Continuous Lattices and Domains, by G. Gierz, K. H. Hofmann, K. Keimel, J. D. Lawson, M. W. Mislove and D. S. Scott, Encyclopedia of Mathemtics and its Applications 93, Cambridge University Press, 2003.

In order to give the book under review the right appreciation it is necessary to first say a few words about its predecessor, the monograph *A Compendium of Continuous Lattices* which was published by the same six authors in 1980. The *Compendium*, as it is commonly called, contains a rather complete and very readable account of the theory of continuous lattices as is was known by that time. It has been a standard reference in its field for more than two decades. Let us briefly recall the main object of study of the Compendium: A *continuous lattice* is a partially ordered set which is *complete*, i.e. every subset has a least upper bound, and *continuous*, i.e. every element is the directed supremum of elements which are *way below* it, where x is way below y ($x \ll y$) if every directed set having a least upper bound above y contains an element above x. Continuous lattices have a rich and satisfying theory with many applications in algebra, topology, analysis and other mathematical disciplines. However, in many situations the completeness axiom turns out to be too restrictive. For example, the elements of a partial order might represent bits of information all of which are consistent, individually, but which may contradict each other and hence have no common upper bound. Nevertheless, in such a situation it is still reasonable to assume that every *directed* subset has least upper bound. A partial orders satisfying the latter, weaker, completeness property is called a *directed complete partial order* (dcpo). Continuous dcpos, called *domains*, were first studied by Dana Scott in the 1970s as a mathematical foundation for the semantics of programming languages. Today, domain theory is a well-established discipline in mathematics and theoretical computer science.

In *Continuous Lattices and Domains* the six authors give domain theory the prominent role it deserves. They extend the Compendium by generalising substantial parts of the theory of continuous lattices to domains and include a wealth of new domain-theoretic results. Some of these new results are of purely mathematical character, but many of them have been motivated by new developments in theoretical computer science and have important applications there, in particular in the theory of program semantics. Let me briefly highlight some of this new material. In Chapter II continuous functions and function spaces are treated in much greater depth as in the Compendium. For example, the cartesian closed categories of FS- and bifinite domains are presented (II-2) and the Isbell topology, which generalises the compact open topology, is discussed (II-4). Chapter IV contains a full treatment of the theory of Lawson Duality of domains (IV-2) and new sections on domain equations and recursive data types (IV-7) as well as powerdomains (IV-8). Chapter V has a new section on domain environments, which is about the representation of a topological space as a subspace of a domain (V-6). Domain environments can be used, for example, to introduce a notion of computability on certain classes of spaces (this is not worked out in the book). Finally, in Chapter VI the authors included a section on stably compact spaces (VI-6), a class of T_0 spaces generalising compact Hausdorff spaces.

Despite all these extensions and additions (there are many more than those mentioned above) the style of presentation has remained the same as in the Compendium: Definitions are well motivated, proofs are clear and detailed, results are illustrated by many examples and each section concludes with plenty of exercises and notes giving historical and bibliographical background information. The “old notes” of the Compendium are kept and are supplemented by “new notes” that clearly say what is different from the Compendium and what is new. Presenting the material in such detail had the inevitable consequence that some important developments in domain theory, for example the theory of effective domains and synthetic domain theory, were left outside, simply because of the sheer volume of the material. I think that the authors decided wisely to give a detailed exposition priority over complete coverage, and it is my hope that experts in the respective fields will be inspired by this monograph to write in a similar style about the topics not covered here. We are in dire need of books that present the (matured) results of the vastly growing research areas on the borderline of mathematics and computer science in a comprehensive, consistent and detailed way. *Continuous Lattices and Domains* is such a book – it provides a reliable source of information for experts in the field, but also for researches and graduate students working on related areas who wish to consolidate their knowledge in the theory of continuous lattices and domains.

Ulrich Berger