context of order-theoretic domain theory, and in the context of the wider classes of non-Hausdorff topological spaces now being used in semantics.

Modelling nondeterminism in domain theory. The purpose of this subproject is to work on some current issues in domain theory as applied to nondeterminism. Nondeterminism is of fundamental interest to computer science for two reasons. In some situations, even although the underlying computations are deterministic, it is not helpful to model them in detail. An example is communication over the internet where the order of arrival of messages depends on specific physical circumstances whose details are not relevant to the correctness of the programs at hand. Another example, of a different kind, is where programs involve a random choice, as, for example, in some cryptographic algorithms. In this project we are interested in modelling combinations of these forms of nondeterminism using a domain-theoretic approach.

Semantics and logic of probabilistic-nondeterministic computation. Probabilistic algorithms play an important role in complexity theory and algorithmics (e.g. efficient prime facorisations). Thus it is important to develop denotational models and logics for reasoning about such programs.

1.8.3 Exact real number computation

Partial differential equations. Domain theory has been applied successfully to obtain solutions of ordinary differential equations (initial and boundary value problems). Using domain-theoretic techniques, one obtains algorithms that approximate the solution of a differential equation up to any desired degree of accurracy, and free of round-off errors. Moreover, if approximations to a solution have been computed, this in particular implies existence of solutions.

The goal is to study different types of partial differential equations (parabolic, elliptic and hyperbolic), first on the basis of examples, and later more general, to investigate to what extent domain theory can be used to approximate solutions.

Hybrid systems. Hybrid Systems consist of a digital control that interacts with a continuous environment. The verification task for hybrid systems is notoriously difficult, as one needs guaranteed approximations for the values of the continuous variables, whose trajectories are usually modelled with differential equations. The task within the project is to extend existing domain-theoretic techniques to include hybrid systems with probabilistic transitional behaviour, so that one can establish quantitative guarantees with respect to the likelihood of events.

Verification and efficient implementation of exact real arithmetic, with applications to dynamical and hybrid systems. We want to improve the efficiency of the iRRAM library for exact real arithmetic, and extend Lester's work on verification of exact arithmetic in Haskell to the more efficient, but also more complicated language `C++`. Here problems from hybrid automata are used as reference for defining the demands the library should meet at the end, involving fast numerical solutions of differential equations as well as very large state spaces.

Development of the Abstract Stone Duality calculus into a high-level language for the iRRAM implementation of exact real arithmetic, in order to provide an interface with constructive analysis. We aim to develop a simple high-level language as an interface between mathematical problems in constructive real analysis and existing implementations (principally Müller's iRRAM) of exact real arithmetic. The focus of this topic will be the development of a standard for this interface that has a firm basis in a mathematical theory (Taylor's Abstract Stone Duality).

Such a language will provide a framework in which to develop sophisticated problems in exact real analysis. It will also facilitate deeper communication between the mathematical and programming communities who are working in constructive real analysis and who already interact through the forum of the conference *Computability and Complexity in Analysis*.

1.8.4 Continuous data and coalgebras

Coalgebraic properties of Markov transition systems Markov transition systems may be perceived as a coalgebra for the subprobability functor, so it becomes interesting to study these coalgebras from a categorical and measure-theoretic point of view. These transition systems are used for interpreting modal logics and coalgebraic logics as their generalizations, the semantics giving different properties depending on the kind of morphisms used in the base category. We will focus on randomized morphisms and investigate problems of bisimilarity, logical equivalence and behavioral equivalence under this notion of morphism, which actually turn out to be closely related to the Giry monad which has recently found some interest. Congruences are closely related, and we will need to look into the lattice structure of congruences for classification purposes.

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Representation of continuous functions on final coalgebras. The topic of the research planned is representations for continuous functions on final coalgebras, ie. on infinite objects. This will cover not only final coalgebras for endofunctors on the category of sets, but also those in slice categories over a given set (such as a datatype of first order formulas).

The problems are all connected with the representation of continuous functions on infinite (non-wellfounded) objects, such as streams, non-terminating automata, and non-wellfounded derivations in reasoning systems. These all (with certain constraints) have a natural topology, arising from their construction as final coalgebras for functors in certain categories. Although it is clear how to define functions *to* such objects, it needs to be clarified how to define functions *on* them.

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1.8.5 Program extraction

Induction and coinduction for program extraction. We plan to formalise constructive proofs in various areas of mathematics from which provably correct programs can be extracted. Initially, we want to look at problems in real number computation, but it is envisaged to move on to more general structures such as metric spaces, topological spaces and dynamical and non-deterministic systems.

In the formalisation of infinitary objects such as real numbers and real functions inductive and coinductive definitions play a prominent role. They give rise to wellfounded and non-wellfounded data structures such as lists, streams and trees and corresponding (co)recursive computation schemes. (Co)inductive definitions have been successfully used to characterise various notions of continuity and to extract (lazy and memoized) algorithms for exact real number computation. It is planned to extend this to algorithms in topology and related areas.

We plan to collaborate with other groups in this project on the following further topics: (i) study of inductive/coinductive notion of continuity from a type theoretic and category-theoretic perspective, (ii) computational content of problems involving non-determinism, (iii) implementation and (partial) automatisation of the formal proof and program extraction process in an interactive theorem prover.

The main reason why we are thinking that the method of program extraction has a realistic potential for nontrivial and practically interesting applications is the observation that in order to obtain executable programs it is often not necessary to formalise a proof completely: usually, by far the largest part of a proof only contributes to the truth, but not the algorithmic content of a theorem and hence can be done informally and imported via axioms. This observation has been made and is being exploited as well in Data Mining, another topic of this project to which ours bears a close relationship.

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Extraction of programs from classical proofs. It is well known that from carefully crafted mathematical proofs we can read off an algorithm together with a proof of its correctness. However, in common, not specially preprocessed, "non-constructive" proofs the computational information is contained implicitly and special methods are needed to automatically extract a correct algorithm. Two such methods are "refined A-translation", which has been developed and investigated in the Munich logic group in the last 15 years, and "Dialectica interpretation", suggested by Kurt Gödel more than 50 years ago. These methods differ by design and by scope of application. The goal of the project is to compare the behaviour of the two methods using the following criteria: (i) applicability, (ii) efficiency of extracted program, and (iii) readability of extracted program.

In the last years methods for extracting computational content from non-constructive proofs have received a lot of renewed interest. One reason is that although many classical proofs can be redone constructively, this can be quite cumbersome. Methods for extraction provide us with means to obtain constructive content, without the need for an expert to rewrite the proof.

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Constructive analysis of principles related to Zorn's Lemma. The extraction of programs from proofs works even for many non-constructive, so-called classical proofs unless they contain invocations of principles, such as the axiom of choice, which postulate the existence of ideal objects. Within this project we intend to systematically prepare a large class of classical proofs, usually done with the axiom of choice and mainly taken from commutative algebra, for program extraction. Our main tool shall be a suitable induction principle serving as a substitute for Zorn's lemma, the relevant form of the axiom of choice in algebra.

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1.8.6 Proof mining

"Proof Mining" refers to the activity of transforming a prima facie non-constructive proof into a new one from which certain new information can be read off which was not visible beforehand. The main proof-theoretic techniques used in this context are novel forms and extensions of the Goedel-Spector functional interpretation of analysis. An important example is the extraction of effective bounds, but also of qualitative information such as the independence of existence statements of certain parameters coming with the problem. The approach has so far been used very successfully in functional analysis (in particular in approximation theory and metric fixed point theory), geodesic geometry, ergodic theory and topological dynamics.

Proof mining in ergodic theory and 'hard' analysis. The purpose of this subproject is to extend the range of the existing applications to proofs heavily based on weak compactness arguments, nonlinear ergodic theorems in the line of Baillon's classical nonlinear ergodic theorems and to new areas such as ergodic Ramsey theory and geometric group theory. The project is also concerned with further foundational work on the underlying proof theoretic methods such as monotone and bounded functional interpretations. Finally, we intend to explore the obvious connections between 'proof mining' in the sense described above and Terence Tao's program of 'hard' or 'finitary' analysis [246, 247].

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Proof mining for semi-classical proofs using hybrid interpretations. A number of proofs in arithmetic make unnecessary use of classical logic. It is planned to investigate the reasons for this, and also the advantages and disadvantages of using classical logic. Our conjecture is that proofs by contradiction are popular because it is "easier" to look for consequences of 'not A', rather than for statements which entail A. In proving A constructively, one has to look for a provable B which entails A, whereas in a proof by contraction we look for a false statement which is entailed by 'not A'. It also seems that these unneeded application of classical logic are an obstacle for the extraction of bounds, as they obscure the algorithms implicit in (even very simple) proofs.

1.8.7 Constructive theories and their strength

Partiality in total type theory. We are looking for ways to represent partial functions within the framework of Martin-Löf Type Theory or an extension in the sense of predicative topos theory. This would give us a way to represent partial functional programs (e.g. Haskell programs) in a total, logically sound language. A key element of this approach would be a type-theoretic account of continuity in the sense of Brouwer.

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Models of constructive theories for reasoning about continuous spaces. Since the 1970ies various formal systems have been investigated for reasoning about mathematical structures constructively and to extract programs from proof in these systems. Typical examples of such theories are constructive type theories and constructive set theories. To understand the variety of and relation between these systems it has proven worthwhile to construct various models mostly based on categorical notions of models.

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Constructive reverse mathematics. The objective of constructive reverse mathematics lies in the classification of theorems by means of logical principles (a logical principle is an axiom which follows from the law of excluded middle) and set existence principles (a set existence principle is an axiom which follows from the axiom of choice).

We intend to (i) find a sound formal foundation for constructive reverse mathematics, and (ii) classify theorems like the pigeonhole principle and various versions of Brouwer's fan theorem.

2 State of the art and preliminary work (Stand der Forschung und Vorarbeiten)

2.1 Computability and complexity over continuous data structures

2.1.1 Data structures, computability and complexity for basic differential operators

There exists a vast literature on theoretical and numerical results for differential equations. Rigorous results on computational properties of such equations are rather rare.

For ordinary differential equations it is well known that local solutions exist and can be computed as long as the solution is unique (see e.g. [66]). The interesting cases of ordinary differential equations induced by Lipschitz-continuous functions and analytic functions are PSPACE-hard [141] and polynomial time computable [197], respectively. Concerning global solutions it is known that in the general case the problem is Σ_2-hard [220] and even in the case of polynomials Blowup-sets cannot be computed in general [120].

The Dirichlet problem has very recently been shown to be $\sharp$-P complete [43, 219]. For analytic bounds, however, the complexity drops to polynomial time. These results have large influence on the complexity of Riemann mappings both in the simply connected and multiple connected case [43, 219].

Computability of the Neumann and Dirichlet boundary value problems for a simple non-symmetric elliptic differential equation in the one-dimensional case can be found in [54]. More general results on operators in Banach spaces can be found in [77]. Further computability aspects of several differential equations have been discussed e.g. in [261, 259, 260, 258, 257, 1, 217] .

Representations of domains in the Euclidean space have been studied in a variety of papers in the last years. Two different representations are used frequently in literature: Jordan curves and grid representations. Essential differences between the representations are already known [143, 64], but the similarity of these representations suggests that there should exist additional constraints where the representations are better comparable.

Concerning computability, the extension by Mathias Schroeder [227] to admissible representation introduced by Weihrauch and Kreitz [174] allows to investigate many function spaces. Complexity issues of such classes are so far only touched in [228].

2.1.2 Computability and complexity on function spaces, and in higher types in general

Concerning the question how elements of rather large spaces (uncountable and even without countable base) can be represented in a computer or in its theoretical model, a Turing machine, Matthias Schröder has made important progress in recent years. While the original notion of "admissible" representations due to Kreitz and Weihrauch, see [174, 256], applies only to T_0 spaces with countable base, Schröder [227] developed a theory of admissible representation for spaces with countable pseudobase. Furthermore, he developed also the notion of "multirepresentation" and extended the notion of admissibility to this more general kind of representations [226, 225]. These extensions are important for obtaining useful computability notions on certain function spaces and have been applied, e.g., by Zhong and Weihrauch [264] for defining computability on generalized functions, Schwartz test functions and tempered distributions. His ideas gave rise to several natural and rather general categories of spaces on which computations can be performed. These categories are often well behaved, e.g., Cartesian closed, and can also be described using domain theory [14]. Finally, Schröder also worked on the question on which spaces one can define useful complexity notions [228].

In Information-Based Complexity, the complexity of many concrete numerical problems has been analysed, see the monograph [252]. Here, complexity is meant with respect to an algebraic computation model. In this model it is assumed that elementary arithmetic computations on real numbers as well as comparisons of real numbers can be computed with infinite precision in one time step. This is of course a rather strong and simplifying assumption. It is interesting to see when and how much statements about solvability and complexity of a computation problem are affected by these assumptions in comparison to the assumptions made in the Turing machine model, i.e., when computation is performed bitwise. In this context, it looks interesting to look in particular for ill-posed problem because for well-posed problems one can assume that precision is not the main issue and therefore the simplifying assumption mentioned above concerning precision may not be so critical. In fact, Volker Bosserhoff already looked at certain ill-posed problems and analysed their solvability [51]. Note that there are problems that cannot even be solved in the strong model of computation used in Information-Based Complexity. For such problems, other notions of solvability are being considered in Information-Based Complexity, e.g. solvability in the average case setting. In the Turing machine model, some earlier approaches to probabilistic computability and computability in the average case setting were suggested by Ko [142, Chapter 5] and by Parker [210, 211]. Recently, a thorough and systematic study and comparison of the different possibilities of computability in this sense was carried out by Volker Bosserhoff [53]. It was a striking result in Information-Based Complexity that there are problems, namely unbounded linear operators, that cannot be solved in the usual (worst case) setting, but can be solved in the average case setting under a Gaussian measure [262, 145, 255], see also [251]. Traub and Werschulz [251, Chapter 6] asked whether the same is true with respect to the Turing machine model, i.e., if bit-wise computability is considered. Recently, Volker Bosserhoff [52] answered their question in the negative. That means, in the Turing machine model the answer to this question is different from the answer in the computation model used in Information-Based Complexity!

The interest in higher-type computability goes back right to the first years of computability theory. There is a huge amount of literature. First, only total functionals were studied. Then, after Ershov's and Scott's seminal work on domains, interests in the field was revived, as now also partial functionals could be studied. In recent years, U. Berger, J. Longley and D. Normann were the main contributors. In particular, publications [185, 186, 187] are relevant to this subproject. The "canonicity" results mentioned above are e.g. proved in [187].

2.1.3 Kolmogorov complexity and entropy of dynamical systems

The algorithmic randomness notion on the Cantor space $\{0, 1\}^\omega$ of one-way infinite sequences applies also to the set of bi-infinite sequences. Shift dynamical systems and cellular automata live on this space, and cellular automata also on higher-dimensional analogs. In [129] the classical Martin-Löf randomness notion was extended to much more general topological spaces with measure. In [62] this randomness notion was analysed especially in the context of cellular automata. The classical effective randomness notion can not only be defined via Martin-Löf tests but also via Kolmogorov complexity. For a generalisation to more general spaces see [105]. We plan to work on open problems related to randomness and Kolmogorov complexity for cellular automata and shift dynamical systems. Another topic that has been studied for both types of dynamical systems is the question under which circumstances the topological entropy is computable. In fact, the problem to compute the topological entropy is of interest also for many other types of dynamical systems, see, e.g., Milnor [194]. Concerning the computability of the entropy of shift spaces, positive results were obtained by Spandl [242] and for gap shifts by Hertling and Spandl [127]. Spandl obtained results even for the topological pressure, a generalisation of the topological entropy, and looked also at applications in statistical physics [243]. Negative results were obtained by Hurd et al. [131], by Simonsen [236], by Spandl [242] and by Hertling and Spandl [128] who answered a recent question by Simonsen [236] by constructing a shift dynamical system with decidable language whose topological entropy is a non-computable real number. They also characterised completely the real numbers that are the topological entropy of a shift dynamical system with decidable language.

2.1.4 Computability in real algebraic geometry

Algorithmic solutions to given systems of real polynomial in-/equalities have entered textbooks like [70, 11]. They can be considered as (highly nontrivial) generalisations of systems of linear in-/equalities treated, say, using Gaussian elimination and simplex method; while Gröbner Base calculations extend the Euclidean algorithm for polynomial division with remainder from the univariate to the multivariate case. Both pertain strongly to the algebraic model of computation.

We have successfully treated effective solvability of systems of linear equations within the setting of Recursive Analysis [267]. There it turned out that the *rank* of the matrix under consideration—or put differently: the dimension of the solution space—is the crucial ingredient which to know or not distinguishes between computability and discontinuity (and hence uncomputability). Notice that the requirement to know the rank weakens that of non-degeneracy (i.e. of full rank). We similarly have completely characterised effective solvability of systems of linear *in*equalities [55, 265, 266]: here, *full-dimensionality* turns out as the central key to computability. And finally we have analogously characterised polynomial division [144, Section 2.2]: here it is the exact degree of the denominator and an upper bound on the degree of the numerator that are necessary and sufficient to assert computability.

2.2 Domains and computation

2.2.1 Topological methods in semantics

Classical order-theoretic domain theory is now well established with a rich mathematical theory (comprehensively surveyed in [111]), and many applications [87]. Recently, Battenfeld, Schröder and Simpson have extended domain theory to a wider class of topological spaces, *topological domains*, capable of modelling programming features that lie beyond the reach of order-theoretic domain theory (parametric polymorphism; a theory of computability for functional programming languages with effects) [13, 14]. This investigation has also initiated new developments in topology [96, 122]. Within the broad class of spaces applicable to semantics, Jung, Kegelmann and Moshier have identified the *stably compact spaces* as the widest class of spaces known to enjoy a *Stone duality* [135], a feature that allows a principled approach to the development of program logics for programs and data. The stably compact spaces include most classes of continuous domains and all compact Hausdorff spaces, and thus provide a common framework for combining much of classical topology and domain theory.

The development of a non-Hausdorff analogue of topological measure theory, needed for the modelling of probabilistic computation, was originally carried out for order-theoretic domain theory [134]. This construction adapts in a natural way to stably-compact spaces [6]. The situation for general topological domain theory is more subtle, since different non-equivalent approaches to modelling probabilistic computation are available [12]. Although a canonical choice has been proposed by Schröder and Simpson [229], who give a *universal property* that characterises an appropriate *probabilistic powerdomain* in any given category of domains, the instantiation of the approach in the case of general topological domain theory has not been carried out. Underpinning all the above is a general programme of developing non-Hausdorff analogues of classical results from functional analysis. For example, investigations by Tix, Keimel and Plotkin, into modelling combinations of probabilistic and nondeterministic choice, required a substantial development of functional-analytic ideas within order-theoretic domain theory [250, 215, 140].

Other computational effects (nondeterminism, side effects, input/output, exceptions) can be modelled as free algebras [216]. To incorporate this within domain theory requires the development of a domain-theoretic form of *universal algebra*. In order-theoretic domain theory, the explicit construction of free algebras has been carried out by Jung, Moshier and Vickers [136], and extended to wider classes of topological spaces by Keimel and Lawson [138]. In full topological domain theory, the existence of a wide class of free algebras has been proved by Battenfeld [12], but explicit constructions have yet to be given. In some situations, the general algebraic approach to effects also extends to probabilistic choice. For continuous domains [134], as well as in the classical case of compact Hausdorff spaces, the construction of probabilistic powerdomains/powerspaces is characterised as a free algebra construction for an algebraic theory of *convex spaces*. It is an open question whether such a characterisation extends to stably compact spaces, though partial results towards a positive answer have been obtained by Cohen, Escardó and Keimel [65].

2.2.2 Modelling nondeterminism in domain theory

Modelling the combination of the different kinds of nondeterminism mentioned in the Summary is of great interest in computer science, with applications to, for example, distributed cryptographic algorithms. Relevant techniques are taken from automata theory, process calculus and model-checking [235, 63, 177, 74], together with domain theory, where we give more comprehensive references: [250, 215, 254, 196, 117, 118, 119, 140]. But much remains to be done.

The semantics of programs is typically given using domains: the elements of carefully chosen domains are used to model the computations at hand. Nondeterminism is usually handled by one or another form of powerdomain. The idea is that if, in the absence of nondeterminism, one used a domain D then, in its presence, one could instead use a so-called powerdomain, $\mathcal{P}(D)$ whose elements are sets of elements of D.

A number of classical constructions in domain theory are available. For ordinary nondeterminism one has available the so-called lower, upper and (order-) convex powerdomains; the first two are suitable for considerations of partial and total correctness, respectively, and the third for a combination of both. Dijkstra's predicate transformers are closely related: there is a 1-1 correspondence between predicate transformers (viewed as functions to a domain of truth-values) and nondeterministic functions, with range the upper powerdomain. For probabilistic nondeterminism one has available the probabilistic powerdomain, whose elements are subprobability valuations, rather than subsets. All these *classical* powerdomains are described in the textbook [111]. The classical powerdomain constructions have important descriptions as free algebras in varieties of algebras with suitable forms of choice operation. For example, under suitable assumptions, the convex powerdomain $\mathcal{P}(D)$ is the free semilattice over D, equipped with a least element, and the sub-probabilistic powerdomain $\mathcal{V}_{\leq 1}(D)$ is the free barycentric algebra over D, equipped with a least element.

New considerations enter when one has several kinds of nondeterminism at once. Computationally this can correspond to probabilistic computation carried out in the setting of distributed or parallel programs. Algebraically, interactions appear between the differing kinds of nondeterministic choice. Such considerations have begun to attract significant attention, including within the domain-theoretic community. In previous work we have considered the combination of ordinary nondeterminism and a construction (the extended probabilistic powerdomain) closely related to the probabilistic powerdomain. Algebraically the extended probabilistic powerdomain is the free cone rather than the free barycentric algebra. Mathematically it is easier to deal with cones than with convex spaces.

So far, we have developed a domain-theoretic analogue of (a small part of) functional analysis, such as separation theorems and a Banach-Alaoglu theorem, and applied them to characterising the combination powerdomains and the corresponding form of predicate transformer, where the domain of the (extended) positive reals plays the rôle of the truth-values [250, 215, 140]. Interestingly we found that the characterisation of the correct form of predicate transformer, the so-called *healthiness* conditions, is best obtained by a functional characterisation of the relevant powerdomains; these are domain-theoretic analogues of the classical Riesz representation theorem of functional analysis.

2.2.3 Semantics and logic of probabilistic-nondeterministic computation

In the book [193] McIver and Morgan have come up with predicate transformer semantics for a basic imperative language with nondeterministic and probabilistic choice constructs. In [250] Keimel, Plotkin and Tix have developed the mathematical foundations for the direct denotational semantics of such languages. A first step of relating these two approaches has been made in [139] via so-called *Minkowski duality*.

2.3 Exact real number computations

2.3.1 Partial differential equations

The idea of using domains in exact real number computation goes back to an idea of D. Scott, who proposed to use the interval domain consisting of all closed real intervals ordered by reverse inclusion as data type of the reals. Another source of ideas is coming from interval analysis (see e.g. [198]). A. Edalat was one of the first who systematically tried to develop Real Analysis in a domain-theoretic framework (see e.g. [87]). Another important work of these 'first days' was M. Escardo's PhD thesis [95]. By now there is a huge literature in the field. The main contributions of the participants relevant to this research are the development of a computational model for multi-variable differential calculus [90] and a treatment of inverse and implicit functions in domain theory [91].

The field of ordinary differential equations is already quite well studied, and the participants have contributed for example in [93] by giving a domain-theoretic account of the classical Picard method for solving ordinary differential equations, and [92] treats the Euler polygonal method. Linear Boundary Value Problems are studied in [214].

2.3.2 Hybrid systems

Based on the domain-theoretic model for ordinary differential equations, the case of non-probabilistic hybrid automata has been analysed from a domain-theoretic point of view in [94].

Moreover, the participants have contributed to the development of measure and integration theory, which will form a cornerstone for the treatment of probability measures, and the associated computational problems, in [89].

2.3.3 Verification and efficient implementation of exact real arithmetic, with applications to dynamical and hybrid systems

There has been significant progress in the last decade on libraries for numeric computation with exact real (or complex) numbers. Here the term 'real number' is meant literally; this is far more than the (finite) approaches using `double precision` numbers or the (still discrete) number sets that can be dealt with in libraries like `MPFR` or `Mathematica`. Instead, the full power of computable real numbers is available, as described in Computable Analysis and the Type-Two-Theory of Effectivity. Corresponding implementations are `iRRAM`[200], `RealLib`[179, 180] (both written in `C++`), or the programs cited in [182, 181] (written in HASKELL and PVS, both functional languages)

Although all of these package are have been programmed in a very careful way, in their development many errors have been made that led to numerically wrong results. In consequence, the implementation in the functional languages have actually already been verified by the authors using `PVS`. Unfortunately, these implementations are slower by magnitudes than the `iRRAM` or the `RealLib`, which both use many optimizations only available in imperative languages.

Applications such as hybrid automata [167, 168, 169] lead to new challenges for the packages: analysing the reachability of states in hybrid automata tends to produce huge state spaces, where any resulting answers can only be trusted if the software is trustworthy. The size of the state spaces already seems to demand special approaches like [201]. Furthermore, the necessary numeric solution of differential equations further enhances the computational complexity [141, 197].

2.3.4 Development of the Abstract Stone Duality calculus into a high level language for the iRRAM implementation of exact real arithmetic, in order to provide an interface with Constructive Analysis

Constructive real analysis already has a quite long history ([2, 46, 142, 256], to name just a few textbooks), implementations of the corresponding theories can be found in many programming languages:

- Classical imperative programming languages (most prominently: `C`) have been used here to get maximal performance, but the use of these languages for real analysis is quite complicated, as a computation with real numbers is essentially a manipulation of (converging) sequences, i.e. functional objects.
- Object-oriented languages (like `C++`) already are much closer to the intended application, as they allow a natural implementation of concepts corresponding to oracle Turing machines or Type-Two-Turing machines, that usually build the computational backbone of constructive real analysis. The efficiency can still be almost as good as with `C`.

- Functional programming (in `OCaml`, e.g.) obviuosly has the closest connection to real analysis. Additionally, functional programming has always had a clear mathematical basis, from which it has developed over 30 years into a mainstream programming paradigm. The main disadvantage of pure functional programming, though, is a quite high computational overhead.

The results of a small series of competitions in the near past [49, 202, 102] approve the remarks on efficiency. So a combination of the approaches should offer the best results: A firm theoretical anchor in a mathematically justified functional language, e.g. based on ASD, together with an efficient implementation in non-functional language.

ASD [18] is a new paradigm for computable general topology, but more specifically its application to analysis, obtained in collaboration with A. Bauer, led to a symbolic language for the reals. This consists of

- The usual arithmetic operations on the integers, rationals and reals;
- The *strict* relations $<$, $>$ and $\neq$ on the reals, considered as taking values in the Sierpiński space Σ;
- Open subspaces considered as Σ-valued predicates;
- Definition of real numbers as open Dedekind cuts;
- The logical operations $\top$, $\perp$, $\wedge$ and $\vee$ on Σ;
- Existential quantification of Σ-valued predicates over the integers, rationals, reals and open and closed intervals; and
- Universal quantification of such predicates over *closed* bounded intervals, considered as a formulation of the Heine–Borel theorem.

The classical real line provides one model of this language; Bauer and Taylor have provided another whose foundation is computable in principle, demonstrating completeness of the axioms.

Taylor [248, 249] has applied these to the intermediate value theorem and connectedness in elementary real analysis, and Bauer [17] has developed a prototype implementation of it. These demonstate that one may both express problems from analysis very naturally in this language, and compute efficiently with it, in particular using Dedekind cuts instead of the more usual Cauchy sequences [15].

2.4 Continuous data and coalgebras

2.4.1 Coalgebraic properties of Markov transition systems

Markov transition systems are used to provide models for the probabilistic interpretation of modal logics. Comparing models via bisimilarity or behavioral equivalence requires some background work in the category of stochastic relations, and it could be shown that semi-pullbacks exist in this category [88, 78]. This leads to a complete characterization of the relationship between bisimilarity, logical and behavioral equivalence, first in a somewhat specialized setting [75], then in full generality [79]. This could be applied to problems in model checking for continuous time models [80]. Subsequently, this work was generalized to the case in which the modal operators of the logic are replaced by predicate liftings in the sense of [213, 224], and we could investigate conditions under which these relationships of stochastic Kripke models are valid [82], leading to a very general criterion for these models to be bisimilar [85]. The investigation of weak morphisms for Markov transition systems started with Giry's seminal work [113] on the categorical foundations of the measure theoretic parts of probability theory. Panangaden realized its importance for the work on transition systems [209](see also [3]), and helped to apply it for the semantics of modal logics [75]. Kleisli morphisms in this category were investigated in [81] where it was shown that there is a very close connection between these morphisms and randomized congruences; this topis is investigated further in [84], the paper [83] gives a first set of criteria for randomized bisimulations to exist. Congruences are by their very structure closely related to Borel equivalence relations, a currently very active field in Descriptive Set Theory [223, 137]. It could recently be shown through a combination of Barr's final sequence (see [263]) and Kolmogorov's Consistency Theorem [212] that final systems exist for a variety of stochastic coalgebras over general measurable spaces [230], the variety being given by the measure-polynomial functor that drives the coalgebra's dynamics. We explore the consequence for Kripke models which model the semantics of general coalgebraic logics.

2.4.2 Representation of continuous functions on final coalgebras

The topic of representations for continuous functions on infinite streams of discrete data goes back to Brouwer in 1927. It is nowadays possible to write programs in practical programming languages that manipulate these representations in a clean and efficient way, expressing composition and other computationally interesting operations. This is demonstrated in the preliminary publication [110]; a journal publication is being prepared. Infinite streams are simple examples of final coalgebras, for endofunctors of the form $(A\times)$. We have recently seen a way to extend the Brouwerian representations to final coalgebras for functors of a more general form, such as may be used to represent a wide variety of automata. The mathematical ingredients of the representation make use of a form of induction recursion [86], and formal topology [222]. A number of loose ends remain to be tied up, in particular the completeness of the representations, and the expression of important combinators such as composition. This material has so far not been published. Endofunctors on slice categories enable the representation of infinite objects such as non-wellfounded proofs of various kinds, that have been investigated by Mints [195], Buchholz [61] and others in connection with composition, and in which interest has recently flared up in connection with inductive definitions and recursion [60, 253]. A crucial liveness condition is necessary to secure soundness for such proofs, that we can now see, more or less, how to express in a computational form using final and initial algebras for endofunctors on certain slice categories over datatypes of formulas. This topic is a little more speculative, but should connect with a broader tradition of proof-theoretical research.

2.5 Program extraction

2.5.1 Induction and coinduction for program extraction

The theory of inductive and coinductive data types and the technique of program extraction from proofs are well developed and supported by a number of interactive proof systems (e.g. Coq, Isabelle, PX, Agda, Minlog).

Extended case studies in the area of exact real number computation have shown that this approach to program development is feasible and leads to interesting new algorithms (for example lazy algorithms for real number arithmetic with respect to a signed digit representation of the reals), and also to new theoretical concepts and results (e.g. an abstract theory of digit spaces). This work is very recent and not yet published, but we presented it at a number of conferences and workshops (see the web page http://www-compsci.swan.ac.uk/~csulrich/slides.html).

Published papers are [34, 42, 32], on program extraction, and [36] on coinductive definitions. Further relevant publications are [33, 31, 30] on termination proofs which can be applied to programs extracted from proofs.

2.5.2 Extraction of programs from classical proofs

First ideas of using proof-theoretical methods for determining presence of computational content in non-constructive mathematical proofs date back to works of Bernays, Gentzen and Gödel. Kreisel [171] explicitly formulated the idea for "unwinding" proofs to discover implicit computational content.

Gödel's "Dialectica interpretation" [116], also known as "Functional Interpretation", gives the earliest concrete method for extracting programs from proofs in classical arithmetic. Even though very powerful, the method has several drawbacks. One of them, mostly a practical one, is that extracted code tends to be complicated, due to the need to employ higher types. Although we know that the obtained algorithm is correct, it is often hard to understand the operational semantics behind it. The other difficulty with the Dialectica Interpretation concerns contractions, i.e., using an assumption more than once in the proof. For the method to work, one needs to assume that all atomic formulas are decidable and to distinguish cases in order to be able to cover all instances of the assumption in the proof. Since contractions occur in almost any nontrivial mathematical proof, programs extracted by the Dialectica Interpretation usually have many nested branches that additionally obscure the program's operational semantics[1].

If one considers Gödel's Dialectica Interpretation restricted to primitive recursive functions taking arguments of base type, one can show that classical (or weak) proofs of totality of such functions can be transformed into constructive proofs of totality. This fact, also known as closure of intuitionistic arithmetic under the Markov rule, makes it possible to extract computable functions from classical totality proofs. The A-translation method, originally proposed by Harvey Friedman [101], gives an alternative simple syntactical proof of this result. However, if applied in its original form as an extraction method, it still produces too complicated programs. An improvement, known as "refined A-translation", has been proposed and studied by the Munich logic group [35].

Other research groups that are currently investigating A-translation and Dialectica Intuitionistic and are closely related to our research reside in Carnegie Mellon, Pittsburgh (Jeremy Avigad [8, 7]), Technische Universität Darmstadt (Ulrich Kohlenbach [148, 158, 166]), University of Wales Swansea (Ulrich Berger, Monika Seisenberger [41, 35]), as well as individual people in Queen Mary's College, London (Paulo Oliva [37, 205, 208, 125]) and University of Innsbruck (Mircea-Dan Hernest [123, 124, 125]).

Latest research by these groups is devoted to

- Simplification of programs extracted by the two methods [39, 35, 205, 208, 125, 124] (Berger, Hernest, Oliva, Schwichtenberg, Seisenberger);
- Investigation of applicability of the two methods to results in classical mathematical analysis and probability theory [8, 37, 123, 124, 148] (Avigad, Berger, Hernest, Kohlenbach, Oliva, Schwichtenberg, Seisenberger).

An alternative method for program extraction from classical proofs is extraction using control operators. The method was originally proposed by Griffin [121], building on results on Felleisen, Leivant and Sabry. The result was further investigated in two directions:

- Extracting programs in a lambda calculus, extended with control operators (Ariola, Herbelin, Makarov, Murthy, Parigot);
- Extracting control operator programs acting on saturated sets of lambda terms in a model theoretic setting (Krivine, Raffalli).

One main point in which the Munich group is pioneering research on program extraction is the development of the software proof assisting system Minlog[2] [232, 20] that has the only currently known implementation of both refined A-translation and Dialectica. From the authors above, Berger, Hernest and Seisenberger were a part of the Munich group and have contributed to development of Minlog.

The only comparisons involving the two methods that are currently known to us have been done by Hernest and Makarov in their PhD theses [123, 124, 190]. The investigations mainly consider several case studies of comparing programs extracted by refined A-translation on one hand and Dialectica and the control operators method on the other.

The method of refined A-translation has been explored in the Munich logic group for the last 15 years [38, 40, 39, 35, 233]. A number of case studies [32, 35, 39, 40, 126] have shown show that it yields quite efficient, readable and sometimes surprising programs. In particular, fine tuning computational content by using uniform quantifiers (suggested by U. Berger) has turned out very effective in improving extracted programs: code often becomes become shorter, less complicated and more efficient, and even may improve from e.g. quadratic to linear time complexity.

In contrast, practical applications of the Dialectica Interpretation have been studied in the group only for the last 3 years. Being too technical for extracting programs "by hand", a computer implementation of the method is necessary to investigate its usability for automatic extraction. Research efforts of the group up to now concentrated on adapting the method for feasible implementation.

[1] This problem does not occur, though, for Kohlenbach's monotone version of functional interpretation which is the main technique used in the applications discussed in Section 2.6 below.

[2] See `http://www.minlog-system.de`

As code extracted by the Dialectica Interpretation is in general more complicated, the need for optimisation is recognised. An extension of Berger's uniform quantifiers to Dialectica (Hernest's Light Dialectica) was adopted. Using uniformities has greater significance with Dialectica as it not only removes unwanted parameters, but also unnecessary case distinctions generated by contractions. [123, 124, 126]. Consequently, the performed case studies act as a proof of the concept that Dialectica should be regarded as a competitor method for extraction of feasible programs.

2.5.3 Constructive analysis of principles related to Zorn's Lemma

Our principal goal is a logical device with which the computational content of a wide class of proofs in classical algebra can be extracted more or less mechanically. Although such an ambitious programme is not only very promising but also somewhat uncertain, there is a convincing reason for which it is worthwhile: it has already worked well in another case, carried out in [29].

Classical analysis is abundant in proofs using countable choice (CC) or even dependent choice (DC), which proof principles are widely but not generally considered as constructive (we have summarised this in [231]). However, any occurrence of CC causes problems when it comes to extracting the computational content from Gödel and Gentzen's negative translation [106, 115] and Friedman's A-translation [101] of classical proofs with CC: the translation CCN of CC fails to be constructively deducible from CC or other constructive principles.

There nonetheless are at least two ways out of this situation. First, Spector [244] could extend Gödel's Dialectica interpretation [116] to classical analysis by interpreting CCN by bar recursion in finite types[3]. Secondly, quite a clever realiser of CCN was found by Berardi, Bezem, and Coquand [21]. The latter alternative is particularly interesting inasmuch as the realiser is nothing but the computational content of the (negative and A-translated) classical proof of CC with the principle of open induction crucial in this context.

Open induction (OI), studied by Raoult [69] and Coquand [218], is the classical contrapositive of the Nash-Williams minimal-bad-sequence argument: that is, the classical equivalent of DC which occurs in the classical proofs of Kruskal's and related theorems. Since OI constructively implies both CCN and DCN (the negative translation of CC and DC, respectively), one knows not only that the realiser of CCN from [21] is correct but also why it works. Moreover, OI is closed under negative and A-translation; whence it proves the same Σ-formulas classically and constructively.

2.6 Proof mining

This subproject belongs to the area of applied proof theory in which proof-theoretic procedures (so called proof interpretations) are applied to concrete proofs in various parts of mathematics. The aim is to gain new information from given proofs. An example is the extraction of effective bounds from ineffective proofs, but also the extraction of qualitative information like the independence of existence statements from certain parameters coming with the problem (this implies the uniformity of the bounds). This part of proof theory, also called "Proof Mining" [166] (see also [191]) is influenced by ideas of G. Kreisel from the 1950s ([203, 189, 173]). It led to new results in number theory [172, 188] and algebra [73].

2.6.1 Proof mining in ergodic theory and 'hard' analysis

In the last 10-15 years U. Kohlenbach has developed new proof-theoretic techniques (novel forms and extensions of functional interpretation), which he systematically applied in functional analysis and hyperbolic geometry. These efforts led to numerous new effective bounds as well as uniformity results, especially in approximation theory [146, 147, 165, 204], metric fixed point theory [58, 57, 107, 150, 151, 153, 154, 160, 161, 162, 163, 178, 184], ergodic theory [9, 164] and topological dynamics [108]. For an overview see [156] and the recent book [158]. In the course of these research developments not only many new quantitative and qualitative improvements of central results such as e.g. [133, 50, 114] in fixed point theory have been obtained, but also very general metatheorems could be derived that explain these applications (and many well known results)

[3]Note that in recent years Spectors interpretation of CC and DC has been extended by Kohlenbach and others to new base types representing abstract structures such as (nonseparable) metric, hyperbolic, normed and Hilbert spaces (see the next section as well as [154, 109, 183, 158]).

as instances of general logical phenomena. For concrete Polish and compact Polish spaces this has already been done in [146, 148], where it was shown that effective bounds can be extracted that are independent of parameters in compact spaces. In fixed-point theory it surprisingly turned out that such independence results can already be obtained by assuming only the metric to be bounded instead of compactness. In Kohlenbach [154] (and Kohlenbach [155]) general metatheorems are derived which explain this prima facie empirical results for large classes of structures. Recently, these meta-results have been refined by P. Gerhardy and U. Kohlenbach in [109] in such a way that where before subspaces or convex subsets had to be assumed as being bounded one now needs only few local bounds between specific terms. In Leustean [183] these metatheorems have recently been transferred to Gromov's so called δ-hyperbolic spaces as well as $\mathbb{R}$-trees. A systematic treatment of these results can be found in the book [158]. This book also discusses the close relationship between the so-called monotone functional interpretation (the main method behind the aforementioned metatheorems) and Tao's recent program of finitising proofs in analysis (see [246]). This is particularly evident in the applications of proof mining in ergodic theory ([9, 164]) as ergodic theory is one of the main areas referred to in Tao's discussion. J. Gaspar and U. Kohlenbach ([104]) started to investigate the proof theory of various forms of Tao's 'finitary' infinite pigeonhole principle as well as formal versions of his 'correspondence principle'. Very recently, U. Kohlenbach and his group have begun to investigate the applicability of the logical machinery sketched above to proofs based on weak compactness. As a first step in this direction, P. Safarik [221] calibrated in his Diplom thesis the precise computational contribution of uses of sequential compactness in proofs of $\forall\exists$-theorems. In [159], U. Kohlenbach shows that weak compactness arguments for general (nonseparable) Hilbert spaces can be formalised in the formal theories used in the above mentioned metatheorems. As a first application this paper also treats a fixed point theorem due to Browder and outlines further research projects in this direction. Towards the use of 'proof mining' in Ramsey theory, recent work of A. Kreuzer ([175]) shows that the use of fixed sequences of instances of Ramsey's theorem for pairs at most contributes a primitive recursive growth whereas it remains as one of the central problems in the area whether the full use of Ramsey's theorem for pairs implies the totality of the Ackermann function. On the logical side new forms of logical metatheorems have recently been developed by E.M. Briseid in [59] and new fundamental research on functional interpretations as such was done by J. Gaspar in [103].

2.6.2 Proof mining for semi-classical proofs using hybrid interpretations

Although it is well-known that all Π^0_2-theorems of number theory can be proven without the use of classical logic, uses of "proof by contradiction" are ubiquitous in arithmetic (see e.g. [4]). Recent successful case studies in functional analysis, e.g. [165, 158], show that even simple uses of classical logic combined with ineffective principles such as Weak König's Lemma can be a real obstacle for the extraction of computational information from proofs.

One of the most successful case studies has been in approximation theory [165], where U. Kohlenbach and P. Oliva analysed Cheney's proof [68] of uniqueness of the L_1-approximation (for fixed $f \in C[0,1]$ and degree n, there exists a unique polynomial of degree n which best approximation f with respect to the L_1 norm). Although Cheney's proof is from 1965, the computational information obtained from the proof (a modulus of uniqueness), and the derived algorithm for computing best L_1 approximations, had been considered an open problem until recently, and only partial results had been obtained during the 70's [47, 48, 176]. The use of classical logic and ineffective principles (WKL) in Cheney's two-page proof baffled mathematicians for almost four decades.

In order to understand all functional interpretations used in the successful case studies mentioned above, in [205] P. Oliva developed a parametrised functional interpretation, where depending on the choice of two parameters one could obtain not only Gödel's Dialectica interpretation, Kohlenbach's monotone interpretations, and Kreisel's modified realizability, but also Diller-Nahm's variant [76] of the Dialectica interpretation and Stein's family of functional interpretations [245]. Although this unifying framework seemed to be satisfactory, an even better analysis has been produced recently [208, 206, 207] using *linear logic* as a refinement of intuitionistic logic [112].

The use of linear logic turned out to be much more useful than previously thought. Once in the context of linear logic, a multi-modal linear logic (see [72]) allowed for the development of a hybrid functional interpretation [125], where multiple functional interpretations could be applied simultaneously to a single proof. In this way, the strengths of each interpretation could be combined to optimally analyse each proof.

2.7 Constructive theories and their strength

2.7.1 Partiality in total type theory

Capretta has suggested an approach to partial functions using a coinductive definition and setoids [67] . Based on work by Capretta, in yet unpublished work, Altenkirch, Capretta and Uustalu have introduce the partiality monad in a type theory with quotient types. We plan to exploit Observational Type Theory (OTT) [5] which provides extensional concepts like quotient types without loosing desirable computational properties of type theory. Another key element would be the integration of continuity principles building on recent work by Berger on bar recursion in type theory [31].

2.7.2 Models of constructive theories for reasoning about continuous spaces

Constructive type theories and its categorical models have been studied from mid 1980ies onward. T. Streicher was involved in this process via various articles and a book that he wrote on semantics of type theory.

Later attention has turned to intuitionistic and/or constructive set theories like IZF and CZF. Together with S. Awodey, C. Butz and A. Simpson he has been working on the problem of relating toposes, i.e. models of higher order arithmetic, to set theory. In [10] the authors have shown how to build a model of a sufficiently weak constructive set theory BIST around every topos $\mathcal{E}$ such that the small part (the sets) of the model are equivalent to $\mathcal{E}$.

2.7.3 Constructive reverse mathematics

'Classical reverse mathematics' in the tradition of Friedman [100] and Simpson [240] uses a subsystem of second-order arithmetic as a basic formal system. Working with classical logic, one investigates systematically which additional set existence assumptions are needed to prove certain theorems. Moreover, those extra axioms should be as weak as possible; over the base system, the respective proposition should also imply the additional set existence assumptions.

'Constructive mathematics' in the sense of Bishop (BISH)[45, 56] is an informal mathematics based on intuitionistic logic. In this setting, a 'nonconstructive principle' is a proposition which is not acceptable in BISH. As a typical example we mention the 'limited principle of omniscience' (LPO), which says that for every binary sequence, either all elements are zero or else there is one element equal to one. M. Mandelkern [192] showed that in BISH the Bolzano–Weierstraß principle is equivalent to LPO. This constitutes another occurrence of reverse mathematics. He classified a theorem by showing that it is equivalent to a logical principle.

Classical reverse mathematics aims to distinguish theorems with regard to set existence axioms (or function existence axioms), on the base of classical logic. Bishop-style constructive mathematics aims to undertake a classification with regards to logical principles. The weak König lemma (WKL), which says that every infinite binary tree has an infinite branch, is a good example to illustrate the difference. In classical reverse mathematics, WKL is equivalent to a choice axiom $AC^{\vee}$, whereas in BISH it is equivalent to LLPO (which is a weakening of LPO). Note that $AC^{\vee}$ holds in BISH, whereas LLPO holds in any classical system. The aim of 'constructive reverse mathematics' is to combine the use of logical axioms and set existence axioms. A pioneering paper for the development in this field is [132], where the equivalence of WKL to the combination of LLPO and $AC^{\vee}$ is shown, on the base of an appropriate formal system.

Our major constribution was to classify uniform continuity theorems [22, 23, 24, 26, 28, 27, 25].

3 Goals and work programme (Ziele und Arbeitsprogramm)

In this section we delineate the scientific goals of each of the problem areas outlined above.

3.1 Computability and complexity over continuous data structures

3.1.1 Data structures, computability and complexity for basic differential operators

We will aim at the following problems:

1. An exact classsification of Blowup-sets of ordinary differential equations for restricted function classes in terms of the arithmetic hierarchy.

2. The complexity of Dirichlet problems for restricted kinds of boundaries.

3. Complexity issues of elliptic boundary value problems.

4. Complexity of the wave equation.

5. Faster algorithms for Riemann mappings and the related Dirichlet Problem for several classes of domains.

3.1.2 Computability and complexity on function spaces, and in higher types in general

We plan to continue the work already done mainly by Matthias Schröder concerning reasonable representations of quite general spaces and reasonable categories for computations. Here we plan to work both in the framework of Type-Two-Theory of Effectivity, see Weihrauch [256], and in the framework of domain theory. We also plan to investigate the difference between the computation model used in Information-Based Complexity [252] and in Computable Analysis [256], i.e., on the one hand an algebraic computation model with function values given via an oracle, and on the other hand the Turing machine model. We intend to look in particular for the differences caused by the fact that in the Turing machine model computations are performed bitwise while in the other model the basic operations are algebraic with infinite precision. In this context we plan to look in particular at ill-posed problems because it seems that there one may find bigger differences than for well-posed problem where precision is not the main issue. A long-term goal is the development of a reasonable and realistic (bit-)complexity theory for problems over function spaces in the Turing machine model and statements about the bit-complexity of concrete numerical problems over function spaces.

Concerning higher-type computability we will work on an extension of the results of [187]. We interested whether there is a wide class of realisability models that has a larger subcategory in common than just $\mathbb{N}$ and retracts thereof, e.g. that one can close up under the formation of various subspaces and quotients as well as exponentials, thus embracing additional spaces of interest in topology and analysis. This is a very ambitious programme. But we are hopeful that we can make some essential progress.

3.1.3 Kolmogorov complexity and entropy of dynamical systems

Already in the context of the classical randomness notion in combination with cellular automata and shift dynamical systems there are some interesting questions we wish to pursue, for example: which cellular automata of dimension greater than 1 preserve randomness? A finer tool for analysing the complexity of a dynamical system than effective randomness is given by Kolmogorov complexity. The question as to the preservation of randomness leads to the finer question of how the Kolmogorov complexity of points changes under application of a dynamical systems mapping and how it relates to the Kolmogorov complexity of a trajectory. The complexity of a dynamical system can also be measured by its topological entropy. One of the goals of this project is to narrow the gap between the known positive statements saying that under certain circumstances the topological entropy is computable and the known negative statements saying that under certain more general circumstances the entropy is not computable. Another goal is to gain a better understanding of how difficult in a computability-theoretic sense the real numbers can be that are the entropy of a cellular automaton.

3.1.4 Computability in real algebraic geometry

We want to characterize computability of the (set of, or of any single) solution(s) to a given system of real polynomial equations and inequalities. Our above preparatory works indicate that integer quantities like dimension, rank, and cardinality are promising for this purpose.

3.2 Domains and computation

3.2.1 Topological methods in semantics

Broadening domain theory. Two major schools of domain theory are currently established: one based on order theory (continuous domains, dcpos), and a more general school based on topology (topological domains, stably-compact spaces). A further generalisation to *point-free topology* (that is, *locale theory* and *formal topology*) is desirable for two reasons: (i) it will allow a constructive theory to be developed, thus building computational content into semantic definitions; and (ii) there are computational phenomena that are naturally modelled in a point-free setting, but not using topology (for example, *random sequences* [239]). A major challenge that will have to be overcome in developing a point-free domain theory is how to deal with function spaces.

Domain-theoretic functional analysis. The existing development and applications of domain-theoretic functional analysis has mainly been carried out in an order-theoretic setting by Tix, Keimel and Plotkin [250, 215, 140]. The more general (non-Hausdorff) topological theory is more complex. So far, a few core results have been announced [229, 65], with highly technical (and still unpublished) proofs. We aim to develop the mathematical tools needed to successfully establish this theory, which is central to understanding the modelling of probabilistic choice in topology-based domain theory. For example, the key lemma, used by Schröder and Simpson in establishing their universal property for the probabilistic powerdomain [229], can be recast in functional-analytic terms as asserting that certain *cones* are *reflexive*. This suggests more powerful and more general methods of proof. For example, is it possible to characterise the reflexive cones in general? One troubling feature of the theory, as developed so far, is that it is dependent upon non-constructive features of classical set-theory (e.g., the axiom of choice) and is thus not amenable to being used as part of a constructive treatment of computability. It seems likely that a point-free theory will not suffer from this limitation, and we intend to develop this.

Domain-theoretic universal algebra. We shall generalise the explicit construction of free algebras of [136, 138] to the topology-based approaches to domain theory. A further case of interest is to generalise the approach to operations whose *arity* is itself given by a domain (such operations are required to model certain effects, such as side effects with higher-type store). A special case of particular interest is the free convex space construction, which provides an algebraic approach to nondeterministic choice. We hope to prove that the stably-compact convex spaces are exactly the Eilenberg-Moore algebras for the free-convex-space monad, thereby characterising the probabilistic powerdomain, for stably compact spaces, as a free algebra.

Observation-induced effects. It is known that the algebraic characterisation of probabilistic choice does not apply to the general case of arbitrary topological domains [12]. Nevertheless, Schröder and Simpson's universal property for probabilistic powerdomains shows that probabilistic powerdomains are determined, in full generality, by a suitable choice of *observation space*, rather than by equations [229]. This idea offers a general approach to modelling computational effects, and combinations of effects, which appears to offer an alternative to the algebraic approach, and one which arguably applies more widely. We shall develop this approach for other computationally natural examples in topology-based categories of domains.

3.2.2 Modelling nondeterminism in domain theory

In the project we plan to continue the work to the combination of ordinary and probabilistic nondeterminism. We would again aim at characterising the combinations: those of lower, upper and order-convex ordinary nondeterminism with probabilistic nondeterminism in terms of suitable convex sets of probability valuations, or measures. We would also look for functional and for free-algebra characterisations. We anticipate that the mathematics we have developed for the conical case will prove a suitable basis for the harder case of convex spaces.

Beyond that there are a number of related questions. Hoare's CSP has proven a fundamental calculus for communication as has Milner's CCS. However modelling the nondeterminism involved has proven a challenge.

Hoare's CSP seems the easier of the two, so that is where we shall place our efforts; it combines two kinds of nondeterminism, thought of as *external* and *internal* choice. There are available equational axiomatisations of these constructions, which suggest working with an order-convex powerdomain for external choice and an upper powerdomain for internal choice. On the other hand the natural mathematical combination (already somewhat investigated) is the combination of a lower and an upper choice, which seems rather to correspond to a game-theoretical notion: with the opponent being modelled by the upper choice. It therefore seems important to look at such combinations, and their computational counterparts. The challenge is to proceed systematically, but efficiently.

It will be clear to the reader that there is a yet further challenge of combining all three: two kinds of nondeterminism with probabilistic nondeterminism. The opportunity may occur to look at such combinations, but we imagine here only making an initial foray, say into probabilistic (or extended probabilistic) nondeterminism with a chosen combination of the other two, say lower and upper ordinary nondeterminism.

3.2.3 Semantics and logic of probabilistic-nondeterministic computation

First we want to extend the work of [139] to partial correctness for which purpose Minkowski duality has to be adapted. We expect that this allow us to explain why in the partial correctness case predicate transformers for while loops are computed as greatest fixpoints.

Another direction is the extension to higher types. This is a problem because one does not know whether there is a natural class of continuous cpos closed under the probabilistic powerdomain. Therefore, we want to investigate whether the setting of A. Simpson's topological domain theory is more suitable for these purposes.

3.3 Exact real computation

3.3.1 Partial differential equations

The work on partial differential equations will start from analysing concretely given equations of each type, and investigating to what extent classical methods (finite elements, finite differences and fixpoint methods) can be generalised or embedded into a domain-theoretic framework. Based on these preliminary investigations, a more general theory will be developed.

3.3.2 Hybrid systems

There is a large body of literature concerning the classical treatment of probabilistic transitions in the framework of hybrid automata. Broadly speaking, these approaches fall into two categories: the first deals with computational approximations and exact decision procedures for a restricted class of systems, whereas the second allows for general probabilistic transitions, but does so far not support guaranteed approximations of the trajectory of continuous variables. We will start with so-called "piecewise continuous Markov Processes" (that fall within the second category) and embed them into the framework of domain theory, using some of the ideas that have already been developed for systems with restricted dynamics (those of the frist category).

3.3.3 Verification and efficient implementation of exact real arithmetic, with applications to dynamical and hybrid systems

The main aim of this research is to develop an efficient and validated implementation of algorithms for effective reasoning about continuous data types. As field of application we will address reachability problems of hybrid systems.

Here we want to address efficient algorithms for the approximate solving of dynamical and hybrid systems with continuous constraints, i.e. the construction of simulations and bisimulations of such hybrid systems. For the necessary theoretical background we have to continue our studies on higher-type continuous constraints based on advanced research in domain theory, computable analysis and Σ-definability.

On completion of this project we will have achieved a synthesis of our work, such that:

1. The algorithms used in the efficient `iRRAM` have been validated in the manner of those already validated in PVS;
2. We will have provided a smooth integration of our work on the `iRRAM` into a framework suitable for use in hybrid automata; and
3. If time permits, we will attempt to validate the underlying algorithms we have used from the Gnu GMP project.
4. Based on `iRRAM` algorithms we will have implemented approximate continuous constraint solving methods for the formal analysis of dynamical and hybrid systems.

3.3.4 Development of the Abstract Stone Duality calculus into a high level language for the iRRAM implementation of exact real arithmetic, in order to provide an interface with constructive analysis

We view this piece of work as a *module* that would not itself do any real arithmetic, but interact with implementations of arithmetic across a defined *interface*. Besides iRRAM, we would hope that this would also serve other systems of exact real arithmetic, making them interoperable as tools for solving problems in constructive analysis.

Starting from Bauer's prototype language implementation, on the one hand, and iRRAM on the other, the principal intellectual issue is to specify a logical *task* that would be

1. Atomic from the point of view of the mathematical language (ASD), but
2. Encapsulate the capability of the computational system (iRRAM).

This task might be to determine whether a simple function of two variables is positive, the variables being universally or existentially quantified over certain ranges. Computationally, this task might be handled by high precision computation, interval halving, the Interval Newton algorithm or other methods.

The function of our module would be to interpret a mathematical problem specified in the ASD language, resolve it into a number of tasks and manage the application of iRRAM or other systems to them.

For reasons of computational efficiency, iRRAM is implemented in C++, but Bauer's prototype is written in OCaml, as this is much more appropriate for working with formal languages in logic. We intend to continue using both languages in this way, possibly adding others such as Coq that are adapted to manipulation of proofs if parallel work succeeds in extending ASD to higher analysis. Practically, we envisage the two parts would communicate either via standard inter-language interfaces or by automatic generation of C++ from the high-level problem.

3.4 Continuous data and coalgebras

3.4.1 Coalgebraic properties of Markov transition systems

We aim at a clarification of the relationship of randomized morphism and congruences for stochastic Kripke models as generalizations of Markov transition systems, parametrized through a measure polynomial functor. These are the steps to be undertaken

1. Investigate the structure of coalgebras, in particular look into conditions under which semi-pullbacks for randomized morphisms exist. For previous results, selection properties based on the Axiom of Choice have been a valuable tool for these investigations. Given the general nature of these functors, we will have to investigate these selection properties as well. Investigate the relationship to the Giry monad.
2. The structure of congruences (both strong and randomized) will be investigated, in particular we will look into the lattice structure. The relationship of congruences and morphisms will be investigated.

3. Kripke models for related logics will be formulated and investigated. We know that weak relationships between these Kripke models exist for the case of modal logics which permit comparing the expressive power of these logics. These relationships will be extended to the more general forms based on the previous steps.

4. Investigate the consequences of the existence of a final model for the semantics of coalgebraic modal logics. It is possible to develop some form of coinductive reasoning along these lines?

3.4.2 Representation of continuous functions on final coalgebras

Some concrete aims are

1. To describe in a form accessible to programmers working with dataflow models of computation the work on streams.

2. To prove completeness of the representation of continuous functions on final coalgebras for finitary functors on **Set**; to investigate the representation of important combinators such as composition and forms of looping.

3. To connect the algebraic approach we have taken so far with the topics of continuous cut-elimination, and with cyclic proofs as studied by Simpson and Brotherston.

3.5 Program extraction

3.5.1 Induction and coinduction for program extraction

The following investigations are planned:

1. To lay the theoretical foundations for program extraction from proofs involving induction and coinduction (soundness, termination).

2. To explore the scope and the limits of program extraction from proofs.

3. To carry out extended case studies of program extraction from proofs involving coinduction in analysis and related areas.

4. To implement coinduction and program extraction, respectively extend existing implementations by coinduction.

3.5.2 Extraction of programs from classical proofs.

One of the main aims of research in the Munich logic group is formalising and fine-tuning the raw proof-theoretic methods so that they can be applied for fully automated extraction of feasible programs.

The planned project should answer to the following standing questions:

1. Which method gives better extracted programs in what cases with respect to the criteria cited above?

2. Is there a general result that relates the behaviour of the two methods?

3. Can the two methods be extended so that they produce better programs?

4. Can the two methods be combined in a way, in which we can obtain better results than applying each of the methods individually?

The last two questions already received some treatment by Berger [32] (extending A-translation), Kohlenbach [148, 158] (extending Dialectica to accommodate treatment in analysis), Hernest [124] (extending Dialectica) and Oliva [125] (combining Dialectica with the realisability method).

3.5.3 Constructive analysis of principles related to Zorn's Lemma

At the end of [29] an interesting question from [21] was renewed: How can one extract programs from classical proofs that use the full-fledged axiom of choice? More specifically, is there possibly a generalisation of open induction (OI) that is as suited for this purpose as OI is in the case of countable choice (CC) and dependent choice (DC)? We now believe that a big step towards a solution of this problem can be done by tackling it first for proofs with Zorn's lemma (ZL), the preferred form of the axiom of choice in algebra.

To this end we will first isolate a suitable principle of Zorn induction (ZI) among the classical contrapositives of ZL, which presumably new induction principle we hold for fairly promising as a counterpart of OI. In other words, we expect to achieve a satisfying answer to the question quoted above by following the strategy recalled before from [29] but with ZI and ZL in place of OI and DC, respectively. As the first case study we will consider the classical proofs of Hilbert's Nullstellensatz, and of the Positivstellensatz from real algebra.

In addition to the obvious parallels between the already successful strategy followed in [29] and our undertaking, there still is more evidence for the practicability of the latter. First, ZL has been said to be constructively neutral [19], just as DC is usually seen. Secondly, when dealing with ZI and ZL we do not expect to lose by the lack of the well-studied domains, such as the integers or finite lists thereof, as they are typical for OI and DC. On the contrary, we expect to even gain considerable profit from moving to algebra: unlike analysis, this basically is an equational theory whose objects are intrinsically finite.

The finitary character of algebra is reflected in a striking way by the form of ZL that is known as Teichmüller's and/or Tukey's lemma (TL). In the prime invocation of TL, to ensure the existence of a maximal ideal of a nontrivial ring, the principal hypothesis that the proper ideals form a set "of finite character" rests upon nothing but the characteristic property of ideals that a ring element belongs to an ideal precisely when it is a linear combination of finitely many elements of this ideal. We therefore hold an induction principle which we will first extract from TL for particularly promising as an alternative of ZI.

3.6 Proof mining

3.6.1 Proof mining in ergodic theory and 'hard' analysis

This subproject aims at enlarging the links between proof theory and functional analysis discovered so far to new fields of applications:

We will extend the range of the applications obtained so far to treat theorems in nonlinear analysis that are based on weak compactness arguments. In particular, we aim at obtaining effective quantitative versions (in the sense of Tao's notion of 'metastability') for the nonlinear ergodic theorems due to Baillon, Bruck, Reich and others.

Another topic is to further explore 'proof mining' in the context of ergodic Ramsey theory, i.e. the logical analysis of ergodic theoretic and topological proofs of combinatorial theorems such as Szemeredi's theorem and the Hale-Jewett theorem extending the line of research from [108].

In recent essays on his Blog, T. Tao discussed a program of 'finitary analysis' which by means of 'correspondence principles' develops analysis on the level of finitary versions on analytical principles. We believe that there is a close connection between this program and the kind of proof theoretic transformations (in particular the monotone functional interpretation) and Tao's approach which we intend to explore systematically. J. Gaspar recently found a counterexample to Tao's original finitisation of the infinite pigeonhole principle. Tao's corrected formulation prompted by this counterexample differs from the version obtained via functional interpretation. We intend to investigate the relationship between the two versions in terms of reverse mathematics continuing the research started in [104]. Another topic is to formulate correspondence principles that do not presuppose any compactntess. First steps in this direction have been done by U. Kohlenbach in [155] but this line of research has a much greater potential to be explored.

The metatheorems found so far are applicable in particular to the class of CAT(0) spaces. These spaces as well as their subclass of $\mathbb{R}$-trees, and the δ-hyperbolic spaces have played a central role in investigations of geometrical aspects of groups in the last 20 years, especially in the work of M. Gromov. In this field many quantitative questions are still open. It is planned to apply the proof-theoretic methods that have been used so far with much success.

In cooperation with P. Oliva (Queen Mary, U of London), we intend to further investigate proof theoretic properties of the main bound extraction techniques and, in particular, the relationship between various forms of functional and realisability interpretations.

3.6.2 Proof mining for semi-classical proofs using hybrid interpretations

The main goal is to look for applications of recently developed machinery in proof mining such as the hybrid functional interpretation in arithmetic. It should be noted that these functional interpretations are normally only applied to constructive systems (with the exception of the Dialectica interpretation, which also interprets Markov principle). Therefore, we aim to combine these different interpretations with embeddings of classical arithmetic into intuitionistic arithmetic. In doing that, we hope to gain a better understanding of why proofs in arithmetic often make use of classical logic, even when the (classical) proofs can be easily turned into constructive ones.

3.7 Constructive theories and their strength

3.7.1 Partiality in total type theory

Our aim will be a formalisation of constructive domain theory based on the partiality monad. This could be exploited in implementations of dependently typed programming languages, e.g. Agda, given access to partial functions without giving up logical consistency.

3.7.2 Models of constructive theories for reasoning about continuous spaces

In collaboration with A. Simpson, T. Streicher intends to construct models for P. Aczel's predicative set theory CZF which are genuinely predicative in the sense that they refute both the powerset axiom and the full separation scheme. They intend to construct such models within a subcategory of sheaves over the exact/regular completion of some model for computation on continuous data types like modest sets over the second Kleene algebra or Scott's graph model. Thus, those models can be expected to be particularly suitable for theories which were designed for constructive reasoning about continuous data types.

3.7.3 Constructive reverse mathematics

We will compare the various formal systems that have been used so far for carrying out constructive reverse mathematics. We will list up criteria for a good formal system like:

- A formal system should be suitable for expressing continuous structures by feasible objects; for example, the unit interval can be largely identified with the Cantor space and the elements of the latter are easy to handle formally.
- It should be possible to compare results obtained in the system with results in yet established theories, like classical reverse mathematics [240].
- The system should allow to produce negative results.

Furthermore, we will classify theorems like: Ramsey's theorem, various versions of Brouwer's fan theorem, and/or the pigeonhole principles. Finally, we will apply proof assistants like MINLOG for verifying the correctness of our equivalence results and for extracting algorithms, if applicable.

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5 Funds requested (Beantragte Mittel)

Für die Durchführung des Projekts sind geplant:

- Individuelle Forschungsaufenthalte der beteiligten deutschen Wissenschaftler an den Partnerhochschulen in Großbritannien.
- Je ein Workshop in Deutschland und Großbritannien.

Beantragt wird die Finanzierung folgender Kosten durch die DFG:

5.1 Aufenthaltskosten

	2009	2010	2011
Anzahl der Wochen	49	68	73

5.2 Reisekosten

Der Berechnung der Flugkosten liegt der in der Anlage beigefügte Kostenvoranschlag eines Reisebüros zugrunde.

	2009	2010	2011
Anzahl der Reisen	23	28	28
Flugkosten	6243,58 Euro	7600,88 Euro	7600,88 Euro

6 Voraussetzungen für die Durchführung des Projekts

Das Projekt wird von folgenden Institutionen und Wissenschaftlern getragen (Eine kurze Darstellung der Qualifikation der teilnehmenden Wissenschaftler ist in der Anlage beigefügt):

6.1 The German team (Zusammensetzung der deutschen Arbeitsgruppe)

- Prof. Dr. Wilfried Buchholz (LMU München)
- Prof. Dr. Ernst-Erich Doberkat (Dortmund)
- Prof. Dr. Peter Hertling (UniBW München)
- Prof. Dr. Klaus Keimel (Darmstadt)
- Prof. Dr. Ulrich Kohlenbach (Darmstadt)
- Prof. Dr. Helmut Schwichtenberg (LMU München)
- Prof. Dr. Dieter Spreen (Siegen)
- Prof. Dr. Thomas Streicher (Darmstadt)
- Prof. Dr. Klaus Weihrauch (Hagen)

- Priv.-Doz. Norbert Th. Müller (Trier)
- Priv.-Doz. Dr. Robert Rettinger (Hagen)
- Priv.-Doz. Dr. Kurt Sieber (Siegen)
- Priv.-Doz. Dr. Martin Ziegler (Paderborn)

- Dr. Ingo D. Battenfeld (Dortmund)
- Dr. Beno van den Berg (Darmstadt)
- Dr. Josef Berger (LMU München)
- Dr. Laurentiu Leustean (Darmstadt)
- Dr. Matthias Schröder (UniBW München)
- Dr. Christoph Schubert (Dortmund)
- Dr. Christoph Spandl (UniBW München)

- Eyvind Briseid (Darmstadt)
- Ruth Dillhage (Hagen)
- Jaime Gaspar (Darmstadt)
- Simon Huber (LMU München)
- Basil Karaidis (LMU München)
- Sven Köhler (Paderborn)
- Alexander Kreuzer (Darmstadt)
- Gisela Krommes (UniBW München)
- Diana Ratiu (LMU München)
- Pavol Safarik (Darmstadt)
- Christof Tacke (Paderborn)
- Trifon Trifonov (LMU München).

6.2 The British team (Zusammensetzung der britischen Arbeitsgruppe)

- Prof. Dr. Peter Aczel (Manchester)
- Prof. Dr. Abbas Edalat (London)
- Prof. Dr. Achim Jung (Birmingham)
- Prof. Dr. Gordon Plotkin (Edinburgh)
- Prof. Dr. John Tucker (Swansea)
- Dr. Thorsten Altenkirch (Nottingham)
- Dr. Edwin Beggs (Swansea)
- Dr. Ulrich Berger (Swansea)
- Dr. Jens Blanck (Swansea)
- Dr. Martín Escardó (Birmingham)
- Dr. Neil Ghani (Glasgow)
- Dr. Peter Hancock (Nottingham)
- Dr. Margarita Korovina (Manchester)
- Dr. Alexander Kurz (Leicester)
- Dr. David Lester (Manchester)
- Dr. John Longley (Edinburgh)
- Dr. Paulo Oliva (London)
- Dr. Dirk Pattinson (London)
- Dr. Bernhard Reus (Brighton)
- Dr. Peter Schuster (Leeds)
- Dr. Monika Seisenberger (Swansea)
- Dr. Anton Setzer (Swansea)
- Dr. Alex Simpson (Edinburgh)
- Dr. Paul Taylor (London)
- Dr. Steve Vickers (Birmingham)
- Assel Altayeva (London).

7 Declararation (Erklärung)

Ein Antrag zur Finanzierung wurde bei keiner anderen Stelle eingereicht. Wenn ich einen solchen Antrag stelle, werde ich die Deutsche Forschungsgemeinschaft unverzüglich benachrichtigen.

Der Vertrauensdozent der Deutschen Forschungsgemeinschaft an der Universität Siegen wird von diesem Antrag unterrichtet.

8 Unterschrift

Siegen, den 24. März 2009

(Prof. Dr. Dieter Spreen)

9 Enclosure (Verzeichnis der Anlagen)

1. Fragebogen für Antragsteller (Prof. Dr. Spreen)
2. Verzeichnis der relevanten Publikationen der letzten Jahre von Dr. U. Berger
3. Verzeichnis der relevanten Publikationen der letzten Jahre von Prof. Dr. D. Spreen
4. Kurze Darstellung der Qualifikation der Projektteilnehmer
5. Kostenvoranschlag eines Reisebüros